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Multi-attribute decision-making by using intuitionistic Fuzzy rough Aczel-Alsina prioritize Aggregation Operator

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ABSTRACT

The management of uncertain and symmetric information is a difficult task. To sort out this issue, multi-attribute decision-making (MADM) is a crucial methodology in decision-making (DM) sciences in which we can select the most suitable and reasonable alternative from uncertain information. Furthermore, this article proposes prioritized operators by allowing the positive real value known as priority degree among the strict priority levels. In addition, Aczel-Alsina (AA) t-norm (TNM) (AATNM) and AA t-conorm (TCNM) (AATCNM) are newly proposed concepts in fuzzy set (FS) theory. However, when experts use AATNM and AATCNM operation for rough FS (RFS), these operational rules fail to aggregate information because it has lower and upper approximations spaces. Thus, an encasement of an intuitionistic FS (IFS) has many chances to lose information. Still, on the other hand, the intuitionistic fuzzy rough (IFR) set (IFRS) has resolved the data loss issue. By taking advantage of IFRS structure, priority degree, and AATNM, AATCNM, we constructed a new operation for IFR values (IFRVs). Also, proposed new aggregation operators (AOs) called IFR Aczel-Alsina prioritized weighted averaging (IFRAAPWA) and IFR Aczel-Alsina prioritized weighted geometric (IFRAAPWG) operators. Some necessary axioms of proposed AOs are also discussed. The MADM approach is developed based on the proposed AOs and utilized in medical diagnosis (MD) problems. The characteristics of the proposed technique are also compared with many present methodologies, which highlight the importance and superiority of suggested work over the prevailing approaches. The effect of parameters on aggregation findings is also investigated, which plays a vital role in outcomes.

Keywords: multi-attribute group decision-making, intuitionistic fuzzy set, priority degree, aggregation operators

1 | INTRODUCTION

In ancient times, doctors prescribed medicine to patients by investigating clinical symptoms and observational bases. The old doctors suggested medicines by observing their ears, eyes, and human specimens and providing the medicines based on the first MD. In the medieval era, medical experts used many uses of diagnostic phases. In the history of the Greeks, they believed that all types of diseases were caused by the disturbance of fluid in the human body, known as humor. But later on, the invention of the microscope revealed both concepts, the pathogenic microorganism and cellular makeup of human tissues. At the end of the 19th century, many advancements in MD methods and instruments were introduced, such as thermometers and stethoscopes for measuring the human body's temperature and investigating the body's blood pressure, respectively. And then, in the field of medicine, clinical laboratories would not be commonly used until the turn of the 20th century.

We need to be aware of any disease or any unexpected change in our human body. This could be an enlarging waistline or even a persistent cough. Symptoms are frequently nothing to worry about, but occasionally they require more investigation. MD is a methodology used to investigate the nature of the disease and separate it from other possible conditions. The word Diagnosis is derived from Greek scientists, which means "knowledge." They appear at the start of the disease, giving uncertain and vague information. In this situation, it is tough to investigate to prescribe accurate medicines for the patients.

The concept of FS theory is defined by Zadeh [1], which is the generalized form of classical set theory (CST) and characterized by membership value (MV) belonging to the unit interval $[0,1]$. Following the same idea as the CST, relations and operations can be defined for FS theory (FST). The FST had many applications in several disciplines. For example, the use of FST in the field of artificial intelligence [2], medical sciences [3], statistical analysis of data [4], and investigation of medical disease [5]. And cluster analysis [6]. Few mathematicians proposed the notion of AOs, such as Fahmi et al. [7], who addressed the notion of fuzzy Einstein AOs and their uses in DM approaches.

