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WASPAS Technique Utilized for Agricultural Robotics System based on Dombi Aggregation Operators under Bipolar Complex Fuzzy Soft Information

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ABSTRACT

This work aims to present some new aggregation operators (AOs) by generalizing the theory of Schweizer-Sklar (SS) aggregation tools. The picture fuzzy set (PFS) is the modified version of intuitionistic fuzzy sets and fuzzy sets. The PFS can handle such situations with three components of human opinion to mitigate the impact of insufficient information on human opinions. By utilizing concepts of prioritization, several mathematicians proposed different AOs. Some realistic operations of SS are presented under the picture fuzzy (PF) information system. We develop a class of new approaches based on picture fuzzy information, including picture fuzzy Schweizer-Sklar prioritized average (PFSSPA) and picture fuzzy Schweizer-Sklar prioritized geometric (PFSSPG) operators. Moreover, we also proposed a series of AOs with specific degrees of weights, such as picture fuzzy Schweizer-Sklar prioritized weighted average (PFSSPWA) and picture fuzzy Schweizer-Sklar prioritized weighted geometric (PFSSPWG) operators. Some realistic characteristics and special cases of currently developed approaches are also presented. To demonstrate the solution to complicated real-life problems, an algorithm for the MADM problem is established under a system of picture-fuzzy information. To check the potential of proposed aggregation approaches, we gave a numerical example to select desirable optimal options by using invented approaches. To reveal the flexibility and applicability of invented approaches, sensitive analysis, and comparative analysis are illustrated by contrasting the findings of existing approaches with currently developed approaches.

Keywords: Fuzzy set, soft set, bipolar complex fuzzy soft set, Dombi aggregation operators, and WASPAS method

1 | INTRODUCTION

By 2025, 9 billion people are expected to inhabit the planet. The nations need to consider new strategies to feed their people in light of the impending severe growth. But it's not that easy. The moment is ideal to introduce disruptive technology like artificial intelligence and robots into agriculture as people shift from rural to urban areas and no one steps forward to manage next-generation farming. Many economic sectors, including customer service, production, shipping, and transportation, have already been invaded by robotics. Fortunately, the next industry to use technology for a significant change is agriculture. But why, of all the technology, robotics? Because it can bridge the gap between the need for labor and the demands of production. As primary producers increasingly use robotics to address a variety of difficulties, the agriculture industry is witnessing a high-tech revolution. To overcome industrial obstacles, the United States, Australia, Japan, and European nations have already embraced robotics. In the upcoming years, it is predicted that the use of robotics in agriculture will increase significantly.

Numerous methods are utilized in the DM problems, including MCDM, MADM, VIKOR, and TOPSIS methods. There is another method known as the WASPAS method. WASPAS is utilized for both weighted aggregations of sum and product. The WASPAS combined both aggregations of sum and product. By the way, there are a lot of hurdles in life which is most of which are tackled but some are related to vagueness and ambiguities. Dilemmas are not tackled easily. For this purpose, researchers try to handle these dilemmas. Zadeh [1] initiated a fuzzy set (FS) which was the first foot for ignoring these dilemmas. FS consists of membership truth grade (MTG) which is lying in the closed interval [0, 1]. FS tackles most dilemmas, after that FS requires more attention. Mardani et al. [2] employed the decision-making (DM) method over fuzzy AOs. Afterward, Atanassov [3] designed intuitionistic FS (IFS) which is associated with MTG and non-membership truth grade (nMTG) and their sum belongs in the closed interval [0, 1]. IFS covers exploring more notions and tackling the limited dilemmas. Xu [4] initiated intuitionistic fuzzy AOs (IFAOs). Moreover, Yu and Xu [5] signified prioritized IFAOs. Liu and Wang [6] demonstrated some improved linguistic intuitionistic fuzzy AOs (IFAOs) and their applications to MADM. Xu and Yager [7] initiated some intuitionistic fuzzy geometric AOs (IFGAOs). Seikh and Mandal [8] dignified intuitionistic fuzzy Dombi (IFD) AOs and their applications to MADM. Gundogdu and Kahraman [9] initiated the extension of WASPAS with spherical fuzzy sets (SFS).

When IFS and interval-valued IFS are successfully engaged to overcome the ambiguities of daily life dilemmas, there is a counter property of each object that was distinguished. To stun this, Zhang [10] initiated the notion of bipolar FS (BFS), where the closed interval [0, 1] variation into a closed interval [-1, 1]. The BFS consists of a pair of positive TG (PTG) and negative TG (NTG). In the BPS, the $PTG \in [0, 1]$, $NTG \in [-1, 0]$ and $PTG + NTG \in [-1, 1]$. The BFS dignified another instrument to demonstrate the ambiguities in decision science. The BFS is smeared in numerous disciplines like medical sciences by Zhang [10], bipolar quantum logic invented computing by Zhang and Peace [11], bio-system regulation by Zhang et al. [12], computational psychiatry by Ahn et al. [13], etc. Now, here demonstrated some AOs that were employed over BFS. Jana et al. [14] signified bipolar fuzzy (BF) Dombi AOs and their applications in MADM. Wei et al. [15] elaborated on BF Hamacher AOs in MADM. Jana et al. [16] signified BF Dombi AOs in MADM. Mahmood et al. [17] invented bipolar neutrosophic Dombi AOs. The BF graph was initiated by Akram [18].

The FS is one of the most significant ideas for the ambiguities and dilemmas. FS consists of one dimensional idea but there are a lot of dilemmas which is not easily tackled by FS. For this, Ramot et al. [19] initiated the combination of FS and Complex, which is known as complex FS (CFS). It is also known as the oversimplification of FS. In the CFS the polar shape is demonstrated by the TG belonging to the unit disc in the complex plane (CP). In view of this notion, Tamir et al. [20] improved the assortment of TG from the unit disc in the CP to a unit square in the CP and signified the CFS in the Cartesian system. Tamir et al. [21] initiated CFS and complex fuzzy (CF) logic. Bi et al. [22], [23] demonstrated the CF arithmetic and geometric AOs. Mahmood et al. [24] elaborated on a complex hesitant FS (CHFS). Ur Rehman et al. [25] invented complex dual hesitant FS (CDHFS). Mahmood et al. [26] initiated the notion of CF N-soft sets. The CFS handles most of the dilemmas which are not exist by FS only, but after this, there are more dilemmas are created which consist of positive and negative portions. For this, Mahmood and Ur Rehman [27] invented the notion of bipolar CFS (BCFS) which is indicated by positive TG (PTG) $\mathcal{P}_{\psi}^{+} = \xi_{\psi}^{+} + \iota\zeta_{\psi}^{+}$ and negative TG (NTG) $\mathcal{Q}_{\psi}^{-} = \xi_{\psi}^{-} + \iota\zeta_{\psi}^{-}$ in a unit square in a CP. BCFS is the combination of BFS which have need PTG and NTG and CFS which have need TG in the 2-dimensional data. Due to these requirements BCFS is played in the significant role in the DM dilemmas. Mahmood and Ur Rehman [28] demonstrated Dombi AOs over BCFS in the MADM. Mahmood et al. [29] initiated the Hamacher AOs of BCFS in the MADM.

The FS tackled all dilemmas in the ambiguities till to was enough limit. Molodtsov [30] initiated a soft set (SS). SS also played a significant role in the vagueness and ambiguities of dilemmas. Maji et al. [31] signified SS with some operations. Ali et al. [32] initiated some novel operations in SS. Moreover, Babitha and Sunil [33] invented SS relations and their functions. The uses of SS in the mining by Herawan and Deris [34]. Mahmood [35] elaborated on the new notion of bipolar SS (BSS) with its applications. Abdullah et al. [36] initiated BSS with application in DM dilemmas. After that, Maji et al. [37] initiated the combination of FS and SS which is known as fuzzy soft set (FSS), and Roy and Maji [38] signified the FSS approach to DM dilemmas. Alcantud [39] initiated FSS-based DM. Selvachandran and Singh [40] elaborated on interval-valued CFSS (IvCFSS) and its applications. Mahmood et al. [41] initiated a new impression known as bipolar complex FSS (BCFSS) with their operations. BCFSS is the combination of BFSS which is needed for PTG and NTG, while CFSS is needed for 2-dimensional data. Mahmood et al. [42] initiated pattern recognition (PR) and medical diagnosis based on BCFSS in MADM. In this manuscript, we introduce WASPAS method for agricultural robotic systems using Dombi AOs based on BCFSS. In section 1, we defined the complete introduction. In section 2, we demonstrated the fundamental prevailing definitions and their properties. In section 3, we introduced some operations on BCFSSs. In section 4, Dombi operators on BCFSNs. In section 5, we demonstrated BCFSDWA AOs, BCFSDOWA AOs, BCFSDHA AOs, BCFSDWG AOs, BCFSDOWG AOs, and BCFSDHG AOs. In section 6, we introduced application of BCFS Dombi AOs. In section 7, we introduced the comparison of our result with prevailing work for supremacy and dominance of our work. In section 8, we initiated the conclusion of our demonstrated work.

2 | PRELIAMINARIES

This segment consists of specific prevailing basic explanations and their properties which we want to discuss.

Definition 1: [1] A FS ψ in the shape of $\psi = \{(\mathbb{t}, \mathcal{P}_{\psi}), \forall\ \mathbb{t} \in \mathbb{K}\}$ on a fixed set $\mathbb{K}$, where, $\mathcal{P}_{\psi}: \mathbb{K} \to [0,1]$ signifies the TG of every element $\mathbb{t} \in \mathbb{K}$.

Definition 2: [10] A BFS ψ over $\mathbb{K}$ is of the shape $\psi = \{(\mathbb{t}, \mathcal{P}_{\psi}^{+}, \mathcal{P}_{\psi}^{-}), \mathbb{t} \in \mathbb{K}\}$, where $\mathcal{P}_{\psi}^{+}: \mathbb{K} \to [0,1]$, $\mathcal{P}_{\psi}^{-}: \mathbb{K} \to [-1,0]$are the PTG and NTG.

Definition 3: [21] A CFS ψ in the shape of $\psi = \{(\mathbb{t}, \mathcal{P}_{\psi}), \mathbb{t} \in \mathbb{K}\} = \{\mathbb{t}, \xi_{\psi} + \iota\zeta_{\psi}, \mathbb{t} \in \mathbb{K}\}$ where $\mathcal{P}_{\psi}$ is Complex TG and ξ_{ψ}, $\zeta_{\psi} \in [0,1]$, and $\iota = \sqrt{-1}$.

Definition 4: [29] A BCFS ψ is of the shape $\psi = \{(\mathbb{t}, \mathcal{P}_{\psi}^{+}, \mathcal{Q}_{\psi}^{-}), \mathbb{t} \in \mathbb{K}\}$ where $\mathcal{P}_{\psi}^{+} = \xi_{\psi}^{+} + \iota\zeta_{\psi}^{+}$ indicate the PTG and $\mathcal{Q}_{\psi}^{-} = \xi_{\psi}^{-} + \iota\zeta_{\psi}^{-}$ indicate the NTG. The values of $\mathcal{P}_{\psi}^{+}$ and $\mathcal{Q}_{\psi}^{-}$ can be take which is lies the unit square of CP and $\xi_{\psi}^{+}, \zeta_{\psi}^{+} \in [0,1]$ and $\xi_{\psi}^{-}, \zeta_{\psi}^{-} \in [-1,0]$.

Definition 5: [32] Let $\mathbb{K}$ is a fixed set, Θ is a set of parameter, $\theta \subset \Theta$, then the couple (ψ, θ) is known as SS, where $\psi: \theta \to \psi(\mathbb{K})$, $\psi(\mathbb{K})$ is the power set of $\mathbb{K}$.

Definition 6: [29] Let $\mathbb{K}$ is a fixed set, Θ is a set of parameter, $\theta \subset \Theta$, then the couple (ψ, θ) is known as bipolar complex fuzzy soft set (BCFSS) over $\mathbb{K}$, where $\psi: \theta \to \text{BCFS}(\mathbb{K})$, $BCFS(\mathbb{K})$ is the assortment of all BCFSs of $\mathbb{K}$. It is demonstrated as

$$(\psi, \theta) = \psi(\theta_{\text{ȷ}}) = \{(\mathbb{t}_{\text{ĭ}}, \mathcal{P}_{\psi}^{+}, \mathcal{Q}_{\psi}^{-}) |\ \forall \mathbb{t}_{\text{ĭ}} \in \mathbb{K}, \forall \theta_{\text{ȷ}} \in \Theta\}$$

$$= \{(\mathbb{t}_{\text{ĭ}}, \xi_{\psi}^{+} + \iota\zeta_{\psi}^{+}, \xi_{\psi}^{-} + \iota\zeta_{\psi}^{-}) |\ \forall \mathbb{t}_{\text{ĭ}} \in \mathbb{K}, \forall \theta_{\text{ȷ}} \in \Theta\} \quad (1)$$

For easiness in this article, we employ $(\psi, \theta) = \psi_{\theta_{\text{ĭȷ}}} = (\mathcal{P}_{\text{ĭȷ}}^{+}, \mathcal{Q}_{\text{ĭȷ}}^{-}) = (\xi_{\text{ĭȷ}}^{+} + \iota\zeta_{\text{ĭȷ}}^{+}, \xi_{\text{ĭȷ}}^{-} + \iota\zeta_{\text{ĭȷ}}^{-})(\text{ĭ} = 1,2,3,\dots,\mathfrak{m}; \text{ȷ} = 1,2,3,\dots,\mathfrak{n})$ as BCFSS. Here we have $\eta = \eta_1, \eta_2, \dots, \eta_{\mathfrak{m}}$ be the weight vector (WV) of specialists $\mathbb{t}_{\text{ĭ}}$ and $\lambda = \lambda_1, \lambda_2, \dots, \lambda_{\mathfrak{n}}$ be the WV of attributes $\theta_{\text{ȷ}}$ holding $\eta_{\text{ĭ}} \geq 0, \lambda_{\text{ȷ}} \geq 0$ such that $\sum_{\text{ĭ}=1}^{\mathfrak{m}} \eta_{\text{ĭ}} = 1$ and $\sum_{\text{ȷ}=1}^{\mathfrak{n}} \lambda_{\text{ȷ}} = 1$.

3 | BASIC OPERATIONS ON BCFSSS

In this Section, we will signify the Dos in Sect 4.1, and in Sect 4.2 we will demonstrate DOs on BCFSNs.

4.1| DOMBI Operation

Dombi understood operations Dombi product and sum which are particular cases of t-norms and t-conorms provided underneath.

Definition 11: Let $\mathbb{t}_1$, $\mathbb{t}_2$ are double real figures, formerly the Dombi t-norms and t-conorms are specified as trials.

$$Dom(\mathbb{t}_1, \mathbb{t}_2) = \frac{1}{1 + \left\{\left(\frac{1-\mathbb{t}_1}{\mathbb{t}_1}\right)^{\mathrm{t}} + \left(\frac{1-\mathbb{t}_2}{\mathbb{t}_2}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}} \quad (4)$$

$$Dom^{*}(\mathbb{t}_1, \mathbb{t}_2) = 1 - \frac{1}{1 + \left\{\left(\frac{\mathbb{t}_1}{1-\mathbb{t}_1}\right)^{\mathrm{t}} + \left(\frac{\mathbb{t}_2}{1-\mathbb{t}_2}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}} \quad (5)$$

Where $\mathrm{t} \geq 1$ and $(\mathbb{t}_1, \mathbb{t}_2) \in [0,1] \times [0,1]$.

4.2| DOMBI Operations on BCFSNs

In the sub-segment, we will inaugurate the notion of Dombi operation on BCFSNs.

Definition 12: Let $\psi_{\theta_{11}} = (\xi_{11}^{+} + \iota\zeta_{11}^{+}, \xi_{11}^{-} + \iota\zeta_{11}^{-})$ and $\psi_{\theta_{12}} = (\xi_{12}^{+} + \iota\zeta_{12}^{+}, \xi_{12}^{-} + \iota\zeta_{12}^{-})$ are two BCFSSs, then

1. $$\psi_{\theta_{11}} \oplus \psi_{\theta_{12}} = \left(\begin{array}{c} 1 - \dfrac{1}{1+\left\{\left(\frac{\xi_{11}^{+}}{1-\xi_{11}^{+}}\right)^{\mathrm{t}} + \left(\frac{\xi_{12}^{+}}{1-\xi_{12}^{+}}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}} + \iota\left(1 - \dfrac{1}{1+\left\{\left(\frac{\zeta_{11}^{+}}{1-\zeta_{11}^{+}}\right)^{\mathrm{t}} + \left(\frac{\zeta_{12}^{+}}{1-\zeta_{12}^{+}}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}}\right), \\ \dfrac{-1}{1+\left\{\left(\frac{1+\xi_{11}^{-}}{|\xi_{11}^{-}|}\right)^{\mathrm{t}} + \left(\frac{1+\xi_{12}^{-}}{|\xi_{12}^{-}|}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}} + \iota\left(\dfrac{-1}{1+\left\{\left(\frac{1+\zeta_{11}^{-}}{|\zeta_{11}^{-}|}\right)^{\mathrm{t}} + \left(\frac{1+\zeta_{12}^{-}}{|\zeta_{12}^{-}|}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}}\right) \end{array} \right)$$

2. $$\psi_{\theta_{11}} \otimes \psi_{\theta_{12}} = \left(\begin{array}{c} \dfrac{1}{1+\left\{\left(\frac{1-\xi_{11}^{+}}{\xi_{11}^{+}}\right)^{\mathrm{t}} + \left(\frac{1-\xi_{12}^{+}}{\xi_{12}^{+}}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}} + \iota\left(\dfrac{1}{1+\left\{\left(\frac{1-\zeta_{11}^{+}}{\zeta_{11}^{+}}\right)^{\mathrm{t}} + \left(\frac{1-\zeta_{12}^{+}}{\zeta_{12}^{+}}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}}\right), \\ -1 + \dfrac{1}{1+\left\{\left(\frac{|\xi_{11}^{-}|}{1+\xi_{11}^{-}}\right)^{\mathrm{t}} + \left(\frac{|\xi_{11}^{-}|}{1+\xi_{11}^{-}}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}} + \iota\left(-1 + \dfrac{1}{1+\left\{\left(\frac{|\zeta_{11}^{-}|}{1+\zeta_{11}^{-}}\right)^{\mathrm{t}} + \left(\frac{|\zeta_{12}^{-}|}{1+\zeta_{12}^{-}}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}}\right) \end{array} \right)$$

3. $$\mu\psi_{\theta} = \left(\begin{array}{c} 1 - \dfrac{1}{1+\left\{\mu\left(\frac{\xi^{+}}{1-\xi^{+}}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}} + \iota\left(1 - \dfrac{1}{1+\left\{\mu\left(\frac{\zeta^{+}}{1-\zeta^{+}}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}}\right), \\ \dfrac{-1}{1+\left\{\mu\left(\frac{1+\xi^{-}}{|\xi^{-}|}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}} + \iota\left(\dfrac{-1}{1+\left\{\mu\left(\frac{1+\zeta^{-}}{|\zeta^{-}|}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}}\right) \end{array} \right)$$

4. $$\psi_{\theta}{}^{\mu} = \left(\begin{array}{c} \dfrac{1}{1+\left\{\mu\left(\frac{1-\xi^{+}}{\xi^{+}}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}} + \iota\left(\dfrac{1}{1+\left\{\mu\left(\frac{1-\zeta^{+}}{\zeta^{+}}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}}\right), \\ -1 + \dfrac{1}{1+\left\{\mu\left(\frac{|\xi^{-}|}{1+\xi^{-}}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}} + \iota\left(-1 + \dfrac{1}{1+\left\{\mu\left(\frac{|\zeta^{-}|}{1+\zeta^{-}}\right)^{\mathrm{t}}\right\}^{\frac{1}{\mathrm{t}}}}\right) \end{array} \right)$$

5 | BIPOLAR COMPLEX FUZZY SOFT DOMBI AGGREGATION OPERATORS

In this segment, we will design DAAOs and DGAOs with BCFSNs.