The idea of IFS was first presented by Atanassov [8], which is characterized by non-membership value (NMV) along with MV The range of both MV and NMV belongs form the unit interval $[0,1]$. The structure of IFS is a valuable and useful concept in DM sciences characterized by two-dimensional scenarios. Based on the IFS structure, several researchers have provided many applications in the field of DM sciences. De et al. [9] addressed the idea of IFS in medical sciences problems. Furthermore, Xiao [10] diagnosed the pattern reorganization problem. Yang et al. [11] gave the concept of the TOSIS method based on the IFS theory. In addition, many similar measures and AOs were developed by several researchers. For example, the theory of geometric AOs addressed by Xu and Yager [12] and Xu [13] diagnosed the approach of averaging AOs for the IFS framework. Hwang and Yang proposed the idea of a similarity measure for IFS [14], and Sheikh and Mandal developed the notion of Dombi AOs for IFS.

The definition of a rough set (RS) was first introduced by Pawlak [13]. It is stated as a pair of precise sets (PS) which is known as the upper approximation (UA) and lower approximation (LA) of RS. Numerous researchers developed several applications in different areas of DM sciences [14], [15]. After a bit of time, Zhang et al. [16] proved a precise solution to the MADM issue through RSF theory. By combining the thoughts of FS and RS theory, many mathematicians proposed multiple mathematical models for solving

MADM problems. Finally, the RFS was initiated by Dubois and Prade [17]. In addition, Qureshi and Shabir offered the generalized form of FRS for roughness in the quantal module [18].

The notion of intuitionistic FRS (IFRS) was developed by Cornelis et al. [19] and generalized the idea of FRS. By following the same scenario, many mathematicians proposed several real-life applications in the medical field solved by using the IFRS theory. For example, detecting breast cancer using the IFRS given by Chowdhary and Acharjya [20] and Chinram et al. [21] suggested the idea of AOs for IFRS and proposed averaging AOs for IFRSs. Based on the IFRS theory, we developed some new AOs called IFRAAPWA and IFRAAPWG operators because of the following reasons:

1. Due to the combined thought of IFS and RS, the idea of IFRS gives more space to the DM.
2. There is no concept for LA and UA in simple IFS. It means whenever the DM wants to come up with information in the shape of LA and UA, then a simple IFS structure has no idea how to deal with this type of information.
3. The thought of IFRS uses the advanced limitation that the sum of MV and NMV of LA and UA must lie from the unit interval $[0, 1]$.
4. The developed IFRAAPWA and IFRAAPWG operators can incorporate the priority degree of MD and evaluate the alternative from the initial assessment, and that type of axiom does not present in intuitionistic fuzzy (IF) weighted averaging (WA) (IFWA) and IF weighted geometric (WG) (IFWG) operators.
5. The proposed IFRAAPWA and IFRAAPWG operators can handle complex, more advanced information.
6. The proposed AOs cover the drawbacks and limitations of the previous fuzzy structures.

By observing these limitations in previous FS theory, we introduced new AOs more adaptable to handle complex data and give more reliable and relevant results. The motivation for this article is given as follows:

1. To introduce the new IFRAAPWA and IFRAAPWG operators in the IFRS environment.
2. Some necessary hypotheses of the proposed work are discussed.
3. To cover the complex information, the MADM algorithm is proposed.
4. To provide numerical MD problems to understand the proposed approach better.

The article is designed as follows: Section 2 provides some fundamental definitions to better understand proposed AOs. Section 3. discussed the prioritized AOs with numerical examples using IFRS and AA operational rules. Section 4 gave the IFRAAPWA and IFRAAPWG operators; some fundamental properties are also proved in this portion. The MADM algorithm is based on the developed IFRAAPWA, and IFRAAPWG operators are presented in Section 5. To better understand suggested AOs, we offer a numerical example to solve the MD problem using the MADM algorithm given in Section 6. Section 7. shows information about the effects of parameters on final results. Section 8. provides a detailed comparative analysis with currently present AOs. The conclusion of the article is discussed in Section 9.

2 | PRELIMINARIES

This section discussed some basic definitions to help understand the proposed MADM approach.