5.1|Bipolar Complex Fuzzy Soft DOBMI Arithmetic Aggregation Operators

This segment of the article consists of BCFSDWA operator, BCFSDOWA operator, and BCFSDHA operator.

Definition 13: Let $\psi_{\theta_{ij}} = \left(\mathcal{P}^{+}_{ij}, \mathcal{Q}^{-}_{ij}\right) = \left(\xi^{+}_{ij} + \iota\zeta^{+}_{ij}, \xi^{-}_{ij} + \iota\zeta^{-}_{ij}\right)(i = 1,2,\dots,m: j = 1,2,\dots,n)$ is the assortment of BCFSSs, then the BCFSDWA operator is a function $\psi^{n}_{\theta} \to \psi_{\theta}$ such that

$$BCFSDWA_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \psi_{13}, \dots, \psi_{\theta_{mn}}\right) = \oplus^{n}_{j=1}\lambda_{j}\left(\oplus^{m}_{i=1}\eta_{i}\psi_{\theta_{ij}}\right) \quad (6)$$

Where $\eta = \eta_1, \eta_2, \dots, \eta_m$ be the WV of specialists t_i and $\lambda = \lambda_1, \lambda_2, \dots, \lambda_n$ be the WV of attributes θ_j, holding $\eta_i \geq 0, \lambda_j \geq 0$ such that $\sum^{m}_{i=1}\eta_i = 1$ and $\sum^{n}_{j=1}\lambda_j = 1$.

Theorem 2: Let $\psi_{\theta_{ij}} = \left(\mathcal{P}^{+}_{ij}, \mathcal{Q}^{-}_{ij}\right) = \left(\xi^{+}_{ij} + \iota\zeta^{+}_{ij}, \xi^{-}_{ij} + \iota\zeta^{-}_{ij}\right)(i = 1,2,\dots,m: j = 1,2,\dots,n)$ is the assortment of BCFSSs, then by using the BCFSDWA operator their calculated is again a BCFSS and

$$BCFSDWA_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \psi_{13}, \dots, \psi_{\theta_{mn}}\right) = \oplus^{n}_{j=1}\lambda_{j}\left(\oplus^{m}_{i=1}\eta_{i}\psi_{\theta_{ij}}\right)$$

$$= \left(\begin{array}{c} 1 - \dfrac{1}{1 + \left\{\sum^{n}_{j=1}\lambda_j\left(\sum^{m}_{i=1}\eta_i\left(\dfrac{\xi^{+}_{ij}}{1-\xi^{+}_{ij}}\right)\right)^{t}\right\}^{\frac{1}{t}}} + \iota\left(1 - \dfrac{1}{1 + \left\{\sum^{n}_{j=1}\lambda_j\left(\sum^{m}_{i=1}\eta_i\left(\dfrac{\zeta^{+}_{ij}}{1-\zeta^{+}_{ij}}\right)\right)^{t}\right\}^{\frac{1}{t}}}\right), \\ \dfrac{-1}{1 + \left\{\sum^{n}_{j=1}\lambda_j\left(\sum^{m}_{i=1}\eta_i\left(\dfrac{1+\xi^{-}_{ij}}{\left|\xi^{-}_{ij}\right|}\right)\right)^{t}\right\}^{\frac{1}{t}}} + \iota\left(\dfrac{-1}{1 + \left\{\sum^{n}_{j=1}\lambda_j\left(\sum^{m}_{i=1}\eta_i\left(\dfrac{1+\zeta^{-}_{ij}}{\left|\zeta^{-}_{ij}\right|}\right)\right)^{t}\right\}^{\frac{1}{t}}}\right) \end{array}\right)$$

Where $\eta = \eta_1, \eta_2, \dots, \eta_m$ be the WV of specialists t_i and $\lambda = \lambda_1, \lambda_2, \dots, \lambda_n$ be the WV of attributes θ_j, holding $\eta_i \geq 0, \lambda_j \geq 0$ such that $\sum^{m}_{i=1}\eta_i = 1$ and $\sum^{n}_{j=1}\lambda_j = 1$.

Proof: Now, the mathematical induction method is used, as trials:

$$\psi_{\theta_{11}} \oplus \psi_{\theta_{12}} = \left(\begin{array}{c} 1 - \dfrac{1}{1 + \left\{\left(\dfrac{\xi^{+}_{11}}{1-\xi^{+}_{11}}\right)^{t} + \left(\dfrac{\xi^{+}_{12}}{1-\xi^{+}_{12}}\right)^{t}\right\}^{\frac{1}{t}}} + \iota\left(1 - \dfrac{1}{1 + \left\{\left(\dfrac{\zeta^{+}_{11}}{1-\zeta^{+}_{11}}\right)^{t} + \left(\dfrac{\zeta^{+}_{12}}{1-\zeta^{+}_{12}}\right)^{t}\right\}^{\frac{1}{t}}}\right), \\ \dfrac{-1}{1 + \left\{\left(\dfrac{1+\xi^{-}_{11}}{\left|\xi^{-}_{11}\right|}\right)^{t} + \left(\dfrac{1+\xi^{-}_{12}}{\left|\xi^{-}_{12}\right|}\right)^{t}\right\}^{\frac{1}{t}}} + \iota\left(\dfrac{-1}{1 + \left\{\left(\dfrac{1+\xi^{-}_{11}}{\left|\xi^{-}_{11}\right|}\right)^{t} + \left(\dfrac{1+\xi^{-}_{12}}{\left|\xi^{-}_{12}\right|}\right)^{t}\right\}^{\frac{1}{t}}}\right) \end{array}\right)$$

First, we prove that the outcome accessible for $m = 2$ and $n = 2$, as trials:

$$BCFSDWA_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}\right) = \oplus^{2}_{j=1}\lambda_j\left(\oplus^{2}_{i=1}\eta_i\psi_{\theta_{ij}}\right) = \lambda_1\left(\eta_1\psi_{\theta_{11}} \oplus \eta_2\psi_{\theta_{21}}\right) \oplus \lambda_2\left(\eta_1\psi_{\theta_{12}} \oplus \eta_2\psi_{\theta_{22}}\right)$$

$$
= \lambda_1 \begin{pmatrix} 1 - \dfrac{1}{1 + \left\{ \eta_1 \left(\dfrac{\xi_{11}^{+}}{1 - \xi_{11}^{+}} \right)^{t} + \eta_2 \left(\dfrac{\xi_{21}^{+}}{1 - \xi_{21}^{+}} \right)^{t} \right\}^{\frac{1}{t}}} + \iota \left(1 - \dfrac{1}{1 + \left\{ \eta_1 \left(\dfrac{\zeta_{11}^{+}}{1 - \zeta_{11}^{+}} \right)^{t} + \eta_2 \left(\dfrac{\zeta_{21}^{+}}{1 - \zeta_{21}^{+}} \right)^{t} \right\}^{\frac{1}{t}}} \right), \\ \dfrac{-1}{1 + \left\{ \eta_1 \left(\dfrac{1 + \xi_{11}^{-}}{|\xi_{11}^{-}|} \right)^{t} + \eta_2 \left(\dfrac{1 + \xi_{21}^{-}}{|\xi_{21}^{-}|} \right)^{t} \right\}^{\frac{1}{t}}} + \iota \left(\dfrac{-1}{1 + \left\{ \eta_1 \left(\dfrac{1 + \xi_{11}^{-}}{|\xi_{11}^{-}|} \right)^{t} + \eta_2 \left(\dfrac{1 + \xi_{21}^{-}}{|\xi_{21}^{-}|} \right)^{t} \right\}^{\frac{1}{t}}} \right) \end{pmatrix}
$$

$$
\oplus \lambda_2 \begin{pmatrix} 1 - \dfrac{1}{1 + \left\{ \eta_1 \left(\dfrac{\xi_{12}^{+}}{1 - \xi_{12}^{+}} \right)^{t} + \eta_2 \left(\dfrac{\xi_{22}^{+}}{1 - \xi_{22}^{+}} \right)^{t} \right\}^{\frac{1}{t}}} + \iota \left(1 - \dfrac{1}{1 + \left\{ \eta_1 \left(\dfrac{\zeta_{12}^{+}}{1 - \zeta_{12}^{+}} \right)^{t} + \eta_2 \left(\dfrac{\zeta_{22}^{+}}{1 - \zeta_{22}^{+}} \right)^{t} \right\}^{\frac{1}{t}}} \right), \\ \dfrac{-1}{1 + \left\{ \eta_1 \left(\dfrac{1 + \xi_{12}^{-}}{|\xi_{12}^{-}|} \right)^{t} + \eta_2 \left(\dfrac{1 + \xi_{22}^{-}}{|\xi_{22}^{-}|} \right)^{t} \right\}^{\frac{1}{t}}} + \iota \left(\dfrac{-1}{1 + \left\{ \eta_1 \left(\dfrac{1 + \zeta_{12}^{-}}{|\zeta_{12}^{-}|} \right)^{t} + \eta_2 \left(\dfrac{1 + \zeta_{22}^{-}}{|\zeta_{22}^{-}|} \right)^{t} \right\}^{\frac{1}{t}}} \right) \end{pmatrix}
$$

$$
= \lambda_1 \begin{pmatrix} 1 - \dfrac{1}{1 + \left\{ \sum_{\breve{i}=1}^{2} \eta_{\breve{i}} \left(\dfrac{\xi_{\breve{i}1}^{+}}{1 - \xi_{\breve{i}1}^{+}} \right)^{t} \right\}^{\frac{1}{t}}} + \iota \left(1 - \dfrac{1}{1 + \left\{ \sum_{\breve{i}=1}^{2} \eta_{\breve{i}} \left(\dfrac{\zeta_{\breve{i}1}^{+}}{1 - \zeta_{\breve{i}1}^{+}} \right)^{t} \right\}^{\frac{1}{t}}} \right), \\ \dfrac{-1}{1 + \left\{ \sum_{\breve{i}=1}^{2} \eta_{\breve{i}} \left(\dfrac{1 + \xi_{\breve{i}1}^{-}}{|\xi_{\breve{i}1}^{-}|} \right)^{t} \right\}^{\frac{1}{t}}} + \iota \left(\dfrac{-1}{1 + \left\{ \sum_{\breve{i}=1}^{2} \eta_{\breve{i}} \left(\dfrac{1 + \zeta_{\breve{i}1}^{-}}{|\zeta_{\breve{i}1}^{-}|} \right)^{t} \right\}^{\frac{1}{t}}} \right) \end{pmatrix}
$$

$$
\oplus \lambda_2 \begin{pmatrix} 1 - \dfrac{1}{1 + \left\{ \sum_{\breve{i}=1}^{2} \eta_{\breve{i}} \left(\dfrac{\xi_{\breve{i}2}^{+}}{1 - \xi_{\breve{i}2}^{+}} \right)^{t} \right\}^{\frac{1}{t}}} + \iota \left(1 - \dfrac{1}{1 + \left\{ \sum_{\breve{i}=1}^{2} \eta_{\breve{i}} \left(\dfrac{\zeta_{\breve{i}2}^{+}}{1 - \zeta_{\breve{i}2}^{+}} \right)^{t} \right\}^{\frac{1}{t}}} \right), \\ \dfrac{-1}{1 + \left\{ \sum_{\breve{i}=1}^{2} \eta_{\breve{i}} \left(\dfrac{1 + \xi_{\breve{i}2}^{-}}{|\xi_{\breve{i}2}^{-}|} \right)^{t} \right\}^{\frac{1}{t}}} + \iota \left(\dfrac{-1}{1 + \left\{ \sum_{\breve{i}=1}^{2} \eta_{\breve{i}} \left(\dfrac{1 + \zeta_{\breve{i}2}^{-}}{|\zeta_{\breve{i}2}^{-}|} \right)^{t} \right\}^{\frac{1}{t}}} \right) \end{pmatrix}
$$

$$= \begin{pmatrix} 1 - \dfrac{1}{1+\left\{\lambda_1\left(\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{\xi^+_{\breve{\imath}1}}{1-\xi^+_{\breve{\imath}1}}\right)^{t}\right)+\lambda_2\left(\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{\xi^+_{\breve{\imath}2}}{1-\xi^+_{\breve{\imath}2}}\right)^{t}\right)\right\}^{\frac{1}{t}}} + \\ \iota\left(1-\dfrac{1}{1+\left\{\lambda_1\left(\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{\zeta^+_{\breve{\imath}1}}{1-\zeta^+_{\breve{\imath}1}}\right)^{t}\right)+\lambda_2\left(\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{\zeta^+_{\breve{\imath}2}}{1-\zeta^+_{\breve{\imath}2}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right), \\ \dfrac{-1}{1+\left\{\lambda_1\left(\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{1+\xi^-_{\breve{\imath}1}}{\left|\xi^-_{\breve{\imath}1}\right|}\right)^{t}\right)+\lambda_2\left(\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{1+\xi^-_{\breve{\imath}2}}{\left|\xi^-_{\breve{\imath}2}\right|}\right)^{t}\right)\right\}^{\frac{1}{t}}} + \\ \iota\left(\dfrac{-1}{1+\left\{\lambda_1\left(\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{1+\zeta^-_{\breve{\imath}1}}{\left|\zeta^-_{\breve{\imath}1}\right|}\right)^{t}\right)+\lambda_2\left(\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{1+\zeta^-_{\breve{\imath}2}}{\left|\zeta^-_{\breve{\imath}2}\right|}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \dfrac{1}{1+\left\{\sum_{\breve{\jmath}=1}^{2}\lambda_{\breve{\jmath}}\left(\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{\xi^+_{\breve{\imath}\breve{\jmath}}}{1-\xi^+_{\breve{\imath}\breve{\jmath}}}\right)^{t}\right)\right\}^{\frac{1}{t}}} + \iota\left(1-\dfrac{1}{1+\left\{\sum_{\breve{\jmath}=1}^{2}\lambda_{\breve{\jmath}}\left(\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{\zeta^+_{\breve{\imath}\breve{\jmath}}}{1-\zeta^+_{\breve{\imath}\breve{\jmath}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right), \\ \dfrac{-1}{1+\left\{\sum_{\breve{\jmath}=1}^{2}\lambda_{\breve{\jmath}}\left(\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{1+\xi^-_{\breve{\imath}\breve{\jmath}}}{\left|\xi^-_{\breve{\imath}\breve{\jmath}}\right|}\right)^{t}\right)\right\}^{\frac{1}{t}}} + \iota\left(\dfrac{-1}{1+\left\{\sum_{\breve{\jmath}=1}^{2}\lambda_{\breve{\jmath}}\left(\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{1+\zeta^-_{\breve{\imath}\breve{\jmath}}}{\left|\zeta^-_{\breve{\imath}\breve{\jmath}}\right|}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right) \end{pmatrix}$$

Therefore, this outcome is accurate for $m = 2$ and $n = 2$.

Now, we reflect this outcome is valid for $m = \kappa_1$ and $n = \kappa_2$, as trials:

$$= \begin{pmatrix} 1 - \dfrac{1}{1+\left\{\sum_{\breve{\jmath}=1}^{\kappa_2}\lambda_{\breve{\jmath}}\left(\sum_{\breve{\imath}=1}^{\kappa_1}\eta_{\breve{\imath}}\left(\frac{\xi^+_{\breve{\imath}\breve{\jmath}}}{1-\xi^+_{\breve{\imath}\breve{\jmath}}}\right)^{t}\right)\right\}^{\frac{1}{t}}} + \iota\left(1-\dfrac{1}{1+\left\{\sum_{\breve{\jmath}=1}^{\kappa_2}\lambda_{\breve{\jmath}}\left(\sum_{\breve{\imath}=1}^{\kappa_1}\eta_{\breve{\imath}}\left(\frac{\zeta^+_{\breve{\imath}\breve{\jmath}}}{1-\zeta^+_{\breve{\imath}\breve{\jmath}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right), \\ \dfrac{-1}{1+\left\{\sum_{\breve{\jmath}=1}^{\kappa_2}\lambda_{\breve{\jmath}}\left(\sum_{\breve{\imath}=1}^{\kappa_1}\eta_{\breve{\imath}}\left(\frac{1+\xi^-_{\breve{\imath}\breve{\jmath}}}{\left|\xi^-_{\breve{\imath}\breve{\jmath}}\right|}\right)^{t}\right)\right\}^{\frac{1}{t}}} + \iota\left(\dfrac{-1}{1+\left\{\sum_{\breve{\jmath}=1}^{\kappa_2}\lambda_{\breve{\jmath}}\left(\sum_{\breve{\imath}=1}^{\kappa_1}\eta_{\breve{\imath}}\left(\frac{1+\zeta^-_{\breve{\imath}\breve{\jmath}}}{\left|\zeta^-_{\breve{\imath}\breve{\jmath}}\right|}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right) \end{pmatrix}$$