Definition 1. [8] Let Q be the universal set, and an IFRS is is defined as $\text{Z} = \{(q, \text{ø}(q), \text{Ơ}(q)), q \in Q\}$, and it satisfied that $0 \leq \text{ø}(q) + \text{Ơ}(q) \leq 1$. The MV is $\text{ø}(q)$ and $\text{Ơ}(q)$ is the NMV. The hesitancy degree of IFS is defined as $H(Z) = 1 - (\text{ø}(q) + \text{Ơ}(q))$.

Definition 2. [22] Assume A and M show the TNM and TCNM of the AA sum and product. As F and S are pair of IFSs, the union and intersection modification into AA sum and AA product, as:

$$F \oplus S = \left(\left(q, A\left(\text{ø}_F(q), (\text{ø}_S(q))\right), M\left(\text{Ơ}_F(q), \vartheta_S(q)\right)\right), q \in Q\right)$$

$$F \otimes S = \left(\left(q, M\left(\text{ø}_F(q), (\text{ø}_S(q))\right), A\left(\text{Ơ}_F(q), \vartheta_S(q)\right)\right), q \in Q\right)$$

Definition 3. [22] Assume $\text{Z} = ((\text{ø}_Z, \text{Ơ}_Z))$, $\text{Z}_1 = \left((\text{ø}_{Z_1}, \text{Ơ}_{Z_1})\right)$ and $\text{Z}_2 = \left((\text{ø}_{Z_2}, \text{Ơ}_{Z_2})\right)$ be intitionistic fuzzy values (IFVs) and $\text{ঽ} \geq 1$ and $\text{ƀ} > 0$. Thus, IFVs operations are from AATNM and and AATCNM can be described as:

$$Z_1 \oplus Z_2 = \left(\left(1 - e^{-\left((-\log(1-\text{ø}_{Z_1}))^{\text{ঽ}} + (-\log(1-\text{ø}_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right), \left(e^{-\left((-\log(\text{Ơ}_{Z_1}))^{\text{ঽ}} + (-\log(\text{Ơ}_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right),$$

$$Z_1 \otimes Z_2 = \left(\left(e^{-\left((-\log(\text{ø}_{Z_1}))^{\text{ঽ}} + (-\log(\text{ø}_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right), \left(1 - e^{-\left((-\log(1-\text{Ơ}_{Z_1}))^{\text{ঽ}} + (-\log(1-\text{Ơ}_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right),$$

$$\text{ƀ}Z = \left(\left(1 - e^{-\left(\text{ƀ}(-\log(1-\text{ø}_Z))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right), \left(e^{-\left(\text{ƀ}(-\log(\text{Ơ}_Z))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right),$$

$$Z^{\text{ƀ}} = \left(\left(e^{-\left(\text{ƀ}(-\log(\text{ø}_Z))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right), \left(1 - e^{-\left(\lambda(-\log(1-\text{Ơ}_Z))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right).$$

Definition 4. Let Q be the universal set, and an IFRS is defined as $Z = \{(q, (\text{ø}_Z, \text{Ơ}_Z), (\text{ư}_Z, \text{ȸ}_Z)), q \in Q\}$, and it satisfies the conditions $0 \leq ((\text{ø}_Z + \text{Ơ}_Z) + (\text{ư}_Z + \text{ȸ}_Z)) \leq 1$. Where $\text{ø}_Z, \text{ư}_Z$ be the MVs and $\text{Ơ}_Z, \text{ȸ}_Z$ be the NMVs of the IRFS. The hesitancy degree of IFRS is defined as $H(Z) = 1 - ((\text{ø}_Z + \text{Ơ}_Z) + (\text{ư}_Z + \text{ȸ}_Z))$