Next, we reflect this outcome is valid for $m = \kappa_1 + 1$ and $n = \kappa_2 + 1$, as trials:

$$BCFSDWA_{\lambda}\left(\psi_{\theta_{11}},\psi_{\theta_{12}},\psi_{13},\ldots,\psi_{\theta_{\kappa_1\kappa_2}},\psi_{\theta_{(\kappa_1+1)(\kappa_2+1)}}\right)=\oplus_{\check{j}=1}^{\kappa_2}\lambda_{\check{j}}\left(\oplus_{\check{i}=1}^{\kappa_1}\eta_{\check{i}}\psi_{\theta_{\check{i}\check{j}}}\right)\oplus\left(\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\psi_{\theta_{(\kappa_1+1)(\kappa_2+1)}}\right)\right)$$

$$=\begin{pmatrix} 1-\dfrac{1}{1+\left\{\sum_{\check{j}=1}^{\kappa_2}\lambda_{\check{j}}\left(\sum_{\check{i}=1}^{\kappa_1}\eta_{\check{i}}\left(\dfrac{\xi_{\check{i}\check{j}}^{+}}{1-\xi_{\check{i}\check{j}}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\iota\left(1-\dfrac{1}{1+\left\{\sum_{\check{j}=1}^{\kappa_2}\lambda_{\check{j}}\left(\sum_{\check{i}=1}^{\kappa_1}\eta_{\check{i}}\left(\dfrac{\zeta_{\check{i}\check{j}}^{+}}{1-\zeta_{\check{i}\check{j}}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right), \\ \dfrac{-1}{1+\left\{\sum_{\check{j}=1}^{\kappa_2}\lambda_{\check{j}}\left(\sum_{\check{i}=1}^{\kappa_1}\eta_{\check{i}}\left(\dfrac{1+\xi_{\check{i}\check{j}}^{-}}{\left|\xi_{\check{i}\check{j}}^{-}\right|}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\iota\left(\dfrac{-1}{1+\left\{\sum_{\check{j}=1}^{\kappa_2}\lambda_{\check{j}}\left(\sum_{\check{i}=1}^{\kappa_1}\eta_{\check{i}}\left(\dfrac{1+\zeta_{\check{i}\check{j}}^{-}}{\left|\zeta_{\check{i}\check{j}}^{-}\right|}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right) \end{pmatrix}$$

$$\oplus\begin{pmatrix} 1-\dfrac{1}{1+\left\{\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\dfrac{\xi_{\check{i}\check{j}}^{+}}{1-\xi_{\check{i}\check{j}}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\iota\left(1-\dfrac{1}{1+\left\{\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\dfrac{\zeta_{\check{i}\check{j}}^{+}}{1-\zeta_{\check{i}\check{j}}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right), \\ \dfrac{-1}{1+\left\{\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\dfrac{1+\xi_{\check{i}\check{j}}^{-}}{\left|\xi_{\check{i}\check{j}}^{-}\right|}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\iota\left(\dfrac{-1}{1+\left\{\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\dfrac{1+\zeta_{\check{i}\check{j}}^{-}}{\left|\zeta_{\check{i}\check{j}}^{-}\right|}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right) \end{pmatrix}$$

$$=\begin{pmatrix} 1-\dfrac{1}{1+\left\{\sum_{\check{j}=1}^{\kappa_2}\lambda_{\check{j}}\left(\sum_{\check{i}=1}^{\kappa_1}\eta_{\check{i}}\left(\dfrac{\xi_{\check{i}\check{j}}^{+}}{1-\xi_{\check{i}\check{j}}^{+}}\right)^{t}\right)+\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\dfrac{\xi_{\check{i}\check{j}}^{+}}{1-\xi_{\check{i}\check{j}}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+ \\ \iota\left(1-\dfrac{1}{1+\left\{\sum_{\check{j}=1}^{\kappa_2}\lambda_{\check{j}}\left(\sum_{\check{i}=1}^{\kappa_1}\eta_{\check{i}}\left(\dfrac{\zeta_{\check{i}\check{j}}^{+}}{1-\zeta_{\check{i}\check{j}}^{+}}\right)^{t}\right)+\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\dfrac{\zeta_{\check{i}\check{j}}^{+}}{1-\zeta_{\check{i}\check{j}}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right), \\ \dfrac{-1}{1+\left\{\sum_{\check{j}=1}^{\kappa_2}\lambda_{\check{j}}\left(\sum_{\check{i}=1}^{\kappa_1}\eta_{\check{i}}\left(\dfrac{1+\xi_{\check{i}\check{j}}^{-}}{\left|\xi_{\check{i}\check{j}}^{-}\right|}\right)^{t}\right)+\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\dfrac{1+\xi_{\check{i}\check{j}}^{-}}{\left|\xi_{\check{i}\check{j}}^{-}\right|}\right)^{t}\right)\right\}^{\frac{1}{t}}}+ \\ \iota\left(\dfrac{-1}{1+\left\{\sum_{\check{j}=1}^{\kappa_2}\lambda_{\check{j}}\left(\sum_{\check{i}=1}^{\kappa_1}\eta_{\check{i}}\left(\dfrac{1+\zeta_{\check{i}\check{j}}^{-}}{\left|\zeta_{\check{i}\check{j}}^{-}\right|}\right)^{t}\right)+\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\dfrac{1+\zeta_{\check{i}\check{j}}^{-}}{\left|\zeta_{\check{i}\check{j}}^{-}\right|}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right) \end{pmatrix}$$

$$
= \left(\begin{array}{c}
1 - \dfrac{1}{1 + \left\{ \sum_{\breve{\jmath}=1}^{\kappa_2+1} \lambda_{\breve{\jmath}} \left(\sum_{\breve{\imath}=1}^{\kappa_1+1} \eta_{\breve{\imath}} \left(\dfrac{\xi^{+}_{\breve{\imath}\breve{\jmath}}}{1-\xi^{+}_{\breve{\imath}\breve{\jmath}}} \right)^{t} \right) \right\}^{\frac{1}{t}}} + \iota \left(1 - \dfrac{1}{1 + \left\{ \sum_{\breve{\jmath}=1}^{\kappa_2+1} \lambda_{\breve{\jmath}} \left(\sum_{\breve{\imath}=1}^{\kappa_1+1} \eta_{\breve{\imath}} \left(\dfrac{\zeta^{+}_{\breve{\imath}\breve{\jmath}}}{1-\zeta^{+}_{\breve{\imath}\breve{\jmath}}} \right)^{t} \right) \right\}^{\frac{1}{t}}} \right), \\
\dfrac{-1}{1 + \left\{ \sum_{\breve{\jmath}=1}^{\kappa_2+1} \lambda_{\breve{\jmath}} \left(\sum_{\breve{\imath}=1}^{\kappa_1+1} \eta_{\breve{\imath}} \left(\dfrac{1+\xi^{-}_{\breve{\imath}\breve{\jmath}}}{|\xi^{-}_{\breve{\imath}\breve{\jmath}}|} \right)^{t} \right) \right\}^{\frac{1}{t}}} + \iota \left(\dfrac{-1}{1 + \left\{ \sum_{\breve{\jmath}=1}^{\kappa_2+1} \lambda_{\breve{\jmath}} \left(\sum_{\breve{\imath}=1}^{\kappa_1+1} \eta_{\breve{\imath}} \left(\dfrac{1+\zeta^{-}_{\breve{\imath}\breve{\jmath}}}{|\zeta^{-}_{\breve{\imath}\breve{\jmath}}|} \right)^{t} \right) \right\}^{\frac{1}{t}}} \right)
\end{array} \right)
$$

Therefore, the outcome is valid for $m = \kappa_1 + 1$ and $n = \kappa_2 + 1$. Hence, the outcome is accurate $m, n \geq 1$. It is obvious after overhead appearance the aggregated outcomes recognized BCFSDWA operator is again BCFSDWA operator.

Table 1. Bipolar Complex Fuzzy Soft Set Data

$\boldsymbol{\psi_{\theta_{\breve{\imath}\breve{\jmath}}}}$	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$
$\mathfrak{t}_1$	$\begin{pmatrix} 0.7 + \iota 0.8, \\ -0.49 - \iota 0.71 \end{pmatrix}$	$\begin{pmatrix} 0.7 + \iota 0.7, \\ -0.62 - \iota 0.49 \end{pmatrix}$	$\begin{pmatrix} 0.8 + \iota 0.7, \\ -0.71 - \iota 0.62 \end{pmatrix}$
$\mathfrak{t}_2$	$\begin{pmatrix} 0.8 + \iota 0.7, \\ -0.82 - \iota 0.89 \end{pmatrix}$	$\begin{pmatrix} 0.6 + \iota 0.5, \\ -0.72 - \iota 0.82 \end{pmatrix}$	$\begin{pmatrix} 0.8 + \iota 0.8, \\ -0.89 - \iota 0.72 \end{pmatrix}$
$\mathfrak{t}_3$	$\begin{pmatrix} 0.7 + \iota 0.6, \\ -0.57 - \iota 0.88 \end{pmatrix}$	$\begin{pmatrix} 0.7 + \iota 0.6, \\ -0.64 - \iota 0.57 \end{pmatrix}$	$\begin{pmatrix} 0.7 + \iota 0.7, \\ -0.88 - \iota 0.64 \end{pmatrix}$
$\mathfrak{t}_4$	$\begin{pmatrix} 0.6 + \iota 0.7, \\ -0.67 - \iota 0.97 \end{pmatrix}$	$\begin{pmatrix} 0.7 + \iota 0.7, \\ -0.76 - \iota 0.67 \end{pmatrix}$	$\begin{pmatrix} 0.6 + \iota 0.6, \\ -0.97 - \iota 0.76 \end{pmatrix}$

Example 1. Let $\mathfrak{t} = \{\mathfrak{t}_1, \mathfrak{t}_2, \mathfrak{t}_3, \mathfrak{t}_4\}$ be set of specialists who calculate the improvement of football player θ usage the attribute set is given as $\theta = \{\theta_1 = Mind\ set, \theta_2 = Fitness, \theta_3 = Technique\}$. Let $\eta = (0.21, 0.32, 0.22, 0.25)$ be the WV of specialists $\mathfrak{t}_{\breve{\imath}}$ and $\lambda = \{0.35, 0.26, 0.39\}$ denotes the WVs for $\theta_{\breve{\jmath}}$ attributes and $t = 2$. Data is his been assumed in Table 1.

$$BCFSDWA_{\lambda}(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{43}}) = \oplus_{\breve{\jmath}=1}^{3} \lambda_{\breve{\jmath}} \left(\oplus_{\breve{\imath}=1}^{4} \eta_{\breve{\imath}} \psi_{\theta_{\breve{\imath}\breve{\jmath}}} \right)$$

$$
= \left(\begin{array}{c}
1 - \dfrac{1}{1 + \left\{ \sum_{\breve{\jmath}=1}^{3} \lambda_{\breve{\jmath}} \left(\sum_{\breve{\imath}=1}^{4} \eta_{\breve{\imath}} \left(\dfrac{\xi^{+}_{\breve{\imath}\breve{\jmath}}}{1-\xi^{+}_{\breve{\imath}\breve{\jmath}}} \right)^{t} \right) \right\}^{\frac{1}{t}}} + \iota \left(1 - \dfrac{1}{1 + \left\{ \sum_{\breve{\jmath}=1}^{3} \lambda_{\breve{\jmath}} \left(\sum_{\breve{\imath}=1}^{4} \eta_{\breve{\imath}} \left(\dfrac{\zeta^{+}_{\breve{\imath}\breve{\jmath}}}{1-\zeta^{+}_{\breve{\imath}\breve{\jmath}}} \right)^{t} \right) \right\}^{\frac{1}{t}}} \right), \\
\dfrac{-1}{1 + \left\{ \sum_{\breve{\jmath}=1}^{3} \lambda_{\breve{\jmath}} \left(\sum_{\breve{\imath}=1}^{4} \eta_{\breve{\imath}} \left(\dfrac{1+\xi^{-}_{\breve{\imath}\breve{\jmath}}}{|\xi^{-}_{\breve{\imath}\breve{\jmath}}|} \right)^{t} \right) \right\}^{\frac{1}{t}}} + \iota \left(\dfrac{-1}{1 + \left\{ \sum_{\breve{\jmath}=1}^{3} \lambda_{\breve{\jmath}} \left(\sum_{\breve{\imath}=1}^{4} \eta_{\breve{\imath}} \left(\dfrac{1+\zeta^{-}_{\breve{\imath}\breve{\jmath}}}{|\zeta^{-}_{\breve{\imath}\breve{\jmath}}|} \right)^{t} \right) \right\}^{\frac{1}{t}}} \right)
\end{array} \right)
$$

Therefore, the following is obtained

$$BCFSDWA_{\lambda}(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{43}}) = \oplus_{j=1}^{3} \lambda_{j} \left(\oplus_{i=1}^{4} \eta_{i} \psi_{\theta_{ij}}\right) = (0.138211, 0.061370, -0.99973, -0.99974)$$

Theorem 3: Let $\psi_{\theta_{ij}} = (\mathcal{P}_{ij}^{+}, \mathcal{Q}_{ij}^{-}) = (\xi_{ij}^{+} + \iota\zeta_{ij}^{+}, \xi_{ij}^{-} + \iota\zeta_{ij}^{-})(i = 1, 2, \dots, m : j = 1, 2, \dots, n)$ is the assortment of BCFSSs, where $\eta = \eta_1, \eta_2, \dots, \eta_m$ be the WV of specialists t_i and $\lambda = \lambda_1, \lambda_2, \dots, \lambda_n$ be the WV of attributes θ_j, holding $\eta_i \geq 0, \lambda_j \geq 0$ such that $\sum_{i=1}^{m} \eta_i = 1$ and $\sum_{j=1}^{n} \lambda_j = 1$. Then, $BCFSDWA_{\lambda}$ AO satisfies the following properties:

1. (Idempotency): Let $\psi_{\theta_{ij}} = (\mathcal{P}_{ij}^{+}, \mathcal{Q}_{ij}^{-}) = (\xi_{ij}^{+} + \iota\zeta_{ij}^{+}, \xi_{ij}^{-} + \iota\zeta_{ij}^{-})(i = 1, 2, \dots, m : j = 1, 2, \dots, n)$ is the assortment of BCFSSs, and all are the same i.e. $\psi_{\theta_{ij}} = \psi_{\theta}, \forall\, i, j$, then the following is obtained:

$$BCFSDWA_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{ij}}\right) = \psi_{\theta}$$

2. (Boundedness): Let $\psi_{\theta_{ij}}^{-} = \left(\min_{j} \min_{i} \mathcal{P}_{ij}^{+}, \max_{j} \max_{i} \mathcal{Q}_{ij}^{-}\right)$ and $\psi_{\theta_{ij}}^{+} = \left(\max_{j} \max_{i} \mathcal{P}_{ij}^{+}, \min_{j} \min_{i} \mathcal{Q}_{ij}^{-}\right)$ then,

$$\psi_{\theta_{ij}}^{-} \leq BCFSDWA_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{ij}}\right) \leq \psi_{\theta_{ij}}^{+}$$

3. (Monotonicity): Let $\psi_{\theta_{ij}} = (\mathcal{P}_{ij}^{+}, \mathcal{Q}_{ij}^{-}) = (\xi_{ij}^{+} + \iota\zeta_{ij}^{+}, \xi_{ij}^{-} + \iota\zeta_{ij}^{-})(i = 1, 2, \dots, m : j = 1, 2, \dots, n)$ and $\phi_{\theta_{ij}} = (\mathcal{R}_{ij}^{+}, \mathcal{S}_{ij}^{-}) = (\vartheta_{ij}^{+} + \iota\tau_{ij}^{+}, \vartheta_{ij}^{-} + \iota\tau_{ij}^{-})(i = 1, 2, \dots, m : j = 1, 2, \dots, n)$ are two assortments of BCFSSs, then the following is obtained:

$$BCFSDWA_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{ij}}\right) \leq BCFSDWA_{\lambda}\left(\phi_{\theta_{11}}, \phi_{\theta_{12}}, \dots, \phi_{\theta_{ij}}\right)$$

$\psi_{\theta_{ij}} \leq \phi_{\theta_{ij}}, \forall\, i, j$

Definition 14: Let $\psi_{\theta_{ij}} = (\mathcal{P}_{ij}^{+}, \mathcal{Q}_{ij}^{-}) = (\xi_{ij}^{+} + \iota\zeta_{ij}^{+}, \xi_{ij}^{-} + \iota\zeta_{ij}^{-})(i = 1, 2, \dots, m : j = 1, 2, \dots, n)$ is the assortment of BCFSSs, then the BCFSDOWA operator is a mapping $\psi_{\theta}^{n} \to \psi_{\theta}$ such that

$$BCFSDOWA_{\lambda}(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \psi_{13}, \dots, \psi_{\theta_{mn}}) = \oplus_{j=1}^{n} \lambda_{j} \left(\oplus_{i=1}^{m} \eta_{i} \psi_{\theta_{o(ij)}}\right) \quad (7)$$

Where $\eta = \eta_1, \eta_2, \dots, \eta_m$ be the WV of specialists t_i and $\lambda = \lambda_1, \lambda_2, \dots, \lambda_n$ be the WV of attributes θ_j, holding $\eta_i \geq 0, \lambda_j \geq 0$ such that $\sum_{i=1}^{m} \eta_i = 1$ and $\sum_{j=1}^{n} \lambda_j = 1$ and $\psi_{\theta_{o(ij)}} = (\mathcal{P}_{o(ij)}^{+}, \mathcal{Q}_{o(ij)}^{-})$ is the permutation of the $i th$ row and $j th$ major elements of the assortment for $i \times j$ BCFSSs $\psi_{\theta_{ij}} = (\mathcal{P}_{ij}^{+}, \mathcal{Q}_{ij}^{-})$ for $i = 1, 2, \dots, m$ *and* $j = 1, 2, \dots, n$. Now, using equation (7) we can define $BCFSDOWA_{\lambda}$ AOs, as trails:

Theorem 4: Let the set BCFSSs, $\psi_{\theta_{ij}} = (\mathcal{P}_{ij}^{+}, \mathcal{Q}_{ij}^{-}) = (\xi_{ij}^{+} + \iota\zeta_{ij}^{+}, \xi_{ij}^{-} + \iota\zeta_{ij}^{-})$ where $i = 1, 2, \dots, m$ *and* $j = 1, 2, \dots, n$. Let $\eta = \eta_1, \eta_2, \dots, \eta_m$ be the WV of specialists t_i and $\lambda = \lambda_1, \lambda_2, \dots, \lambda_n$ be the WV of attributes θ_j, holding $\eta_i \geq 0, \lambda_j \geq 0$ such that $\sum_{i=1}^{m} \eta_i = 1$ and $\sum_{j=1}^{n} \lambda_j = 1$. Then, the aggregated result for the BCFSDOWA operator is defined as trials:

$$BCFSDOWA_{\lambda}(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \psi_{13}, \dots, \psi_{\theta_{mn}}) = \oplus_{j=1}^{n} \lambda_{j} \left(\oplus_{i=1}^{m} \eta_{i} \psi_{\theta_{o(ij)}}\right)$$

$$= \left(\begin{array}{c} 1 - \dfrac{1}{1 + \left\{ \sum_{ǰ=1}^{ꞥ} \lambda_{ǰ} \left(\sum_{ǐ=1}^{ꬺ} \eta_{ǐ} \left(\dfrac{\xi^{+}_{o(ǐǰ)}}{1 - \xi^{+}_{o(ǐǰ)}} \right)^{ŧ} \right) \right\}^{\frac{1}{ŧ}}} + \iota \left(1 - \dfrac{1}{1 + \left\{ \sum_{ǰ=1}^{ꞥ} \lambda_{ǰ} \left(\sum_{ǐ=1}^{ꬺ} \eta_{ǐ} \left(\dfrac{\zeta^{+}_{o(ǐǰ)}}{1 - \zeta^{+}_{o(ǐǰ)}} \right)^{ŧ} \right) \right\}^{\frac{1}{ŧ}}} \right), \\ \dfrac{-1}{1 + \left\{ \sum_{ǰ=1}^{ꞥ} \lambda_{ǰ} \left(\sum_{ǐ=1}^{ꬺ} \eta_{ǐ} \left(\dfrac{1 + \xi^{-}_{o(ǐǰ)}}{\left|\xi^{-}_{o(ǐǰ)}\right|} \right)^{ŧ} \right) \right\}^{\frac{1}{ŧ}}} + \iota \left(\dfrac{-1}{1 + \left\{ \sum_{ǰ=1}^{ꞥ} \lambda_{ǰ} \left(\sum_{ǐ=1}^{ꬺ} \eta_{ǐ} \left(\dfrac{1 + \zeta^{-}_{o(ǐǰ)}}{\left|\zeta^{-}_{o(ǐǰ)}\right|} \right)^{ŧ} \right) \right\}^{\frac{1}{ŧ}}} \right) \end{array} \right)$$

Where $\psi_{\theta_{o(ǐǰ)}} = \left(\mathcal{P}^{+}_{o(ǐǰ)}, \mathcal{Q}^{-}_{o(ǐǰ)}\right)$ is the permutation of the $ǐth$ row and $ǰth$ major elements of the assortment for $ǐ \times ǰ$ BCFSSs $\psi_{\theta_{ǐǰ}} = \left(\mathcal{P}^{+}_{ǐǰ}, \mathcal{Q}^{-}_{ǐǰ}\right)$ for $ǐ = 1, 2, \dots, ꬺ$ and $ǰ = 1, 2, \dots, ꞥ$.

Proof: The proof is like Theorem 2.

Theorem 5: Let $\psi_{\theta_{ǐǰ}} = \left(\mathcal{P}^{+}_{ǐǰ}, \mathcal{Q}^{-}_{ǐǰ}\right) = \left(\xi^{+}_{ǐǰ} + \iota\zeta^{+}_{ǐǰ}, \xi^{-}_{ǐǰ} + \iota\zeta^{-}_{ǐǰ}\right)(ǐ = 1, 2, \dots, ꬺ: ǰ = 1, 2, \dots, ꞥ)$ is the assortment of BCFSSs, where $\eta = \eta_1, \eta_2, \dots, \eta_{ꬺ}$ be the WV of specialists $ŧ_{ǐ}$ and $\lambda = \lambda_1, \lambda_2, \dots, \lambda_{ꞥ}$ be the WV of attributes $\theta_{ǰ}$, holding $\eta_{ǐ} \geq 0, \lambda_{ǰ} \geq 0$ such that $\sum_{ǐ=1}^{ꬺ} \eta_{ǐ} = 1$ and $\sum_{ǰ=1}^{ꞥ} \lambda_{ǰ} = 1$. Then, $BCFSDOWA_{\lambda}$ AO satisfies the following properties:

1) (Idempotency): Let $\psi_{\theta_{ǐǰ}} = \left(\mathcal{P}^{+}_{ǐǰ}, \mathcal{Q}^{-}_{ǐǰ}\right) = \left(\xi^{+}_{ǐǰ} + \iota\zeta^{+}_{ǐǰ}, \xi^{-}_{ǐǰ} + \iota\zeta^{-}_{ǐǰ}\right)(ǐ = 1, 2, \dots, ꬺ: ǰ = 1, 2, \dots, ꞥ)$ is the assortment of BCFSSs, and all are the same i.e. $\psi_{\theta_{ǐǰ}} = \psi_{\theta}, \forall\, ǐ, ǰ$, then the following is obtained:

$$BCFSDOWA_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{ǐǰ}}\right) = \psi_{\theta}$$

2) (Boundedness): Let $\psi^{-}_{\theta_{ǐǰ}} = \left(\min_{ǰ} \min_{ǐ} \mathcal{P}^{+}_{ǐǰ}, \max_{ǰ} \max_{ǐ} \mathcal{Q}^{-}_{ǐǰ}\right)$ and $\psi^{+}_{\theta_{ǐǰ}} = \left(\max_{ǰ} \max_{ǐ} \mathcal{P}^{+}_{ǐǰ}, \min_{ǰ} \min_{ǐ} \mathcal{Q}^{-}_{ǐǰ}\right)$ then,

$$\psi^{-}_{\theta_{ǐǰ}} \leq BCFSDOWA_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{ǐǰ}}\right) \leq \psi^{+}_{\theta_{ǐǰ}}$$

3) (Monotonicity): Let $\psi_{\theta_{ǐǰ}} = \left(\mathcal{P}^{+}_{ǐǰ}, \mathcal{Q}^{-}_{ǐǰ}\right) = \left(\xi^{+}_{ǐǰ} + \iota\zeta^{+}_{ǐǰ}, \xi^{-}_{ǐǰ} + \iota\zeta^{-}_{ǐǰ}\right)(ǐ = 1, 2, \dots, ꬺ: ǰ = 1, 2, \dots, ꞥ)$ and $\phi_{\theta_{ǐǰ}} = \left(\mathcal{R}^{+}_{ǐǰ}, \mathcal{S}^{-}_{ǐǰ}\right) = \left(\vartheta^{+}_{ǐǰ} + \iota\tau^{+}_{ǐǰ}, \vartheta^{-}_{ǐǰ} + \iota\tau^{-}_{ǐǰ}\right)(ǐ = 1, 2, \dots, ꬺ: ǰ = 1, 2, \dots, ꞥ)$ are two assortments of BCFSSs, then the following is obtained:

$$BCFSDOWA_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{ǐǰ}}\right) \leq BCFSDOWA_{\lambda}\left(\phi_{\theta_{11}}, \phi_{\theta_{12}}, \dots, \phi_{\theta_{ǐǰ}}\right)$$

If $\psi_{\theta_{ǐǰ}} \leq \phi_{\theta_{ǐǰ}}, \forall\, ǐ, ǰ$.

In contrast to BCFSDOWA, which only targets the well-arranged situations of BCFS values slightly more than the weights of the BCFS values themselves, the BCFSDWA operator targets only BCFS values, as can be seen from definitions (15) and (16). The characteristics of the BCFSDWA and BCFSDOWA are combined to define the BCFSDHA operator.

Definition 15: Let $\psi_{\theta_{ǐǰ}} = \left(\mathcal{P}^{+}_{ǐǰ}, \mathcal{Q}^{-}_{ǐǰ}\right) = \left(\xi^{+}_{ǐǰ} + \iota\zeta^{+}_{ǐǰ}, \xi^{-}_{ǐǰ} + \iota\zeta^{-}_{ǐǰ}\right)(ǐ = 1, 2, \dots, ꬺ: ǰ = 1, 2, \dots, ꞥ)$ is the assortment of BCFSSs, then the BCFSDHA operator is a function $\psi^{n}_{\theta} \to \psi_{\theta}$ such that

$$BCFSDHWA_{ƛ}(\psi_{\theta_{11}},\psi_{\theta_{12}},\psi_{13},\ldots,\psi_{\theta_{mn}})=\oplus_{\check{j}=1}^{n}ƛ_{\check{j}}\left(\oplus_{\breve{i}=1}^{m}\eta_{\breve{i}}\psi_{\theta_{o(\breve{i}\check{j})}}\right) \tag{8}$$

Where $\eta=\eta_1,\eta_2,\ldots,\eta_{m}$ be the WV of specialists $\mathfrak{t}_{\breve{i}}$ and $ƛ=ƛ_1,ƛ_2,\ldots,ƛ_{n}$ be the WV of attributes $\theta_{\check{j}}$, holding $\eta_{\breve{i}}\geq 0,ƛ_{\check{j}}\geq 0$ such that $\sum_{\breve{i}=1}^{m}\eta_{\breve{i}}=1$ and $\sum_{\check{j}=1}^{n}ƛ_{\check{j}}=1$. The $\psi_{\theta_{o(\breve{i}\check{j})}}=n\partial_{\breve{i}}\rho_{\check{j}}\psi_{\theta_{\breve{i}\check{j}}}$ is the permutation of the $\breve{i}th$ row and $\check{j}th$ major elements of the assortment for $\breve{i}\times\check{j}$ BCFSSs $\psi_{\theta_{\breve{i}\check{j}}}=\left(\mathcal{P}_{\breve{i}\check{j}}^{+},\mathcal{Q}_{\breve{i}\check{j}}^{-}\right)$ and $'n'$ is the matching coefficient. Now, using equation (8) we can define $BCFSDHA_{ƛ}$ AOs, as trails:

Theorem 6: Assume the set BCFSSs, $\psi_{\theta_{\breve{i}\check{j}}}=\left(\mathcal{P}_{\breve{i}\check{j}}^{+},\mathcal{Q}_{\breve{i}\check{j}}^{-}\right)=\left(\xi_{\breve{i}\check{j}}^{+}+\iota\zeta_{\breve{i}\check{j}}^{+},\xi_{\breve{i}\check{j}}^{-}+\iota\zeta_{\breve{i}\check{j}}^{-}\right)$, Where $\breve{i}=1,2,\ldots,m\ and\ \check{j}=1,2,\ldots,n$. Let $\eta=\eta_1,\eta_2,\ldots,\eta_{m}$ be the WV of specialists $\mathfrak{t}_{\breve{i}}$ and $ƛ=ƛ_1,ƛ_2,\ldots,ƛ_{n}$ be the WV of attributes $\theta_{\check{j}}$, holding $\eta_{\breve{i}}\geq 0,ƛ_{\check{j}}\geq 0$ such that $\sum_{\breve{i}=1}^{m}\eta_{\breve{i}}=1$ and $\sum_{\check{j}=1}^{n}ƛ_{\check{j}}=1$. Then, the aggregated result for the BCFSDOWA operator is defined as trials:

$$BCFSDOWA_{ƛ}(\psi_{\theta_{11}},\psi_{\theta_{12}},\psi_{13},\ldots,\psi_{\theta_{mn}})=\oplus_{\check{j}=1}^{n}ƛ_{\check{j}}\left(\oplus_{\breve{i}=1}^{m}\eta_{\breve{i}}\psi_{\theta_{o(\breve{i}\check{j})}}\right)$$

$$=\left(\begin{array}{c}1-\dfrac{1}{1+\left\{\sum_{\check{j}=1}^{n}ƛ_{\check{j}}\left(\sum_{\breve{i}=1}^{m}\eta_{\breve{i}}\left(\dfrac{\xi_{o(\breve{i}\check{j})}^{+}}{1-\xi_{o(\breve{i}\check{j})}^{+}}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}}+\iota\left(1-\dfrac{1}{1+\left\{\sum_{\check{j}=1}^{n}ƛ_{\check{j}}\left(\sum_{\breve{i}=1}^{m}\eta_{\breve{i}}\left(\dfrac{\zeta_{o(\breve{i}\check{j})}^{+}}{1-\zeta_{o(\breve{i}\check{j})}^{+}}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}}\right),\\ \dfrac{-1}{1+\left\{\sum_{\check{j}=1}^{n}ƛ_{\check{j}}\left(\sum_{\breve{i}=1}^{m}\eta_{\breve{i}}\left(\dfrac{1+\xi_{o(\breve{i}\check{j})}^{-}}{\left|\xi_{o(\breve{i}\check{j})}^{-}\right|}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}}+\iota\left(\dfrac{-1}{1+\left\{\sum_{\check{j}=1}^{n}ƛ_{\check{j}}\left(\sum_{\breve{i}=1}^{m}\eta_{\breve{i}}\left(\dfrac{1+\zeta_{o(\breve{i}\check{j})}^{-}}{\left|\zeta_{o(\breve{i}\check{j})}^{-}\right|}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}}\right)\end{array}\right)$$

Where $\psi_{\theta_{o(\breve{i}\check{j})}}=n\partial_{\breve{i}}\rho_{\check{j}}\psi_{\theta_{\breve{i}\check{j}}}$ is the permutation of the $\breve{i}th$ row and $\check{j}th$ major elements of the assortment for $\breve{i}\times\check{j}$ BCFSSs $\psi_{\theta_{\breve{i}\check{j}}}=\left(\mathcal{P}_{\breve{i}\check{j}}^{+},\mathcal{Q}_{\breve{i}\check{j}}^{-}\right)$ and $'n'$ is the matching coefficient.

Proof: The proof theorem is like Theorem 2.

Theorem 7: Let $\psi_{\theta_{\breve{i}\check{j}}}=\left(\mathcal{P}_{\breve{i}\check{j}}^{+},\mathcal{Q}_{\breve{i}\check{j}}^{-}\right)=\left(\xi_{\breve{i}\check{j}}^{+}+\iota\zeta_{\breve{i}\check{j}}^{+},\xi_{\breve{i}\check{j}}^{-}+\iota\zeta_{\breve{i}\check{j}}^{-}\right)(\breve{i}=1,2,\ldots,m:\check{j}=1,2,\ldots,n)$ is the assortment of BCFSSs, where $\eta=\eta_1,\eta_2,\ldots,\eta_{m}$ be the WV of specialists $\mathfrak{t}_{\breve{i}}$ and $ƛ=ƛ_1,ƛ_2,\ldots,ƛ_{n}$ be the WV of attributes $\theta_{\check{j}}$, holding $\eta_{\breve{i}}\geq 0,ƛ_{\check{j}}\geq 0$ such that $\sum_{\breve{i}=1}^{m}\eta_{\breve{i}}=1$ and $\sum_{\check{j}=1}^{n}ƛ_{\check{j}}=1$. Then, $BCFSDHWA_{ƛ}$ AO satisfies the following properties:

1. (Idempotency): Let $\psi_{\theta_{\breve{i}\check{j}}}=\left(\mathcal{P}_{\breve{i}\check{j}}^{+},\mathcal{Q}_{\breve{i}\check{j}}^{-}\right)=\left(\xi_{\breve{i}\check{j}}^{+}+\iota\zeta_{\breve{i}\check{j}}^{+},\xi_{\breve{i}\check{j}}^{-}+\iota\zeta_{\breve{i}\check{j}}^{-}\right)(\breve{i}=1,2,\ldots,m:\check{j}=1,2,\ldots,n)$ is the assortment of BCFSSs, and all are the same i.e. $\psi_{\theta_{\breve{i}\check{j}}}=\psi_{\theta},\forall\,\breve{i},\check{j}$, then the following is obtained:

$$BCFSDHWA_{ƛ}\left(\psi_{\theta_{11}},\psi_{\theta_{12}},\ldots,\psi_{\theta_{\breve{i}\check{j}}}\right)=\psi_{\theta}$$

2. (Boundedness): Let $\psi_{\theta_{\breve{i}\check{j}}}^{-}=\left(\min_{\check{j}}\min_{\breve{i}}\mathcal{P}_{\breve{i}\check{j}}^{+},\max_{\check{j}}\max_{\breve{i}}\mathcal{Q}_{\breve{i}\check{j}}^{-}\right)$ and $\psi_{\theta_{\breve{i}\check{j}}}^{+}=\left(\max_{\check{j}}\max_{\breve{i}}\mathcal{P}_{\breve{i}\check{j}}^{+},\min_{\check{j}}\min_{\breve{i}}\mathcal{Q}_{\breve{i}\check{j}}^{-}\right)$ then,

$$\psi_{\theta_{\breve{i}\check{j}}}^{-}\leq BCFSDHWA_{ƛ}\left(\psi_{\theta_{11}},\psi_{\theta_{12}},\ldots,\psi_{\theta_{\breve{i}\check{j}}}\right)\leq\psi_{\theta_{\breve{i}\check{j}}}^{+}$$

3. (Monotonicity): Let $\psi_{\theta_{ĭj̆}} = \left(\mathcal{P}_{ĭj̆}^{+}, \mathcal{Q}_{ĭj̆}^{-}\right) = \left(\xi_{ĭj̆}^{+} + \iota\zeta_{ĭj̆}^{+}, \xi_{ĭj̆}^{-} + \iota\zeta_{ĭj̆}^{-}\right)(ĭ = 1,2,\dots,\mathfrak{m}: j̆ = 1,2,\dots,\mathfrak{n})$ and $\phi_{\theta_{ĭj̆}} = \left(\mathcal{R}_{ĭj̆}^{+}, \mathcal{S}_{ĭj̆}^{-}\right) = \left(\vartheta_{ĭj̆}^{+} + \iota\tau_{ĭj̆}^{+}, \vartheta_{ĭj̆}^{-} + \iota\tau_{ĭj̆}^{-}\right)(ĭ = 1,2,\dots,\mathfrak{m}: j̆ = 1,2,\dots,\mathfrak{n})$ are two assortments of BCFSSs, then the following is obtained:

$$BCFSDHWA_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{ĭj̆}}\right) \le BCFSDHWA_{\lambda}\left(\phi_{\theta_{11}}, \phi_{\theta_{12}}, \dots, \phi_{\theta_{ĭj̆}}\right)$$

If $\psi_{\theta_{ĭj̆}} \le \phi_{\theta_{ĭj̆}}, \forall\, ĭ, j̆$.

Remark 1.

1. When $\lambda = \left(\frac{1}{\mathfrak{n}}, \frac{1}{\mathfrak{n}}, \frac{1}{\mathfrak{n}}, \dots, \frac{1}{\mathfrak{n}}\right)$ then the BCFSDHA operators covert into the BCFSDWA.
2. When $\partial = \left(\frac{1}{\mathfrak{n}}, \frac{1}{\mathfrak{n}}, \frac{1}{\mathfrak{n}}, \dots, \frac{1}{\mathfrak{n}}\right)$ then the BCFSDHA operators covert into the BCFSDOA.

5.2 | Bipolar Complex Fuxxy Soft DOMBI Geometric Aggregation Operators

BIPOLAR COMPLEX FUZZY SOFT DOMBI GEOMETRIC AGGREGATION OPERATORS

This segment of the article consists of BCFSDWG operator, BCFSDOWG operator, and BCFSDHG operator.

Definition 16: Let $\psi_{\theta_{ĭj̆}} = \left(\mathcal{P}_{ĭj̆}^{+}, \mathcal{Q}_{ĭj̆}^{-}\right) = \left(\xi_{ĭj̆}^{+} + \iota\zeta_{ĭj̆}^{+}, \xi_{ĭj̆}^{-} + \iota\zeta_{ĭj̆}^{-}\right)(ĭ = 1,2,\dots,\mathfrak{m}: j̆ = 1,2,\dots,\mathfrak{n})$ is the assortment of BCFSSs, then the BCFSDWG operator is a function $\psi_{\theta}^{n} \to \psi_{\theta}$ such that

$$BCFSDWG_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \psi_{13}, \dots, \psi_{\theta_{\mathfrak{mn}}}\right) = \otimes_{j̆=1}^{\mathfrak{n}}\left(\otimes_{ĭ=1}^{\mathfrak{m}}\left(\psi_{\theta_{ĭj̆}}\right)^{\eta_{ĭ}}\right)^{\lambda_{j̆}} \tag{9}$$

Where $\eta = \eta_1, \eta_2, \dots, \eta_{\mathfrak{m}}$ be the WV of specialists $\mathfrak{t}_{ĭ}$ and $\lambda = \lambda_1, \lambda_2, \dots, \lambda_{\mathfrak{n}}$ be the WV of attributes $\theta_{j̆}$, holding $\eta_{ĭ} \ge 0, \lambda_{j̆} \ge 0$ such that $\sum_{ĭ=1}^{\mathfrak{m}} \eta_{ĭ} = 1$ and $\sum_{j̆=1}^{\mathfrak{n}} \lambda_{j̆} = 1$.

Theorem 8: Let $\psi_{\theta_{ĭj̆}} = \left(\mathcal{P}_{ĭj̆}^{+}, \mathcal{Q}_{ĭj̆}^{-}\right) = \left(\xi_{ĭj̆}^{+} + \iota\zeta_{ĭj̆}^{+}, \xi_{ĭj̆}^{-} + \iota\zeta_{ĭj̆}^{-}\right)(ĭ = 1,2,\dots,\mathfrak{m}: j̆ = 1,2,\dots,\mathfrak{n})$ is the assortment of BCFSSs, then by using the BCFSDWG operator their calculated is again a BCFSS and

$$BCFSDWG_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \psi_{13}, \dots, \psi_{\theta_{\mathfrak{mn}}}\right) = \otimes_{j̆=1}^{\mathfrak{n}}\left(\otimes_{ĭ=1}^{\mathfrak{m}}\left(\psi_{\theta_{ĭj̆}}\right)^{\eta_{ĭ}}\right)^{\lambda_{j̆}}$$

$$= \left(\begin{array}{c} \dfrac{1}{1 + \left\{\sum_{j̆=1}^{\mathfrak{n}} \lambda_{j̆}\left(\sum_{ĭ=1}^{\mathfrak{m}} \eta_{ĭ}\left(\frac{1 - \xi_{ĭj̆}^{+}}{\xi_{ĭj̆}^{+}}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}} + \iota\left(\dfrac{1}{1 + \left\{\sum_{j̆=1}^{\mathfrak{n}} \lambda_{j̆}\left(\sum_{ĭ=1}^{\mathfrak{m}} \eta_{ĭ}\left(\frac{1 - \zeta_{ĭj̆}^{+}}{\zeta_{ĭj̆}^{+}}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}}\right), \\ -1 + \dfrac{1}{1 + \left\{\sum_{j̆=1}^{\mathfrak{n}} \lambda_{j̆}\left(\sum_{ĭ=1}^{\mathfrak{m}} \eta_{ĭ}\left(\frac{\left|\xi_{ĭj̆}^{-}\right|}{1 + \xi_{ĭj̆}^{-}}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}} + \iota\left(-1 + \dfrac{1}{1 + \left\{\sum_{j̆=1}^{\mathfrak{n}} \lambda_{j̆}\left(\sum_{ĭ=1}^{\mathfrak{m}} \eta_{ĭ}\left(\frac{\left|\zeta_{ĭj̆}^{-}\right|}{1 + \zeta_{ĭj̆}^{-}}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}}\right) \end{array}\right)$$

Where $\eta = \eta_1, \eta_2, \dots, \eta_{\mathfrak{m}}$ be the WV of specialists $\mathfrak{t}_{ĭ}$ and $\lambda = \lambda_1, \lambda_2, \dots, \lambda_{\mathfrak{n}}$ be the WV of attributes $\theta_{j̆}$, holding $\eta_{ĭ} \ge 0, \lambda_{j̆} \ge 0$ such that $\sum_{ĭ=1}^{\mathfrak{m}} \eta_{ĭ} = 1$ and $\sum_{j̆=1}^{\mathfrak{n}} \lambda_{j̆} = 1$.