Definition 5. Assume A and M show the TNM and TCNM of the Aczel-Alsina sum and product. As F and S are pair of IFSs, the union and intersection modification into AA sum and AA product, as:

$$F \oplus S = \left\{\begin{matrix} (q, A\{\text{ø}_F(q), \text{ø}_S(q)\}, M\{\text{Ơ}_F(q), \text{Ơ}_S(q)\}), \\ (q, A\{\text{ư}_F(q), \text{ư}_S(q)\}, M\{\text{ȸ}_F(q), \text{ȸ}_S(q)\}), \end{matrix} q \in Q\right\}$$

$$F \otimes S = \left\{\begin{matrix} (q, M\{\text{ø}_F(q), \text{ø}_S(q)\}, A\{\text{Ơ}_F(q), \text{Ơ}_S(q)\}), \\ (q, M\{\text{ư}_F(q), \text{ư}_S(q)\}, A\{\text{ȸ}_F(q), \text{ȸ}_S(q)\}), \end{matrix} q \in Q\right\}$$

Definition 6. Let $Z = \{(\text{ø}_Z, \text{Ơ}_Z), (\text{ư}_Z, \text{ȸ}_Z)\}$, $Z_1 = \{(\text{ø}_{Z_1}, \text{Ơ}_{Z_1}), (\text{ư}_{Z_1}, \text{ȸ}_{Z_1})\}$ and $Z_2 = \{(\text{ø}_{Z_2}, \text{Ơ}_{Z_2}), (\text{ư}_{Z_2}, \text{ȸ}_{Z_2})\}$ can be IFRVs and $\text{ঽ} \geq 1$ and $\text{ƀ} > 0$. So, the operation of AATNM and AATCNM for IFRVs can be described below:

$$Z_1 \oplus Z_2 = \left\{\begin{matrix}\left(\left(1 - e^{-\left(\left(-\log(1-\text{ø}_{Z_1})\right)^{\text{ঌ}} + \left(-\log(1-\text{ø}_{Z_2})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right), \left(e^{-\left(\left(-\log(\text{ơ}_{Z_1})\right)^{\text{ঌ}} + \left(-\log(\text{ơ}_{Z_2})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right)\right), \\ \left(\left(1 - e^{-\left(\left(-\log(1-\text{ư}_{Z_1})\right)^{\text{ঌ}} + \left(-\log(1-\text{ư}_{Z_2})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right), \left(e^{-\left(\left(-\log(\text{ȸ}_{Z_1})\right)^{\text{ঌ}} + \left(-\log(\text{ȸ}_{Z_2})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right)\right)\end{matrix}\right\},$$

$$Z_1 \otimes Z_2 = \left\{\begin{matrix}\left(\left(e^{-\left(\left(-\log(\text{ø}_{Z_1})\right)^{\text{ঌ}} + \left(-\log(\text{ø}_{Z_2})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right), \left(1 - e^{-\left(\left(-\log(1-\text{ơ}_{Z_1})\right)^{\text{ঌ}} + \left(-\log(1-\text{ơ}_{Z_2})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right)\right), \\ \left(\left(e^{-\left(\left(-\log(\text{ư}_{Z_1})\right)^{\text{ঌ}} + \left(-\log(\text{ư}_{Z_2})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right), \left(1 - e^{-\left(\left(-\log(1-\text{ȸ}_{Z_1})\right)^{\text{ঌ}} + \left(-\log(1-\text{ȸ}_{Z_2})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right)\right)\end{matrix}\right\},$$

$$\text{ƀ}Z = \left\{\begin{matrix}\left(\left(1 - e^{-\left(\text{ƀ}\left(-\log(1-\text{ø}_{Z})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right), \left(e^{-\left(\text{ƀ}\left(-\log(\text{ơ}_{Z})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right)\right), \\ \left(\left(1 - e^{-\left(\text{ƀ}\left(-\log(1-\text{ư}_{Z})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right), \left(e^{-\left(\text{ƀ}\left(-\log(\text{ȸ}_{Z})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right)\right)\end{matrix}\right\},$$

$$Z^{\lambda} = \left\{\begin{matrix}\left(\left(e^{-\left(\text{ƀ}\left(-\log(\text{ø}_{Z})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right), \left(1 - e^{-\left(\text{ƀ}\left(-\log(1-\text{ơ}_{Z})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right)\right), \\ \left(\left(e^{-\left(\text{ƀ}\left(-\log(\text{ư}_{Z})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right), \left(1 - e^{-\left(\text{ƀ}\left(-\log(1-\text{ȸ}_{Z})\right)^{\text{ঌ}}\right)^{\frac{1}{\text{ঌ}}}}\right)\right)\end{matrix}\right\}.$$