Proof: Now, the mathematical induction technique is utilized, as trials:

$$\psi_{\theta_{11}}\otimes\psi_{\theta_{12}}=\begin{pmatrix}\dfrac{1}{1+\left\{\left(\frac{1-\xi_{11}^{+}}{\xi_{11}^{+}}\right)^{t}+\left(\frac{1-\xi_{12}^{+}}{\xi_{12}^{+}}\right)^{t}\right\}^{\frac{1}{t}}}+\iota\left(\dfrac{1}{1+\left\{\left(\frac{1-\zeta_{11}^{+}}{\zeta_{11}^{+}}\right)^{t}+\left(\frac{1-\zeta_{12}^{+}}{\zeta_{12}^{+}}\right)^{t}\right\}^{\frac{1}{t}}}\right),\\ -1+\dfrac{1}{1+\left\{\left(\frac{|\xi_{11}^{-}|}{1+\xi_{11}^{-}}\right)^{t}+\left(\frac{|\xi_{12}^{-}|}{1+\xi_{12}^{-}}\right)^{t}\right\}^{\frac{1}{t}}}+\iota\left(-1+\dfrac{1}{1+\left\{\left(\frac{|\xi_{11}^{-}|}{1+\xi_{11}^{-}}\right)^{t}+\left(\frac{|\xi_{12}^{-}|}{1+\xi_{12}^{-}}\right)^{t}\right\}^{\frac{1}{t}}}\right)\end{pmatrix}$$

First, we prove that the result accessible for $m = 2$ and $n = 2$, as trials:

$$BCFSDWG_{\lambda}\left(\psi_{\theta_{11}},\psi_{\theta_{12}}\right)=\otimes_{\breve{j}=1}^{2}\left(\otimes_{\breve{\imath}=1}^{2}\left(\psi_{\theta_{\breve{\imath}\breve{j}}}\right)^{\eta_{\breve{\imath}}}\right)^{\lambda_{\breve{j}}}=\left(\left(\psi_{\theta_{11}}\right)^{\eta_{1}}\otimes\left(\psi_{\theta_{21}}\right)^{\eta_{2}}\right)^{\lambda_{1}}\otimes\left(\left(\psi_{\theta_{12}}\right)^{\eta_{1}}\otimes\left(\psi_{\theta_{22}}\right)^{\eta_{2}}\right)^{\lambda_{2}}$$

$$=\lambda_{1}\begin{pmatrix}\dfrac{1}{1+\left\{\eta_{1}\left(\frac{1-\xi_{11}^{+}}{\xi_{11}^{+}}\right)^{t}+\eta_{2}\left(\frac{1-\xi_{21}^{+}}{\xi_{21}^{+}}\right)^{t}\right\}^{\frac{1}{t}}}+\iota\left(\dfrac{1}{1+\left\{\eta_{1}\left(\frac{1-\zeta_{11}^{+}}{\zeta_{11}^{+}}\right)^{t}+\eta_{2}\left(\frac{1-\zeta_{21}^{+}}{\zeta_{21}^{+}}\right)^{t}\right\}^{\frac{1}{t}}}\right),\\ -1+\dfrac{1}{1+\left\{\eta_{1}\left(\frac{|\xi_{11}^{-}|}{1+\xi_{11}^{-}}\right)^{t}+\eta_{2}\left(\frac{|\xi_{21}^{-}|}{1+\xi_{21}^{-}}\right)^{t}\right\}^{\frac{1}{t}}}+\iota\left(-1+\dfrac{1}{1+\left\{\eta_{1}\left(\frac{|\zeta_{11}^{-}|}{1+\zeta_{11}^{-}}\right)^{t}+\eta_{2}\left(\frac{|\zeta_{21}^{-}|}{1+\zeta_{21}^{-}}\right)^{t}\right\}^{\frac{1}{t}}}\right)\end{pmatrix}$$

$$\otimes\lambda_{2}\begin{pmatrix}\dfrac{1}{1+\left\{\eta_{1}\left(\frac{1-\xi_{12}^{+}}{\xi_{12}^{+}}\right)^{t}+\eta_{2}\left(\frac{1-\xi_{22}^{+}}{\xi_{22}^{+}}\right)^{t}\right\}^{\frac{1}{t}}}+\iota\left(\dfrac{1}{1+\left\{\eta_{1}\left(\frac{1-\zeta_{12}^{+}}{\zeta_{12}^{+}}\right)^{t}+\eta_{2}\left(\frac{1-\zeta_{22}^{+}}{\zeta_{22}^{+}}\right)^{t}\right\}^{\frac{1}{t}}}\right),\\ -1+\dfrac{1}{1+\left\{\eta_{1}\left(\frac{|\xi_{12}^{-}|}{1+\xi_{12}^{-}}\right)^{t}+\eta_{2}\left(\frac{|\xi_{22}^{-}|}{1+\xi_{22}^{-}}\right)^{t}\right\}^{\frac{1}{t}}}+\iota\left(-1+\dfrac{1}{1+\left\{\eta_{1}\left(\frac{|\zeta_{12}^{-}|}{1+\zeta_{12}^{-}}\right)^{t}+\eta_{2}\left(\frac{|\zeta_{22}^{-}|}{1+\zeta_{22}^{-}}\right)^{t}\right\}^{\frac{1}{t}}}\right)\end{pmatrix}$$

$$=\lambda_{1}\begin{pmatrix}\dfrac{1}{1+\left\{\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{1-\xi_{\breve{\imath}1}^{+}}{\xi_{\breve{\imath}1}^{+}}\right)^{t}\right\}^{\frac{1}{t}}}+\iota\left(\dfrac{1}{1+\left\{\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{1-\zeta_{\breve{\imath}1}^{+}}{\zeta_{\breve{\imath}1}^{+}}\right)^{t}\right\}^{\frac{1}{t}}}\right),\\ -1+\dfrac{1}{1+\left\{\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{|\xi_{\breve{\imath}1}^{-}|}{1+\xi_{\breve{\imath}1}^{-}}\right)^{t}\right\}^{\frac{1}{t}}}+\iota\left(-1+\dfrac{1}{1+\left\{\sum_{\breve{\imath}=1}^{2}\eta_{\breve{\imath}}\left(\frac{|\zeta_{\breve{\imath}1}^{-}|}{1+\xi_{\breve{\imath}1}^{-}}\right)^{t}\right\}^{\frac{1}{t}}}\right)\end{pmatrix}$$

$$\otimes \lambda_2 \begin{pmatrix} \frac{1}{1+\left\{\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{1-\xi_{\check{i}2}^{+}}{\xi_{\check{i}2}^{+}}\right)^{t}\right\}^{\frac{1}{t}}} + \iota\left(\frac{1}{1+\left\{\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{1-\zeta_{\check{i}2}^{+}}{\zeta_{\check{i}2}^{+}}\right)^{t}\right\}^{\frac{1}{t}}}\right), \\ -1+\frac{1}{1+\left\{\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{|\xi_{\check{i}2}^{-}|}{1+\xi_{\check{i}2}^{-}}\right)^{t}\right\}^{\frac{1}{t}}} + \iota\left(-1+\frac{1}{1+\left\{\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{|\zeta_{\check{i}2}^{-}|}{1+\xi_{\check{i}2}^{-}}\right)^{t}\right\}^{\frac{1}{t}}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{1+\left\{\lambda_1\left(\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{1-\xi_{\check{i}1}^{+}}{\xi_{\check{i}1}^{+}}\right)^{t}\right)+\lambda_2\left(\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{1-\xi_{\check{i}2}^{+}}{\xi_{\check{i}2}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}} + \\ \iota\left(\frac{1}{1+\left\{\lambda_1\left(\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{1-\zeta_{\check{i}1}^{+}}{\zeta_{\check{i}1}^{+}}\right)^{t}\right)+\lambda_2\left(\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{1-\zeta_{\check{i}2}^{+}}{\zeta_{\check{i}2}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right), \\ -1+\frac{1}{1+\left\{\lambda_1\left(\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{|\xi_{\check{i}1}^{-}|}{1+\xi_{\check{i}1}^{-}}\right)^{t}\right)+\lambda_2\left(\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{|\xi_{\check{i}2}^{-}|}{1+\xi_{\check{i}2}^{-}}\right)^{t}\right)\right\}^{\frac{1}{t}}} + \\ \iota\left(-1+\frac{1}{1+\left\{\lambda_1\left(\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{|\zeta_{\check{i}1}^{-}|}{1+\zeta_{\check{i}1}^{-}}\right)^{t}\right)+\lambda_2\left(\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{|\zeta_{\check{i}2}^{-}|}{1+\zeta_{\check{i}2}^{-}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{1+\left\{\sum_{\check{j}=1}^{2}\lambda_{\check{j}}\left(\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{1-\xi_{\check{i}\check{j}}^{+}}{\xi_{\check{i}\check{j}}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}} + \iota\left(\frac{1}{1+\left\{\sum_{\check{j}=1}^{2}\lambda_{\check{j}}\left(\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{1-\zeta_{\check{i}\check{j}}^{+}}{\zeta_{\check{i}\check{j}}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right), \\ -1+\frac{1}{1+\left\{\sum_{\check{j}=1}^{2}\lambda_{\check{j}}\left(\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{|\xi_{\check{i}\check{j}}^{-}|}{1+\xi_{\check{i}\check{j}}^{-}}\right)^{t}\right)\right\}^{\frac{1}{t}}} + \iota\left(-1+\frac{1}{1+\left\{\sum_{\check{j}=1}^{2}\lambda_{\check{j}}\left(\sum_{\check{i}=1}^{2}\eta_{\check{i}}\left(\frac{|\zeta_{\check{i}\check{j}}^{-}|}{1+\zeta_{\check{i}\check{j}}^{-}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right) \end{pmatrix}$$

Therefore, this outcome is true for $\mathfrak{m} = 2$ and $\mathfrak{n} = 2$.

Now, we consider this outcome is valid for $\mathfrak{m} = \kappa_1$ and $\mathfrak{n} = \kappa_2$, as trials:

$$BCFSDWG_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \psi_{13}, \dots, \psi_{\theta_{\kappa_1\kappa_2}}\right) = \otimes_{\check{j}=1}^{\kappa_2}\left(\otimes_{\check{i}=1}^{\kappa_1}\left(\psi_{\theta_{\check{i}\check{j}}}\right)^{\eta_{\check{i}}}\right)^{\lambda_{\check{j}}}$$

$$
=\begin{pmatrix}
\dfrac{1}{1+\left\{\sum_{\breve{j}=1}^{\kappa_2}\lambda_{\breve{j}}\left(\sum_{\breve{i}=1}^{\kappa_1}\eta_{\breve{i}}\left(\dfrac{1-\xi^{+}_{\breve{i}\breve{j}}}{\xi^{+}_{\breve{i}\breve{j}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\iota\left(\dfrac{1}{1+\left\{\sum_{\breve{j}=1}^{\kappa_2}\lambda_{\breve{j}}\left(\sum_{\breve{i}=1}^{\kappa_1}\eta_{\breve{i}}\left(\dfrac{1-\zeta^{+}_{\breve{i}\breve{j}}}{\zeta^{+}_{\breve{i}\breve{j}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right), \\
-1+\dfrac{1}{1+\left\{\sum_{\breve{j}=1}^{\kappa_2}\lambda_{\breve{j}}\left(\sum_{\breve{i}=1}^{\kappa_1}\eta_{\breve{i}}\left(\dfrac{\left|\xi^{-}_{\breve{i}\breve{j}}\right|}{1+\xi^{-}_{\breve{i}\breve{j}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\iota\left(-1+\dfrac{1}{1+\left\{\sum_{\breve{j}=1}^{\kappa_2}\lambda_{\breve{j}}\left(\sum_{\breve{i}=1}^{\kappa_1}\eta_{\breve{i}}\left(\dfrac{\left|\zeta^{-}_{\breve{i}\breve{j}}\right|}{1+\zeta^{-}_{\breve{i}\breve{j}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right)
\end{pmatrix}
$$

Next, we consider this outcome to be valid for $m=\kappa_1+1$ and $n=\kappa_2+1$, as trials:

$$
BCFSDWG_{\lambda}\left(\psi_{\theta_{11}},\psi_{\theta_{12}},\psi_{13},\ldots,\psi_{\theta_{\kappa_1\kappa_2}},\psi_{\theta_{(\kappa_1+1)(\kappa_2+1)}}\right)=\otimes_{\breve{j}=1}^{\kappa_2}\left(\otimes_{\breve{i}=1}^{\kappa_1}\left(\psi_{\theta_{\breve{i}\breve{j}}}\right)^{\eta_{\breve{i}}}\right)^{\lambda_{\breve{j}}}\otimes\left(\left(\left(\psi_{\theta_{(\kappa_1+1)(\kappa_2+1)}}\right)^{\eta_{\kappa_1+1}}\right)^{\lambda_{\kappa_2+1}}\right)
$$

$$
=\begin{pmatrix}
\dfrac{1}{1+\left\{\sum_{\breve{j}=1}^{\kappa_2}\lambda_{\breve{j}}\left(\sum_{\breve{i}=1}^{\kappa_1}\eta_{\breve{i}}\left(\dfrac{1-\xi^{+}_{\breve{i}\breve{j}}}{\xi^{+}_{\breve{i}\breve{j}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\iota\left(\dfrac{1}{1+\left\{\sum_{\breve{j}=1}^{\kappa_2}\lambda_{\breve{j}}\left(\sum_{\breve{i}=1}^{\kappa_1}\eta_{\breve{i}}\left(\dfrac{1-\zeta^{+}_{\breve{i}\breve{j}}}{\zeta^{+}_{\breve{i}\breve{j}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right), \\
-1+\dfrac{1}{1+\left\{\sum_{\breve{j}=1}^{\kappa_2}\lambda_{\breve{j}}\left(\sum_{\breve{i}=1}^{\kappa_1}\eta_{\breve{i}}\left(\dfrac{\left|\xi^{-}_{\breve{i}\breve{j}}\right|}{1+\xi^{-}_{\breve{i}\breve{j}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\iota\left(-1+\dfrac{1}{1+\left\{\sum_{\breve{j}=1}^{\kappa_2}\lambda_{\breve{j}}\left(\sum_{\breve{i}=1}^{\kappa_1}\eta_{\breve{i}}\left(\dfrac{\left|\zeta^{-}_{\breve{i}\breve{j}}\right|}{1+\zeta^{-}_{\breve{i}\breve{j}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right)
\end{pmatrix}
$$

$$
\otimes\begin{pmatrix}
\dfrac{1}{1+\left\{\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\dfrac{1-\xi^{+}_{\breve{i}\breve{j}}}{\xi^{+}_{\breve{i}\breve{j}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\iota\left(\dfrac{1}{1+\left\{\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\dfrac{1-\zeta^{+}_{\breve{i}\breve{j}}}{\zeta^{+}_{\breve{i}\breve{j}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right), \\
-1+\dfrac{1}{1+\left\{\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\dfrac{\left|\xi^{-}_{\breve{i}\breve{j}}\right|}{1+\xi^{-}_{\breve{i}\breve{j}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\iota\left(-1+\dfrac{1}{1+\left\{\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\dfrac{\left|\zeta^{-}_{\breve{i}\breve{j}}\right|}{1+\zeta^{-}_{\breve{i}\breve{j}}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right)
\end{pmatrix}
$$

$$=\begin{pmatrix}\dfrac{1}{1+\left\{\sum_{\text{ĵ}=1}^{\kappa_2}\lambda_{\text{ĵ}}\left(\sum_{\text{ĭ}=1}^{\kappa_1}\eta_{\text{ĭ}}\left(\frac{1-\xi_{\text{ĭĵ}}^{+}}{\xi_{\text{ĭĵ}}^{+}}\right)^{t}\right)+\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\frac{1-\xi_{\text{ĭĵ}}^{+}}{\xi_{\text{ĭĵ}}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\\ \iota\left(\dfrac{1}{1+\left\{\sum_{\text{ĵ}=1}^{\kappa_2}\lambda_{\text{ĵ}}\left(\sum_{\text{ĭ}=1}^{\kappa_1}\eta_{\text{ĭ}}\left(\frac{1-\zeta_{\text{ĭĵ}}^{+}}{\zeta_{\text{ĭĵ}}^{+}}\right)^{t}\right)+\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\frac{1-\zeta_{\text{ĭĵ}}^{+}}{\zeta_{\text{ĭĵ}}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right),\\ -1+\dfrac{1}{1+\left\{\sum_{\text{ĵ}=1}^{\kappa_2}\lambda_{\text{ĵ}}\left(\sum_{\text{ĭ}=1}^{\kappa_1}\eta_{\text{ĭ}}\left(\frac{|\xi_{\text{ĭĵ}}^{-}|}{1+\xi_{\text{ĭĵ}}^{-}}\right)^{t}\right)+\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\frac{|\xi_{\text{ĭĵ}}^{-}|}{1+\xi_{\text{ĭĵ}}^{-}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\\ \iota\left(-1+\dfrac{1}{1+\left\{\sum_{\text{ĵ}=1}^{\kappa_2}\lambda_{\text{ĵ}}\left(\sum_{\text{ĭ}=1}^{\kappa_1}\eta_{\text{ĭ}}\left(\frac{|\zeta_{\text{ĭĵ}}^{-}|}{1+\zeta_{\text{ĭĵ}}^{-}}\right)^{t}\right)+\lambda_{\kappa_2+1}\left(\eta_{\kappa_1+1}\left(\frac{|\zeta_{\text{ĭĵ}}^{-}|}{1+\zeta_{\text{ĭĵ}}^{-}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right)\end{pmatrix}$$

$$=\begin{pmatrix}\dfrac{1}{1+\left\{\sum_{\text{ĵ}=1}^{\kappa_2+1}\lambda_{\text{ĵ}}\left(\sum_{\text{ĭ}=1}^{\kappa_1+1}\eta_{\text{ĭ}}\left(\frac{1-\xi_{\text{ĭĵ}}^{+}}{\xi_{\text{ĭĵ}}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\iota\left(\dfrac{1}{1+\left\{\sum_{\text{ĵ}=1}^{\kappa_2+1}\lambda_{\text{ĵ}}\left(\sum_{\text{ĭ}=1}^{\kappa_1+1}\eta_{\text{ĭ}}\left(\frac{1-\zeta_{\text{ĭĵ}}^{+}}{\zeta_{\text{ĭĵ}}^{+}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right),\\ -1+\dfrac{1}{1+\left\{\sum_{\text{ĵ}=1}^{\kappa_2+1}\lambda_{\text{ĵ}}\left(\sum_{\text{ĭ}=1}^{\kappa_1+1}\eta_{\text{ĭ}}\left(\frac{|\xi_{\text{ĭĵ}}^{-}|}{1+\xi_{\text{ĭĵ}}^{-}}\right)^{t}\right)\right\}^{\frac{1}{t}}}+\iota\left(-1+\dfrac{1}{1+\left\{\sum_{\text{ĵ}=1}^{\kappa_2+1}\lambda_{\text{ĵ}}\left(\sum_{\text{ĭ}=1}^{\kappa_1+1}\eta_{\text{ĭ}}\left(\frac{|\zeta_{\text{ĭĵ}}^{-}|}{1+\zeta_{\text{ĭĵ}}^{-}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right)\end{pmatrix}$$

Therefore, the outcome is valid for $\text{m}=\kappa_1+1$ and $\text{n}=\kappa_2+1$. Hence, the outcome is true $\text{m},\text{n}\geq 1$. It is obvious from the above expression that the aggregated outcomes invented BCFSDWG operator is again BCFSDWG operator.

Table 2. Bipolar Complex Fuzzy Soft Set Data

$\boldsymbol{\psi_{\theta_{\text{ĭĵ}}}}$	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$
$\mathfrak{t}_1$	$\begin{pmatrix}0.7+\iota 0.8,\\-0.49-\iota 0.71\end{pmatrix}$	$\begin{pmatrix}0.7+\iota 0.7,\\-0.62-\iota 0.49\end{pmatrix}$	$\begin{pmatrix}0.8+\iota 0.7,\\-0.71-\iota 0.62\end{pmatrix}$
$\mathfrak{t}_2$	$\begin{pmatrix}0.8+\iota 0.7,\\-0.82-\iota 0.89\end{pmatrix}$	$\begin{pmatrix}0.6+\iota 0.5,\\-0.72-\iota 0.82\end{pmatrix}$	$\begin{pmatrix}0.8+\iota 0.8,\\-0.89-\iota 0.72\end{pmatrix}$
$\mathfrak{t}_3$	$\begin{pmatrix}0.7+\iota 0.6,\\-0.57-\iota 0.88\end{pmatrix}$	$\begin{pmatrix}0.7+\iota 0.6,\\-0.64-\iota 0.57\end{pmatrix}$	$\begin{pmatrix}0.7+\iota 0.7,\\-0.88-\iota 0.64\end{pmatrix}$
$\mathfrak{t}_4$	$\begin{pmatrix}0.6+\iota 0.7,\\-0.67-\iota 0.97\end{pmatrix}$	$\begin{pmatrix}0.7+\iota 0.7,\\-0.76-\iota 0.67\end{pmatrix}$	$\begin{pmatrix}0.6+\iota 0.6,\\-0.97-\iota 0.76\end{pmatrix}$

Example 2. Let $\mathfrak{t}=\{\mathfrak{t}_1,\mathfrak{t}_2,\mathfrak{t}_3,\mathfrak{t}_4\}$ be set of specialists who calculate the progress of basketball player θ using the attribute set is given as $\theta=\{\theta_1=Mind\ set,\theta_2=Fitness,\theta_3=Technique\}$. Let $\eta=$

(0.22, 0.31, 0.23, 0.24) be the WV of specialists $\mathfrak{t}_{\breve{\imath}}$ and $\lambda = \{0.33, 0.27, 0.4\}$ denotes the WVs for $\theta_{\breve{\jmath}}$ attributes and $\mathrm{t} = 2$. Data is he has been given in Table 2.

$$BCFSDWG_{\lambda}(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{43}}) = \otimes_{\breve{\jmath}=1}^{3}\left(\otimes_{\breve{\imath}=1}^{4}\left(\psi_{\theta_{\breve{\imath}\breve{\jmath}}}\right)^{\eta_{\breve{\imath}}}\right)^{\lambda_{\breve{\jmath}}}$$

$$= \begin{pmatrix} \dfrac{1}{1+\left\{\sum_{\breve{\jmath}=1}^{3}\lambda_{\breve{\jmath}}\left(\sum_{\breve{\imath}=1}^{4}\eta_{\breve{\imath}}\left(\dfrac{1-\xi_{\breve{\imath}\breve{\jmath}}^{+}}{\xi_{\breve{\imath}\breve{\jmath}}^{+}}\right)^{\mathrm{t}}\right)\right\}^{\frac{1}{\mathrm{t}}}} + \iota\left(\dfrac{1}{1+\left\{\sum_{\breve{\jmath}=1}^{3}\lambda_{\breve{\jmath}}\left(\sum_{\breve{\imath}=1}^{4}\eta_{\breve{\imath}}\left(\dfrac{1-\zeta_{\breve{\imath}\breve{\jmath}}^{+}}{\zeta_{\breve{\imath}\breve{\jmath}}^{+}}\right)^{\mathrm{t}}\right)\right\}^{\frac{1}{\mathrm{t}}}}\right), \\ -1+\dfrac{1}{1+\left\{\sum_{\breve{\jmath}=1}^{3}\lambda_{\breve{\jmath}}\left(\sum_{\breve{\imath}=1}^{4}\eta_{\breve{\imath}}\left(\dfrac{|\xi_{\breve{\imath}\breve{\jmath}}^{-}|}{1+\xi_{\breve{\imath}\breve{\jmath}}^{-}}\right)^{\mathrm{t}}\right)\right\}^{\frac{1}{\mathrm{t}}}} + \iota\left(-1+\dfrac{1}{1+\left\{\sum_{\breve{\jmath}=1}^{3}\lambda_{\breve{\jmath}}\left(\sum_{\breve{\imath}=1}^{4}\eta_{\breve{\imath}}\left(\dfrac{|\zeta_{\breve{\imath}\breve{\jmath}}^{-}|}{1+\zeta_{\breve{\imath}\breve{\jmath}}^{-}}\right)^{\mathrm{t}}\right)\right\}^{\frac{1}{\mathrm{t}}}}\right) \end{pmatrix}$$

Therefore, the following is obtained

$$BCFSDWG_{\lambda}(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{43}}) = \otimes_{\breve{\jmath}=1}^{3}\left(\otimes_{\breve{\imath}=1}^{4}\left(\psi_{\theta_{\breve{\imath}\breve{\jmath}}}\right)^{\eta_{\breve{\imath}}}\right)^{\lambda_{\breve{\jmath}}} = (0.999789, 0.999625, -0.00102, -0.00106)$$

Theorem 9: Let $\psi_{\theta_{\breve{\imath}\breve{\jmath}}} = \left(\mathcal{P}_{\breve{\imath}\breve{\jmath}}^{+}, \mathcal{Q}_{\breve{\imath}\breve{\jmath}}^{-}\right) = \left(\xi_{\breve{\imath}\breve{\jmath}}^{+} + \iota\zeta_{\breve{\imath}\breve{\jmath}}^{+}, \xi_{\breve{\imath}\breve{\jmath}}^{-} + \iota\zeta_{\breve{\imath}\breve{\jmath}}^{-}\right)(\breve{\imath} = 1, 2, \dots, \mathfrak{m}: \breve{\jmath} = 1, 2, \dots, \mathfrak{n})$ is the assortment of BCFSSs, where $\eta = \eta_1, \eta_2, \dots, \eta_{\mathfrak{m}}$ be the WV of specialists $\mathfrak{t}_{\breve{\imath}}$ and $\lambda = \lambda_1, \lambda_2, \dots, \lambda_{\mathfrak{n}}$ be the WV of attributes $\theta_{\breve{\jmath}}$, holding $\eta_{\breve{\imath}} \geq 0, \lambda_{\breve{\jmath}} \geq 0$ such that $\sum_{\breve{\imath}=1}^{\mathfrak{m}}\eta_{\breve{\imath}} = 1$ and $\sum_{\breve{\jmath}=1}^{\mathfrak{n}}\lambda_{\breve{\jmath}} = 1$. Then, $BCFSDWG_{\lambda}$ AO satisfies the following properties:

1. (Idempotency): Let $\psi_{\theta_{\breve{\imath}\breve{\jmath}}} = \left(\mathcal{P}_{\breve{\imath}\breve{\jmath}}^{+}, \mathcal{Q}_{\breve{\imath}\breve{\jmath}}^{-}\right) = \left(\xi_{\breve{\imath}\breve{\jmath}}^{+} + \iota\zeta_{\breve{\imath}\breve{\jmath}}^{+}, \xi_{\breve{\imath}\breve{\jmath}}^{-} + \iota\zeta_{\breve{\imath}\breve{\jmath}}^{-}\right)(\breve{\imath} = 1, 2, \dots, \mathfrak{m}: \breve{\jmath} = 1, 2, \dots, \mathfrak{n})$ is the assortment of BCFSSs, and all are the same i.e. $\psi_{\theta_{\breve{\imath}\breve{\jmath}}} = \psi_{\theta}, \forall\, \breve{\imath}, \breve{\jmath}$, then the following is obtained:

$$BCFSDWG_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{\breve{\imath}\breve{\jmath}}}\right) = \psi_{\theta}$$

2. (Boundedness): Let $\psi_{\theta_{\breve{\imath}\breve{\jmath}}}^{-} = \left(\min_{\breve{\jmath}}\min_{\breve{\imath}}\mathcal{P}_{\breve{\imath}\breve{\jmath}}^{+}, \max_{\breve{\jmath}}\max_{\breve{\imath}}\mathcal{Q}_{\breve{\imath}\breve{\jmath}}^{-}\right)$ and $\psi_{\theta_{\breve{\imath}\breve{\jmath}}}^{+} = \left(\max_{\breve{\jmath}}\max_{\breve{\imath}}\mathcal{P}_{\breve{\imath}\breve{\jmath}}^{+}, \min_{\breve{\jmath}}\min_{\breve{\imath}}\mathcal{Q}_{\breve{\imath}\breve{\jmath}}^{-}\right)$ then,

$$\psi_{\theta_{\breve{\imath}\breve{\jmath}}}^{-} \leq BCFSDWG_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{\breve{\imath}\breve{\jmath}}}\right) \leq \psi_{\theta_{\breve{\imath}\breve{\jmath}}}^{+}$$

3. (Monotonicity): Let $\psi_{\theta_{\breve{\imath}\breve{\jmath}}} = \left(\mathcal{P}_{\breve{\imath}\breve{\jmath}}^{+}, \mathcal{Q}_{\breve{\imath}\breve{\jmath}}^{-}\right) = \left(\xi_{\breve{\imath}\breve{\jmath}}^{+} + \iota\zeta_{\breve{\imath}\breve{\jmath}}^{+}, \xi_{\breve{\imath}\breve{\jmath}}^{-} + \iota\zeta_{\breve{\imath}\breve{\jmath}}^{-}\right)(\breve{\imath} = 1, 2, \dots, \mathfrak{m}: \breve{\jmath} = 1, 2, \dots, \mathfrak{n})$ and $\phi_{\theta_{\breve{\imath}\breve{\jmath}}} = \left(\mathcal{R}_{\breve{\imath}\breve{\jmath}}^{+}, \mathcal{S}_{\breve{\imath}\breve{\jmath}}^{-}\right) = \left(\vartheta_{\breve{\imath}\breve{\jmath}}^{+} + \iota\tau_{\breve{\imath}\breve{\jmath}}^{+}, \vartheta_{\breve{\imath}\breve{\jmath}}^{-} + \iota\tau_{\breve{\imath}\breve{\jmath}}^{-}\right)(\breve{\imath} = 1, 2, \dots, \mathfrak{m}: \breve{\jmath} = 1, 2, \dots, \mathfrak{n})$ are two assortments of BCFSSs, then the following is obtained:

$$BCFSDWG_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{\breve{\imath}\breve{\jmath}}}\right) \leq BCFSDWG_{\lambda}\left(\phi_{\theta_{11}}, \phi_{\theta_{12}}, \dots, \phi_{\theta_{\breve{\imath}\breve{\jmath}}}\right)$$

If $\psi_{\theta_{\breve{\imath}\breve{\jmath}}} \leq \phi_{\theta_{\breve{\imath}\breve{\jmath}}}, \forall\, \breve{\imath}, \breve{\jmath}$.

Definition 17: Let $\psi_{\theta_{\breve{\imath}\breve{\jmath}}} = \left(\mathcal{P}_{\breve{\imath}\breve{\jmath}}^{+}, \mathcal{Q}_{\breve{\imath}\breve{\jmath}}^{-}\right) = \left(\xi_{\breve{\imath}\breve{\jmath}}^{+} + \iota\zeta_{\breve{\imath}\breve{\jmath}}^{+}, \xi_{\breve{\imath}\breve{\jmath}}^{-} + \iota\zeta_{\breve{\imath}\breve{\jmath}}^{-}\right)(\breve{\imath} = 1, 2, \dots, \mathfrak{m}: \breve{\jmath} = 1, 2, \dots, \mathfrak{n})$ is the assortment of BCFSSs, then the BCFSDOWG operator is a function $\psi_{\theta}^{n} \to \psi_{\theta}$ such that

$$BCFSDOWG_{ƛ}(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \psi_{13}, \dots, \psi_{\theta_{ɱn}}) = \otimes_{ǰ=1}^{n} \left(\otimes_{ĭ=1}^{ɱ} \left(\psi_{\theta_{o(ĭǰ)}}\right)^{\eta_{ĭ}}\right)^{ƛ_{ǰ}} \quad (11)$$

Where $\eta = \eta_1, \eta_2, \dots, \eta_{ɱ}$ be the WV of specialists $t_{ĭ}$ and $ƛ = ƛ_1, ƛ_2, \dots, ƛ_{n}$ be the WV of attributes $\theta_{ǰ}$, holding $\eta_{ĭ} \geq 0, ƛ_{ǰ} \geq 0$ such that $\sum_{ĭ=1}^{ɱ} \eta_{ĭ} = 1$ and $\sum_{ǰ=1}^{n} ƛ_{ǰ} = 1$ and $\psi_{\theta_{o(ĭǰ)}} = \left(\mathcal{P}^{+}_{o(ĭǰ)}, \mathcal{Q}^{-}_{o(ĭǰ)}\right)$ is the permutation of the $ĭth$ row and $ǰth$ major elements of the assortment for $ĭ \times ǰ$ BCFSSs $\psi_{\theta_{ĭǰ}} = \left(\mathcal{P}^{+}_{ĭǰ}, \mathcal{Q}^{-}_{ĭǰ}\right)$ for $ĭ = 1, 2, \dots, ɱ$ *and* $ǰ = 1, 2, \dots, n$.

Now, using equation (11) we can define $BCFSDOWG_{ƛ}$ AOs, as trials:

Theorem 10: Let the set BCFSSs, $\psi_{\theta_{ĭǰ}} = \left(\mathcal{P}^{+}_{ĭǰ}, \mathcal{Q}^{-}_{ĭǰ}\right) = \left(\xi^{+}_{ĭǰ} + \iota\zeta^{+}_{ĭǰ}, \xi^{-}_{ĭǰ} + \iota\zeta^{-}_{ĭǰ}\right)$ where $ĭ = 1, 2, \dots, ɱ$ *and* $ǰ = 1, 2, \dots, n$. Let $\eta = \eta_1, \eta_2, \dots, \eta_{ɱ}$ be the WV of specialists $t_{ĭ}$ and $ƛ = ƛ_1, ƛ_2, \dots, ƛ_{n}$ be the WV of attributes $\theta_{ǰ}$, holding $\eta_{ĭ} \geq 0, ƛ_{ǰ} \geq 0$ such that $\sum_{ĭ=1}^{ɱ} \eta_{ĭ} = 1$ and $\sum_{ǰ=1}^{n} ƛ_{ǰ} = 1$. Then, the aggregated outcome for the BCFSDOWG operator is defined as trials:

$$BCFSDOWG_{ƛ}(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \psi_{13}, \dots, \psi_{\theta_{ɱn}}) = \otimes_{ǰ=1}^{n} \left(\otimes_{ĭ=1}^{ɱ} \left(\psi_{\theta_{o(ĭǰ)}}\right)^{\eta_{ĭ}}\right)^{ƛ_{ǰ}}$$

$$= \left(\begin{array}{c} \dfrac{1}{1 + \left\{\sum_{ǰ=1}^{n} ƛ_{ǰ}\left(\sum_{ĭ=1}^{ɱ} \eta_{ĭ}\left(\dfrac{1-\xi^{+}_{o(ĭǰ)}}{\xi^{+}_{o(ĭǰ)}}\right)^{t}\right)\right\}^{\frac{1}{t}}} + \iota\left(\dfrac{1}{1 + \left\{\sum_{ǰ=1}^{n} ƛ_{ǰ}\left(\sum_{ĭ=1}^{ɱ} \eta_{ĭ}\left(\dfrac{1-\zeta^{+}_{o(ĭǰ)}}{\zeta^{+}_{o(ĭǰ)}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right), \\ -1 + \dfrac{1}{1 + \left\{\sum_{ǰ=1}^{n} ƛ_{ǰ}\left(\sum_{ĭ=1}^{ɱ} \eta_{ĭ}\left(\dfrac{\left|\xi^{-}_{o(ĭǰ)}\right|}{1+\xi^{-}_{o(ĭǰ)}}\right)^{t}\right)\right\}^{\frac{1}{t}}} + \iota\left(-1 + \dfrac{1}{1 + \left\{\sum_{ǰ=1}^{n} ƛ_{ǰ}\left(\sum_{ĭ=1}^{ɱ} \eta_{ĭ}\left(\dfrac{\left|\zeta^{-}_{o(ĭǰ)}\right|}{1+\zeta^{-}_{o(ĭǰ)}}\right)^{t}\right)\right\}^{\frac{1}{t}}}\right) \end{array}\right)$$

Where $\psi_{\theta_{o(ĭǰ)}} = \left(\mathcal{P}^{+}_{o(ĭǰ)}, \mathcal{Q}^{-}_{o(ĭǰ)}\right)$ is the permutation of the $ĭth$ row and $ǰth$ major elements of the assortment for $ĭ \times ǰ$ BCFSSs $\psi_{\theta_{ĭǰ}} = \left(\mathcal{P}^{+}_{ĭǰ}, \mathcal{Q}^{-}_{ĭǰ}\right)$ for $ĭ = 1, 2, \dots, ɱ$ *and* $ǰ = 1, 2, \dots, n$.

Proof: The proof is like Theorem 8.