Example 1. Suppose $Z = \{(0.7, 0.1), (0.1, 0.2)\}$, $Z_1 = \{(0.2, 0.1), (0.4, 0.1)\}$ and $Z_2 = \{(0.5, 0.4), (0.8, 0.1)\}$ be IFRVs so, $\text{ঌ} = 5$ and $\text{ƀ} = 3$, so we calculate:

$$Z_1 \oplus Z_2 = \left\{\begin{matrix}\left(\left(1 - e^{-\left((-\log(1-0.2))^5 + (-\log(1-0.5))^5\right)^{\frac{1}{5}}}\right), \left(e^{-\left((-\log(0.1))^5 + (-\log(0.4))^5\right)^{\frac{1}{5}}}\right)\right), \\ \left(\left(1 - e^{-\left((-\log(1-0.4))^5 + (-\log(1-0.8))^5\right)^{\frac{1}{5}}}\right), \left(e^{-\left((-\log(0.1))^5 + (-\log(0.1))^5\right)^{\frac{1}{5}}}\right)\right)\end{matrix}\right\}$$

$$= \{(0.2601, 0.3671), (0.5031, 0.3170)\}$$

$$Z_1 \otimes Z_2 = \left\{\begin{matrix}\left(\left(e^{-\left((-\log(0.2))^5 + (-\log(0.5))^5\right)^{\frac{1}{5}}}\right), \left(1 - e^{-\left((-\log(1-0.1))^5 + (-\log(1-0.4))^5\right)^{\frac{1}{5}}}\right)\right), \\ \left(\left(e^{-\left((-\log(0.4))^5 + (-\log(0.8))^5\right)^{\frac{1}{5}}}\right), \left(1 - e^{-\left((-\log(1-0.1))^5 + (-\log(1-0.1))^5\right)^{\frac{1}{5}}}\right)\right)\end{matrix}\right\}$$

$$= \{(0.4961, 0.1990), (0.6717, 0.0512)\}$$

$$\text{ƀ}Z = \left\{\begin{matrix}\left(\left(1 - e^{-\left(3(-\log(1-0.7))^5\right)^{\frac{1}{5}}}\right), \left(e^{-\left(3(-\log(0.1))^5\right)^{\frac{1}{5}}}\right)\right), \\ \left(\left(1 - e^{-\left(3(-\log(1-0.1))^5\right)^{\frac{1}{5}}}\right), \left(e^{-\left(3(-\log(0.2))^5\right)^{\frac{1}{5}}}\right)\right)\end{matrix}\right\}$$

$$= \{(0.4787, 0.2877), (0.0554, 0.4186)\}$$

$$Z^{\text{ƀ}} = \left\{\begin{matrix}\left(\left(e^{-\left(3(-\log(0.7))^5\right)^{\frac{1}{5}}}\right), \left(1 - e^{-\left(3(-\log(1-0.1))^5\right)^{\frac{1}{5}}}\right)\right), \\ \left(\left(e^{-\left(3(-\log(0.1))^5\right)^{\frac{1}{5}}}\right), \left(1 - e^{-\left(3(-\log(1-0.2))^5\right)^{\frac{1}{5}}}\right)\right)\end{matrix}\right\}$$