Theorem 11: Let $\psi_{\theta_{ĭǰ}} = \left(\mathcal{P}^{+}_{ĭǰ}, \mathcal{Q}^{-}_{ĭǰ}\right) = \left(\xi^{+}_{ĭǰ} + \iota\zeta^{+}_{ĭǰ}, \xi^{-}_{ĭǰ} + \iota\zeta^{-}_{ĭǰ}\right)(ĭ = 1, 2, \dots, ɱ: ǰ = 1, 2, \dots, n)$ is the assortment of BCFSSs, where $\eta = \eta_1, \eta_2, \dots, \eta_{ɱ}$ be the WV of specialists $t_{ĭ}$ and $ƛ = ƛ_1, ƛ_2, \dots, ƛ_{n}$ be the WV of attributes $\theta_{ǰ}$, holding $\eta_{ĭ} \geq 0, ƛ_{ǰ} \geq 0$ such that $\sum_{ĭ=1}^{ɱ} \eta_{ĭ} = 1$ and $\sum_{ǰ=1}^{n} ƛ_{ǰ} = 1$. Then, $BCFSDOWG_{ƛ}$ AO satisfies the following properties:

1. (Idempotency): Let $\psi_{\theta_{ĭǰ}} = \left(\mathcal{P}^{+}_{ĭǰ}, \mathcal{Q}^{-}_{ĭǰ}\right) = \left(\xi^{+}_{ĭǰ} + \iota\zeta^{+}_{ĭǰ}, \xi^{-}_{ĭǰ} + \iota\zeta^{-}_{ĭǰ}\right)(ĭ = 1, 2, \dots, ɱ: ǰ = 1, 2, \dots, n)$ is the assortment of BCFSSs, and all are the same i.e. $\psi_{\theta_{ĭǰ}} = \psi_{\theta}, \forall\, ĭ, ǰ$, then the following is obtained:

$$BCFSDOWG_{ƛ}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{ĭǰ}}\right) = \psi_{\theta}$$

2. (Boundedness): Let $\psi^{-}_{\theta_{ĭǰ}} = \left(\min_{ǰ}\min_{ĭ} \mathcal{P}^{+}_{ĭǰ}, \max_{ǰ}\max_{ĭ} \mathcal{Q}^{-}_{ĭǰ}\right)$ and $\psi^{+}_{\theta_{ĭǰ}} = \left(\max_{ǰ}\max_{ĭ} \mathcal{P}^{+}_{ĭǰ}, \min_{ǰ}\min_{ĭ} \mathcal{Q}^{-}_{ĭǰ}\right)$ then,

$$\psi^{-}_{\theta_{ĭǰ}} \leq BCFSDOWG_{ƛ}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{ĭǰ}}\right) \leq \psi^{+}_{\theta_{ĭǰ}}$$

3. (Monotonicity): Let $\psi_{\theta_{ĭĵ}} = \left(\mathcal{P}^{+}_{ĭĵ}, \mathcal{Q}^{-}_{ĭĵ}\right) = \left(\xi^{+}_{ĭĵ} + \iota\zeta^{+}_{ĭĵ}, \xi^{-}_{ĭĵ} + \iota\zeta^{-}_{ĭĵ}\right)(ĭ = 1,2,\dots,ꝳ : ĵ = 1,2,\dots,ꞥ)$ and $\phi_{\theta_{ĭĵ}} = \left(\mathcal{R}^{+}_{ĭĵ}, \mathcal{S}^{-}_{ĭĵ}\right) = \left(\vartheta^{+}_{ĭĵ} + \iota\tau^{+}_{ĭĵ}, \vartheta^{-}_{ĭĵ} + \iota\tau^{-}_{ĭĵ}\right)(ĭ = 1,2,\dots,ꝳ : ĵ = 1,2,\dots,ꞥ)$ are two assortments of BCFSSs, then the following is obtained:

$$BCFSDOWG_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \dots, \psi_{\theta_{ĭĵ}}\right) \leq BCFSDOWG_{\lambda}\left(\phi_{\theta_{11}}, \phi_{\theta_{12}}, \dots, \phi_{\theta_{ĭĵ}}\right)$$

If $\psi_{\theta_{ĭĵ}} \leq \phi_{\theta_{ĭĵ}}, \forall\, ĭ, ĵ$.

We can see from definitions (18) and (19) that the BCFSDWG operator solely targets BCFS values, whereas BCFSDOWG only targets ordered situations of BCFS standards slightly more than the weights of the BCFS standards themselves. By combining the qualities of the BCFSDWG and BCFSDOWG, the BCFSDHG operator is defined below.

Definition 18: Let $\psi_{\theta_{ĭĵ}} = \left(\mathcal{P}^{+}_{ĭĵ}, \mathcal{Q}^{-}_{ĭĵ}\right) = \left(\xi^{+}_{ĭĵ} + \iota\zeta^{+}_{ĭĵ}, \xi^{-}_{ĭĵ} + \iota\zeta^{-}_{ĭĵ}\right)(ĭ = 1,2,\dots,ꝳ : ĵ = 1,2,\dots,ꞥ)$ is the assortment of BCFSSs, then the BCFSDHG operator is a function $\psi_{\theta}^{n} \to \psi_{\theta}$ such that

$$BCFSDHWG_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \psi_{13}, \dots, \psi_{\theta_{ꝳꞥ}}\right) = \otimes_{ĵ=1}^{ꞥ}\left(\otimes_{ĭ=1}^{ꝳ}\left(\psi_{\theta_{o(ĭĵ)}}\right)^{\eta_{ĭ}}\right)^{\lambda_{ĵ}} \tag{12}$$

Where $\eta = \eta_1, \eta_2, \dots, \eta_{ꝳ}$ be the WV of specialists $ȶ_{ĭ}$ and $\lambda = \lambda_1, \lambda_2, \dots, \lambda_{ꞥ}$ be the WV of attributes $\theta_{ĵ}$, holding $\eta_{ĭ} \geq 0, \lambda_{ĵ} \geq 0$ such that $\sum_{ĭ=1}^{ꝳ}\eta_{ĭ} = 1$ and $\sum_{ĵ=1}^{ꞥ}\lambda_{ĵ} = 1$. The $\psi_{\theta_{o(ĭĵ)}} = n\partial_{ĭ}\rho_{ĵ}\psi_{\theta_{ĭĵ}}$ is the permutation of the $ĭth$ row and $ĵth$ major elements of the assortment for $ĭ \times ĵ$ BCFSSs $\psi_{\theta_{ĭĵ}} = \left(\mathcal{P}^{+}_{ĭĵ}, \mathcal{Q}^{-}_{ĭĵ}\right)$ and $'n'$ is the matching coefficient. Now, using equation (12) we can define $BCFSDHG_{\lambda}$ AOs, as trials:

Theorem 12: Let the set BCFSSs, $\psi_{\theta_{ĭĵ}} = \left(\mathcal{P}^{+}_{ĭĵ}, \mathcal{Q}^{-}_{ĭĵ}\right) = \left(\xi^{+}_{ĭĵ} + \iota\zeta^{+}_{ĭĵ}, \xi^{-}_{ĭĵ} + \iota\zeta^{-}_{ĭĵ}\right)$, Where $ĭ = 1,2,\dots,ꝳ$ *and* $ĵ = 1,2,\dots,ꞥ$. Let $\eta = \eta_1, \eta_2, \dots, \eta_{ꝳ}$ be the WV of specialists $ȶ_{ĭ}$ and $\lambda = \lambda_1, \lambda_2, \dots, \lambda_{ꞥ}$ be the WV of attributes $\theta_{ĵ}$, holding $\eta_{ĭ} \geq 0, \lambda_{ĵ} \geq 0$ such that $\sum_{ĭ=1}^{ꝳ}\eta_{ĭ} = 1$ and $\sum_{ĵ=1}^{ꞥ}\lambda_{ĵ} = 1$. Then, the aggregated outcome for the BCFSDHWG operator is defined as trials:

$$BCFSDHWG_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \psi_{13}, \dots, \psi_{\theta_{ꝳꞥ}}\right) = \otimes_{ĵ=1}^{ꞥ}\left(\otimes_{ĭ=1}^{ꝳ}\left(\psi_{\theta_{o(ĭĵ)}}\right)^{\eta_{ĭ}}\right)^{\lambda_{ĵ}}$$

$$= \left(\begin{array}{c} \dfrac{1}{1 + \left\{\sum_{ĵ=1}^{ꞥ}\lambda_{ĵ}\left(\sum_{ĭ=1}^{ꝳ}\eta_{ĭ}\left(\dfrac{1-\xi^{+}_{o(ĭĵ)}}{\xi^{+}_{o(ĭĵ)}}\right)^{ŧ}\right)\right\}^{\frac{1}{ŧ}}} + \iota\left(\dfrac{1}{1 + \left\{\sum_{ĵ=1}^{ꞥ}\lambda_{ĵ}\left(\sum_{ĭ=1}^{ꝳ}\eta_{ĭ}\left(\dfrac{1-\zeta^{+}_{o(ĭĵ)}}{\zeta^{+}_{o(ĭĵ)}}\right)^{ŧ}\right)\right\}^{\frac{1}{ŧ}}}\right), \\ -1 + \dfrac{1}{1 + \left\{\sum_{ĵ=1}^{ꞥ}\lambda_{ĵ}\left(\sum_{ĭ=1}^{ꝳ}\eta_{ĭ}\left(\dfrac{\left|\xi^{-}_{o(ĭĵ)}\right|}{1+\xi^{-}_{o(ĭĵ)}}\right)^{ŧ}\right)\right\}^{\frac{1}{ŧ}}} + \iota\left(-1 + \dfrac{1}{1 + \left\{\sum_{ĵ=1}^{ꞥ}\lambda_{ĵ}\left(\sum_{ĭ=1}^{ꝳ}\eta_{ĭ}\left(\dfrac{\left|\zeta^{-}_{o(ĭĵ)}\right|}{1+\zeta^{-}_{o(ĭĵ)}}\right)^{ŧ}\right)\right\}^{\frac{1}{ŧ}}}\right) \end{array}\right)$$

Where $\psi_{\theta_{o(ĭĵ)}} = n\partial_{ĭ}\rho_{ĵ}\psi_{\theta_{ĭĵ}}$ is the permutation of the $ĭth$ row and $ĵth$ major elements of the assortment for $ĭ \times ĵ$ BCFSSs $\psi_{\theta_{ĭĵ}} = \left(\mathcal{P}^{+}_{ĭĵ}, \mathcal{Q}^{-}_{ĭĵ}\right)$ and $'n'$ is the matching coefficient.

Proof: The proof theorem is like Theorem 8.

Theorem 13: Let $\psi_{\theta_{ĭj̆}} = \left(\mathcal{P}_{ĭj̆}^{+}, \mathcal{Q}_{ĭj̆}^{-}\right) = \left(\xi_{ĭj̆}^{+} + \iota\zeta_{ĭj̆}^{+}, \xi_{ĭj̆}^{-} + \iota\zeta_{ĭj̆}^{-}\right)(ĭ = 1, 2, \ldots, m: j̆ = 1, 2, \ldots, n)$ is the assortment of BCFSSs, where $\eta = \eta_1, \eta_2, \ldots, \eta_m$ be the WV of specialists $t_ĭ$ and $\lambda = \lambda_1, \lambda_2, \ldots, \lambda_n$ be the WV of attributes $\theta_j̆$, holding $\eta_ĭ \geq 0, \lambda_j̆ \geq 0$ such that $\sum_{ĭ=1}^{m} \eta_ĭ = 1$ and $\sum_{j̆=1}^{n} \lambda_j̆ = 1$. Then, $BCFSDHWG_{\lambda}$ AO satisfies the following properties:

1. (Idempotency): Let $\psi_{\theta_{ĭj̆}} = \left(\mathcal{P}_{ĭj̆}^{+}, \mathcal{Q}_{ĭj̆}^{-}\right) = \left(\xi_{ĭj̆}^{+} + \iota\zeta_{ĭj̆}^{+}, \xi_{ĭj̆}^{-} + \iota\zeta_{ĭj̆}^{-}\right)(ĭ = 1, 2, \ldots, m: j̆ = 1, 2, \ldots, n)$ is the assortment of BCFSSs, and all are the same i.e. $\psi_{\theta_{ĭj̆}} = \psi_{\theta}, \forall\, ĭ, j̆$, then the following is obtained:

$$BCFSDHWG_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \ldots, \psi_{\theta_{ĭj̆}}\right) = \psi_{\theta}$$

2. (Boundedness): Let $\psi_{\theta_{ĭj̆}}^{-} = \left(\min_{j̆} \min_{ĭ} \mathcal{P}_{ĭj̆}^{+}, \max_{j̆} \max_{ĭ} \mathcal{Q}_{ĭj̆}^{-}\right)$ and $\psi_{\theta_{ĭj̆}}^{+} = \left(\max_{j̆} \max_{ĭ} \mathcal{P}_{ĭj̆}^{+}, \min_{j̆} \min_{ĭ} \mathcal{Q}_{ĭj̆}^{-}\right)$ then,

$$\psi_{\theta_{ĭj̆}}^{-} \leq BCFSDHWG_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \ldots, \psi_{\theta_{ĭj̆}}\right) \leq \psi_{\theta_{ĭj̆}}^{+}$$

3. (Monotonicity): Let $\psi_{\theta_{ĭj̆}} = \left(\mathcal{P}_{ĭj̆}^{+}, \mathcal{Q}_{ĭj̆}^{-}\right) = \left(\xi_{ĭj̆}^{+} + \iota\zeta_{ĭj̆}^{+}, \xi_{ĭj̆}^{-} + \iota\zeta_{ĭj̆}^{-}\right)(ĭ = 1, 2, \ldots, m: j̆ = 1, 2, \ldots, n)$ and $\phi_{\theta_{ĭj̆}} = \left(\mathcal{R}_{ĭj̆}^{+}, \mathcal{S}_{ĭj̆}^{-}\right) = \left(\vartheta_{ĭj̆}^{+} + \iota\tau_{ĭj̆}^{+}, \vartheta_{ĭj̆}^{-} + \iota\tau_{ĭj̆}^{-}\right)(ĭ = 1, 2, \ldots, m: j̆ = 1, 2, \ldots, n)$ are two assortments of BCFSSs, then the following is obtained:

$$BCFSDHWG_{\lambda}\left(\psi_{\theta_{11}}, \psi_{\theta_{12}}, \ldots, \psi_{\theta_{ĭj̆}}\right) \leq BCFSDHWG_{\lambda}\left(\phi_{\theta_{11}}, \phi_{\theta_{12}}, \ldots, \phi_{\theta_{ĭj̆}}\right)$$

If $\psi_{\theta_{ĭj̆}} \leq \phi_{\theta_{ĭj̆}}, \forall\, ĭ, j̆$.

Remark 2.

1. When $\lambda = \left(\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$ then the BCFSDHG operators covert into the BCFSDWG.
2. When $\partial = \left(\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$ then the BCFSDHG operators covert into the BCFSDOG.

6 | THE WASPAS USING BCFS INFORMATION

Now, here we want to discuss the applications of agricultural robotics systems. There are ten applications of agricultural robotics systems but we discuss only four applications:

Autonomous Precision Seedings

Sowing seeds is the first step in beginning a farming operation. To disperse seeds, farmers have long used their hands. A 'broadcast spreader' connected to a tracker was used by farmers to sprinkle their crops while cutting-edge equipment was in operation. Even though the procedure became simple, these additional characteristics wasted all of the seeds by dispersing them widely. To help, autonomous precision sowing is fortunately available. The mechanism uses a combination of robotics and geo-mapping to place the seeds exactly where they must be for proper growth.

Multi-Talented Robots for Harvesting

Humans are frequently replaced by robots to complete repetitive activities. At picking and harvesting, they operate in this manner. To obtain useful food products, harvesting must be done, which is a tedious task. Robots are assuming control of the process to relieve people of these tiresome jobs. While fundamental food crops like wheat and barley may be planted and harvested by robots with ease, other tasks, like picking fruit and vegetables, require multi-skilled robots.

Micro-Spraying Robots

The majority of the pesticide repellant that is sprayed on the plants ends up damaging the soil. There is less of a possibility for the future plantation to escape the chemicals even when the land is continuously churned to alter its texture. Additionally, it harms the ecosystem. So, to focus on the effects, farmers are deploying micro-spraying robots. Micro-spraying robots can detect weeds using cutting-edge computer technology and then spray a precise drop of herbicide onto them.

Robotic Automation Process (RPA)

Plan devotes often choose nursery planting. We can obtain the necessary crops for our daily consumption by cultivating them in our backyards. To regularly groom and water them, though, is rather a hassle. The robotic automation process performs at its peak in this situation. All nursery planting tasks, such as watering at regular intervals and harvesting ripe vegetables or fruits, are handled by robotic process automation.

6.1| Algorithm WASPAS Method

The weighted sum (WS) and weighted product (WP) techniques are two well-known methodologies that are integrated into the WAPAS method. Here, we have the set of specialists $\mathfrak{t} = \{\mathfrak{t}_1, \mathfrak{t}_2, \dots, \mathfrak{t}_{\mathfrak{m}}\}$ and $\theta = \{\theta_1, \theta_2, \dots, \theta_{\mathfrak{n}}\}$ is the set of attributes. There are some WVs corresponding sets of above, then $\eta = (\eta_1, \eta_2, \dots, \eta_{\mathfrak{m}})$ be the WV of specialists $\mathfrak{t}_{\check{\imath}}$ and $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_{\mathfrak{n}}\}$ denotes the WVs for $\theta_{\check{\jmath}}$ attributes and $\mathrm{t} = 2$, where $(\check{\imath} = 1, 2, \dots, \mathfrak{m}: \check{\jmath} = 1, 2, \dots, \mathfrak{n})$. Following is a description of the WASPAS method's computational process for DM problems containing solely favorable criteria. First, we have a table:

Step 1. Let $\psi_{\theta_{\check{\imath}\check{\jmath}}} = (\mathcal{P}^+_{\check{\imath}\check{\jmath}}, \mathcal{Q}^-_{\check{\imath}\check{\jmath}}) = (\xi^+_{\check{\imath}\check{\jmath}} + \iota\zeta^+_{\check{\imath}\check{\jmath}}, \xi^-_{\check{\imath}\check{\jmath}} + \iota\zeta^-_{\check{\imath}\check{\jmath}})(\check{\imath} = 1, 2, \dots, \mathfrak{m}: \check{\jmath} = 1, 2, \dots, \mathfrak{n})$, then determine the option performance rating for each attribute as trials:

$$\mathcal{P}^+_{0\check{\jmath}} = \max_{\check{\imath}} \xi^+_{\check{\imath}\check{\jmath}} + \iota \max_{\check{\imath}} \zeta^+_{\check{\imath}\check{\jmath}}, \mathcal{Q}^-_{0\check{\jmath}} = \max_{\check{\imath}} \xi^-_{\check{\imath}\check{\jmath}} + \iota \max_{\check{\imath}} \zeta^-_{\check{\imath}\check{\jmath}} \qquad (13)$$

Where $\psi_{\theta_{0\check{\jmath}}}$ denotes the optimal performance rating for each $\check{\jmath} - th$ attribute, $\psi_{\theta_{\check{\imath}\check{\jmath}}}$ signified the performance rating of $\check{\imath} - th$ alternative in relation to the $\check{\jmath} - th$ attribute.

Table 3. Bipolar Complex Fuzzy Soft Set

$\psi_{\theta_{\check{\imath}\check{\jmath}}}$	θ_1	θ_2	…	$\theta_{\mathfrak{n}}$
$\mathfrak{t}_1$	$\begin{pmatrix}\xi^+_{11} + \iota\zeta^+_{11}, \\ \xi^-_{11} + \iota\zeta^-_{11}\end{pmatrix}$	$\begin{pmatrix}\xi^+_{12} + \iota\zeta^+_{12}, \\ \xi^-_{12} + \iota\zeta^-_{12}\end{pmatrix}$	…	$\begin{pmatrix}\xi^+_{1\mathfrak{n}} + \iota\zeta^+_{1\mathfrak{n}}, \\ \xi^-_{1\mathfrak{n}} + \iota\zeta^-_{1\mathfrak{n}}\end{pmatrix}$
$\mathfrak{t}_2$	$\begin{pmatrix}\xi^+_{21} + \iota\zeta^+_{21}, \\ \xi^-_{21} + \iota\zeta^-_{21}\end{pmatrix}$	$\begin{pmatrix}\xi^+_{22} + \iota\zeta^+_{22}, \\ \xi^-_{22} + \iota\zeta^-_{22}\end{pmatrix}$	…	$\begin{pmatrix}\xi^+_{2\mathfrak{n}} + \iota\zeta^+_{2\mathfrak{n}}, \\ \xi^-_{2\mathfrak{n}} + \iota\zeta^-_{2\mathfrak{n}}\end{pmatrix}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\mathfrak{t}_{\mathfrak{m}}$	$\begin{pmatrix}\xi^+_{\mathfrak{m}1} + \iota\zeta^+_{\mathfrak{m}1}, \\ \xi^-_{\mathfrak{m}1} + \iota\zeta^-_{\mathfrak{m}1}\end{pmatrix}$	$\begin{pmatrix}\xi^+_{\mathfrak{m}2} + \iota\zeta^+_{\mathfrak{m}2}, \\ \xi^-_{\mathfrak{m}2} + \iota\zeta^-_{\mathfrak{m}2}\end{pmatrix}$	…	$\begin{pmatrix}\xi^+_{\mathfrak{m}\mathfrak{n}} + \iota\zeta^+_{\mathfrak{m}\mathfrak{n}}, \\ \xi^-_{\mathfrak{m}\mathfrak{n}} + \iota\zeta^-_{\mathfrak{m}\mathfrak{n}}\end{pmatrix}$

Step 2. Making the normalized matrix, as trials:

$$r_{\check{\imath}\check{\jmath}} = \frac{\psi_{\theta_{\check{\imath}\check{\jmath}}}}{\psi_{\theta_{0\check{\jmath}}}} = \frac{\left(\mathcal{P}^{+}_{\check{\imath}\check{\jmath}}, \mathcal{Q}^{-}_{\check{\imath}\check{\jmath}}\right)}{\left(\mathcal{P}^{+}_{0\check{\jmath}}, \mathcal{Q}^{-}_{0\check{\jmath}}\right)} = \frac{\left(\xi^{+}_{\check{\imath}\check{\jmath}} + \iota\zeta^{+}_{\check{\imath}\check{\jmath}}, \xi^{-}_{\check{\imath}\check{\jmath}} + \iota\zeta^{-}_{\check{\imath}\check{\jmath}}\right)}{\left(1 + \max_{\check{\imath}} \xi^{+}_{\check{\imath}\check{\jmath}} + \iota\left(1 + \max_{\check{\imath}} \zeta^{+}_{\check{\imath}\check{\jmath}}\right), 1 - \max_{\check{\imath}} \xi^{-}_{\check{\imath}\check{\jmath}} + \iota\left(1 - \max_{\check{\imath}} \zeta^{-}_{\check{\imath}\check{\jmath}}\right)\right)} \tag{14}$$

Where $r_{\check{\imath}\check{\jmath}}$ signified the normalized performance rating of $\check{\imath} - th$ alternative in relation to the $\check{\jmath} - th$ attribute.

Step 3. Calculate the significance of each alternative invented on the WS method

$$\mathbb{Q}^{WS}_{\check{\imath}} = \left(\begin{array}{c} 1 - \dfrac{1}{1 + \left\{\sum_{\check{\jmath}=1}^{\mathfrak{n}} \lambda_{\check{\jmath}} \left(\sum_{\check{\imath}=1}^{\mathfrak{m}} \eta_{\check{\imath}} \left(\dfrac{r^{+}_{\check{\imath}\check{\jmath}}}{1 - r^{+}_{\check{\imath}\check{\jmath}}}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}} + \iota\left(1 - \dfrac{1}{1 + \left\{\sum_{\check{\jmath}=1}^{\mathfrak{n}} \lambda_{\check{\jmath}} \left(\sum_{\check{\imath}=1}^{\mathfrak{m}} \eta_{\check{\imath}} \left(\dfrac{r^{+}_{\check{\imath}\check{\jmath}}}{1 - r^{+}_{\check{\imath}\check{\jmath}}}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}}\right), \\ \dfrac{-1}{1 + \left\{\sum_{\check{\jmath}=1}^{\mathfrak{n}} \lambda_{\check{\jmath}} \left(\sum_{\check{\imath}=1}^{\mathfrak{m}} \eta_{\check{\imath}} \left(\dfrac{1 + r^{-}_{\check{\imath}\check{\jmath}}}{\left|r^{-}_{\check{\imath}\check{\jmath}}\right|}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}} + \iota\left(\dfrac{-1}{1 + \left\{\sum_{\check{\jmath}=1}^{\mathfrak{n}} \lambda_{\check{\jmath}} \left(\sum_{\check{\imath}=1}^{\mathfrak{m}} \eta_{\check{\imath}} \left(\dfrac{1 + r^{-}_{\check{\imath}\check{\jmath}}}{\left|r^{-}_{\check{\imath}\check{\jmath}}\right|}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}}\right) \end{array}\right)$$

Step 4. Calculate the significance of each alternative invented on the WP method

$$\mathbb{Q}^{WP}_{\check{\imath}} = \left(\begin{array}{c} \dfrac{1}{1 + \left\{\sum_{\check{\jmath}=1}^{\mathfrak{n}} \lambda_{\check{\jmath}} \left(\sum_{\check{\imath}=1}^{\mathfrak{m}} \eta_{\check{\imath}} \left(\dfrac{1 - r^{+}_{\check{\imath}\check{\jmath}}}{r^{+}_{\check{\imath}\check{\jmath}}}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}} + \iota\left(\dfrac{1}{1 + \left\{\sum_{\check{\jmath}=1}^{\mathfrak{n}} \lambda_{\check{\jmath}} \left(\sum_{\check{\imath}=1}^{\mathfrak{m}} \eta_{\check{\imath}} \left(\dfrac{1 - r^{+}_{\check{\imath}\check{\jmath}}}{r^{+}_{\check{\imath}\check{\jmath}}}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}}\right), \\ -1 + \dfrac{1}{1 + \left\{\sum_{\check{\jmath}=1}^{\mathfrak{n}} \lambda_{\check{\jmath}} \left(\sum_{\check{\imath}=1}^{\mathfrak{m}} \eta_{\check{\imath}} \left(\dfrac{\left|r^{-}_{\check{\imath}\check{\jmath}}\right|}{1 + r^{-}_{\check{\imath}\check{\jmath}}}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}} + \iota\left(\dfrac{-1}{1 + \left\{\sum_{\check{\jmath}=1}^{\mathfrak{n}} \lambda_{\check{\jmath}} \left(\sum_{\check{\imath}=1}^{\mathfrak{m}} \eta_{\check{\imath}} \left(\dfrac{\left|r^{-}_{\check{\imath}\check{\jmath}}\right|}{1 + r^{-}_{\check{\imath}\check{\jmath}}}\right)^{\mathfrak{t}}\right)\right\}^{\frac{1}{\mathfrak{t}}}}\right) \end{array}\right)$$

Step 5. Calculate the overall importance of each alternative $\mathbb{Q}_{\check{\imath}}$ as trials:

$$\mathbb{Q}_{\check{\imath}} = \frac{1}{2}\left(\mathbb{Q}^{WS}_{\check{\imath}} + \mathbb{Q}^{WP}_{\check{\imath}}\right) \tag{17}$$

6.2| Numerical Example

Example 3: A robotic system works in the agricultural field and satisfies all observations in agriculture. Let $\mathfrak{t} = \{\mathfrak{t}_1, \mathfrak{t}_2, \mathfrak{t}_3, \mathfrak{t}_4\}$ be set of specialists who calculate the progress of Robots θ using the attribute set given as $\theta = \{\theta_1 = Autonomous\ precision\ seeding, \theta_2 = Multi - talented\ robots\ for\ harvesting, \theta_3 = Micro - spraying\ robots, \theta_4 = Robotic\ automation\ process\}$. Let $\eta = (0.22, 0.31, 0.23, 0.24)$ be the WV of specialists $\mathfrak{t}_{\check{\imath}}$ and $\lambda = \{0.31, 0.24, 0.23, 0.22\}$ denotes the WVs for $\theta_{\check{\jmath}}$ attributes and $\mathfrak{t} = 2$. Data has been given in Table 4.

Table 4. Bipolar Complex Fuzzy Soft Set Data

$\psi_{\theta_{\mathfrak{i}\mathfrak{j}}}$	θ_1	θ_2	θ_3	θ_4
$\mathfrak{t}_1$	$\begin{pmatrix}0.7+\iota 0.8,\\ -0.3-\iota 0.6\end{pmatrix}$	$\begin{pmatrix}0.7+\iota 0.7,\\ -0.4-\iota 0.5\end{pmatrix}$	$\begin{pmatrix}0.8+\iota 0.7,\\ -0.6-\iota 0.1\end{pmatrix}$	$\begin{pmatrix}0.6+\iota 0.6,\\ -0.3-\iota 0.4\end{pmatrix}$
$\mathfrak{t}_2$	$\begin{pmatrix}0.8+\iota 0.7,\\ -0.1-\iota 0.5\end{pmatrix}$	$\begin{pmatrix}0.6+\iota 0.5,\\ -0.1-\iota 0.3\end{pmatrix}$	$\begin{pmatrix}0.8+\iota 0.8,\\ -0.5-\iota 0.2\end{pmatrix}$	$\begin{pmatrix}0.6+\iota 0.5,\\ -0.1-\iota 0.3\end{pmatrix}$
$\mathfrak{t}_3$	$\begin{pmatrix}0.7+\iota 0.6,\\ -0.3-\iota 0.4\end{pmatrix}$	$\begin{pmatrix}0.7+\iota 0.6,\\ -0.5-\iota 0.2\end{pmatrix}$	$\begin{pmatrix}0.7+\iota 0.7,\\ -0.4-\iota 0.3\end{pmatrix}$	$\begin{pmatrix}0.7+\iota 0.6,\\ -0.3-\iota 0.4\end{pmatrix}$
$\mathfrak{t}_4$	$\begin{pmatrix}0.6+\iota 0.7,\\ -0.2-\iota 0.3\end{pmatrix}$	$\begin{pmatrix}0.7+\iota 0.7,\\ -0.4-\iota 0.1\end{pmatrix}$	$\begin{pmatrix}0.6+\iota 0.6,\\ -0.3-\iota 0.4\end{pmatrix}$	$\begin{pmatrix}0.7+\iota 0.7,\\ -0.4-\iota 0.5\end{pmatrix}$

Step 1. By equation 13 we get

$$\mathcal{P}_{0,1}^{+}=\max_{1}(0.7,0.8,0.7,0.6)+\iota\max_{1}(0.8,0.7,0.6,0.7)=0.8+\iota 0.8$$

$$\mathcal{Q}_{0,1}^{-}=\max_{1}(-0.3,-0.1,-0.3,-0.2)+\iota\max_{1}(-0.6,-0.5,-0.4,-0.3)=-0.1+\iota(-0.3)$$

$$\mathcal{P}_{0,2}^{+}=\max_{2}(0.7,0.6,0.7,0.7)+\iota\max_{2}(0.7,0.5,0.6,0.7)=0.7+\iota 0.7$$

$$\mathcal{Q}_{0,2}^{-}=\max_{2}(-0.4,-0.1,-0.5,-0.4)+\iota\max_{2}(-0.5,-0.3,-0.2,-0.1)=-0.1+\iota(-0.1)$$

$$\mathcal{P}_{0,3}^{+}=\max_{3}(0.8,0.8,0.7,0.6)+\iota\max_{3}(0.7,0.8,0.7,0.6)=0.8+\iota 0.8$$

$$\mathcal{Q}_{0,3}^{-}=\max_{3}(-0.6,-0.5,-0.4,-0.3)+\iota\max_{3}(-0.1,-0.2,-0.3,-0.4)=-0.3+\iota(-0.1)$$

$$\mathcal{P}_{0,4}^{+}=\max_{4}(0.6,0.6,0.7,0.7)+\iota\max_{4}(0.6,0.5,0.6,0.7)=0.7+\iota 0.7$$

$$\mathcal{Q}_{0,4}^{-}=\max_{4}(-0.3,-0.1,-0.3,-0.4)+\iota\max_{4}(-0.4,-0.3,-0.4,-0.5)=-0.1+\iota(-0.3)$$

Step 2. By equation 14 we have specified in Table 5.

Table 5. Normalized Matrix

$\psi_{\theta_{\mathfrak{i}\mathfrak{j}}}$	θ_1	θ_2	θ_3	θ_4
$\mathfrak{t}_1$	$\begin{pmatrix}0.388+\iota 0.444,\\ -0.272-\iota 0.461\end{pmatrix}$	$\begin{pmatrix}0.411+\iota 0.411,\\ -0.363-\iota 454\end{pmatrix}$	$\begin{pmatrix}0.444+\iota 0.388,\\ -0.461-\iota 0.090\end{pmatrix}$	$\begin{pmatrix}0.352+\iota 0.352,\\ -0.272-\iota 0.307\end{pmatrix}$
$\mathfrak{t}_2$	$\begin{pmatrix}0.444+\iota 0.388,\\ -0.09-\iota 0.384\end{pmatrix}$	$\begin{pmatrix}0.352+\iota 0.294,\\ -0.090-\iota 0.363\end{pmatrix}$	$\begin{pmatrix}0.444+\iota 0.444,\\ -0.384-\iota 0.181\end{pmatrix}$	$\begin{pmatrix}0.352+\iota 0.294,\\ -0.090-\iota 0.230\end{pmatrix}$
$\mathfrak{t}_3$	$\begin{pmatrix}0.388+\iota 0.333,\\ -0.272-\iota 0.307\end{pmatrix}$	$\begin{pmatrix}0.411+\iota 0.352,\\ -0.454-\iota 0.181\end{pmatrix}$	$\begin{pmatrix}0.388+\iota 0.388,\\ -0.307-\iota 0.272\end{pmatrix}$	$\begin{pmatrix}0.411+\iota 0.352,\\ -0.272-\iota 0.307\end{pmatrix}$
$\mathfrak{t}_4$	$\begin{pmatrix}0.333+\iota 0.388,\\ -0.181-\iota 0.230\end{pmatrix}$	$\begin{pmatrix}0.411+\iota 0.411,\\ -0.363-\iota 0.090\end{pmatrix}$	$\begin{pmatrix}0.333+\iota 0.333,\\ -0.230-\iota 0.363\end{pmatrix}$	$\begin{pmatrix}0.411+\iota 0.411,\\ -0.363-\iota 0.384\end{pmatrix}$

Step 3. By equation 15 we have the values of $\mathbb{Q}_{\mathfrak{i}}^{WS}$ as trials:

$\mathbb{Q}_1^{WS}=(0.005445,0.004794,-0.62825,-0.97558)$,
$\mathbb{Q}_2^{WS}=(0.006266,0.0033,-0.93667,-0.93514)$,
$\mathbb{Q}_3^{WS}=(0.006848,0.004794,-0.97505,-0.72494)$,
$\mathbb{Q}_4^{WS}=(0.004884,0.002571,-0.81335,-0.95391)$

Step 4. By equation 16 we have the values of $\mathbb{Q}_{\mathfrak{i}}^{WP}$ as trials:

$\mathbb{Q}_1^{WP}=(0.9871,0.988622,-0.000006,-0.00012)$,
$\mathbb{Q}_2^{WP}=(0.988771,0.983532,-0.00004,-0.00009)$,
$\mathbb{Q}_3^{WP}=(0.989722,0.988622,-0.00013,-0.000008)$,
$\mathbb{Q}_4^{WP}=(0.985621,0.978944,-0.00001,-0.00006)$

Step 5. Score values of

$\mathbb{Q}_{\mathfrak{i}}^{WS}=(-0.2734,-0.34056,-0.297087,-0.31495)$ and
$\mathbb{Q}_{\mathfrak{i}}^{WP}=(0.618899,0.618043,0.619552,0.616124)$, then by equation 17, we have
$\mathbb{Q}_1=0.172751$, $\mathbb{Q}_2=0.138741$, $\mathbb{Q}_3=0.1612323$, $\mathbb{Q}_4=0.150586$, then their rating is

$$\mathbb{Q}_1>\mathbb{Q}_3>\mathbb{Q}_4>\mathbb{Q}_2$$

There is the best performance of $\mathbb{Q}_1$ robot.

7| COMPARATIVE ANALYSIS

To demonstrate the correctness and superiority of the exhibited work, this section contains a comparison of the invented work with existing methodologies.

We imply a contrast between the presented work and Seikh and Mandal [8] and Jana et al. [14]. The intuitionistic fuzzy (IF) Dombi AOs were denoted by Seikh and Mandal [8] and the BF Dombi AOs were shown by Jana et al. [14]. Assume that Table 5 has the data. The information found in Table 5 is 2-dimensional and is available in the context of BCFSSs (both PTG and NTG have phase and amplitude terms). As far as we are aware, PTG and NTG are the only types of one-dimensional data that Jana et al. [14] can handle. This makes it evident that the Dombi operator presented by Jana et al. [14] is incapable of retaining data that includes the amplitude term, which means that Jana et al. [14] are unable to determine the WASPAS in the presence of BCFSSs. Additionally, Seikh and Mandal [8] only can deal with one-dimensional data (i.e., phase term) with TG and nTG when both TG and nTG belong to the [0, 1]. We received no information from Seikh and Mandal [8] regarding the negative aspect and amplitude term of these grades. This makes it obvious that Seikh and Mandal [8] are unable to hold the data that provides the negative aspect amplitude term, i.e., they are unable to resolve the WASPAS problems in the context of BCFSSs. Purely, the problems with WASPAS can be fixed thanks to our proven work. This suggests that the signified work is more adaptable and influential than the current work. The BCFS Dombi AOs are more general than the existing work because they become bipolar fuzzy soft Dombi AOs when we set the amplitude term equal to zero in both PTG and NTG, and fuzzy soft dombi AOs when we set the amplitude term equal to zero in PTG and ignore the NTG part. Table 6 details the score values and evaluation outcomes for the demonstrated and prevalent work.

Table 6. Score values and rating outcomes of demonstrated and prevailing work

Methods	**Score values**	**Rating**
Seikh and Mandal [8]	Failed	Failed
Jana et al. [14]	Failed	Failed
Proposed work	$\mathbb{Q}_1 = 0.172751$, $\mathbb{Q}_2 = 0.138741$, $\mathbb{Q}_3 = 0.1612323$, $\mathbb{Q}_4 = 0.150586$	$\mathbb{Q}_1 > \mathbb{Q}_3 > \mathbb{Q}_4 > \mathbb{Q}_2$

8 | CONCLUSION

One of the most important and prevalent methods for resolving problems involving irrelevant and inconsistent information in everyday life is the theory of BCFS information. In this study, we elaborated AOs for BCFSSs that use Tamir et al. [20]'s CFS assessment rather than Ramot et al. [43]'s CFS assessment. Below, it is discussed how this study most significantly affected:

1. We explained how BCFS Dombi weighted averaging (BCFSDWA), BCFS Dombi ordered weighted averaging (BCFSDOWA), BCFS Dombi hybrid averaging (BCFSDHA), BCFS Dombi

weighted geometric (BCFSDWG), BCFS Dombi ordered weighted geometric (BCFSDOWG), and BCFS Dombi hybrid geometric (BCFSDHG) AOs work, as well as their associated properties.

2. A WASPAS is a component of the DM process that was used to identify the best prime from among the expert and attribute families. Based on the aforementioned study, we applied the theoretical WASPAS method by applying the BCFSSs.
3. As a final step, we interpret a numerical model to support the method that has been shown to work. To highlight the value and applicability of our research, we compare it to existing research.

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