$$= \{(0.8245, 0.0554), (0.2877, 0.1137)\}$$

Theorem 1. Let $Z = \{(\text{ø}_Z, \text{Ơ}_Z), (\text{ư}_Z, \text{ȸ}_Z)\}$, $Z_1 = \{(\text{ø}_{Z_1}, \text{Ơ}_{Z_1}), (\text{ư}_{Z_1}, \text{ȸ}_{Z_1})\}$ and $Z_2 = \{(\text{ø}_{Z_2}, \text{Ơ}_{Z_2}), (\text{ư}_{Z_2}, \text{ȸ}_{Z_2})\}$ be IFRVs, so we define a few operational laws below:

$$Z_1 \oplus Z_2 = Z_2 \oplus Z_1$$

$$Z_1 \otimes Z_2 = Z_2 \otimes Z_1$$

$$\text{ꞗ}(Z_1 \oplus Z_2) = \text{ꞗ}Z_1 \oplus \text{ꞗ}Z_2,\ \text{ꞗ} > 0$$

$$(\text{ꞗ}_1 + \text{ꞗ}_2)Z = \text{ꞗ}_1 Z + \text{ꞗ}_2 Z,\ \text{ꞗ}_1, \text{ꞗ}_2 > 0$$

$$(Z_1 \otimes Z_2)^{\text{ꞗ}} = Z_1^{\text{ꞗ}} \otimes Z_2^{\text{ꞗ}},\ \text{ꞗ} > 0$$

$$Z^{\text{ꞗ}_1} \otimes Z^{\text{ꞗ}_2} = Z^{(\text{ꞗ}_1 + \text{ꞗ}_2)}$$

Now we can utilization three IFRVs Z , $Z_1$and Z_2 and $\text{ꞗ}, \text{ꞗ}_1, \text{ꞗ}_2 > 0$ to prove the above properties:

Proof 1.

$$Z_1 \oplus Z_2 = \left\{ \begin{matrix} \left(\left(1 - e^{-\left((-\log(1-\text{ø}_{Z_1}))^{\text{ℷ}} + (-\log(1-\text{ø}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(e^{-\left((-\log(\text{Ơ}_{Z_1}))^{\text{ℷ}} + (-\log(\text{Ơ}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right), \\ \left(\left(1 - e^{-\left((-\log(1-\text{ư}_{Z_1}))^{\text{ℷ}} + (-\log(1-\text{ư}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(e^{-\left((-\log(\text{ȸ}_{Z_1}))^{\text{ℷ}} + (-\log(\text{ȸ}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right) \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} \left(\left(1 - e^{-\left((-\log(1-\text{ø}_{Z_2}))^{\text{ℷ}} + (-\log(1-\text{ø}_{Z_1}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(e^{-\left((-\log(\text{Ơ}_{Z_2}))^{\text{ℷ}} + (-\log(\text{Ơ}_{Z_1}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right), \\ \left(\left(1 - e^{-\left((-\log(1-\text{ư}_{Z_2}))^{\text{ℷ}} + (-\log(1-\text{ư}_{Z_1}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(e^{-\left((-\log(\text{ȸ}_{Z_2}))^{\text{ℷ}} + (-\log(\text{ȸ}_{Z_1}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right) \end{matrix} \right\}$$

$$= Z_2 \oplus Z_1$$

Proof 2.

$$Z_1 \otimes Z_2 = \left\{ \begin{matrix} \left(\left(e^{-\left((-\log(\text{ø}_{Z_1}))^{\text{ℷ}} + (-\log(\text{ø}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left((-\log(1-\text{Ơ}_{Z_1}))^{\text{ℷ}} + (-\log(1-\text{Ơ}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right), \\ \left(\left(e^{-\left((-\log(\text{ư}_{Z_1}))^{\text{ℷ}} + (-\log(\text{ư}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left((-\log(1-\text{ȸ}_{Z_1}))^{\text{ℷ}} + (-\log(1-\text{ȸ}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right) \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} \left(\left(e^{-\left((-\log(\text{ø}_{Z_2}))^{\text{ℷ}} + (-\log(\text{ø}_{Z_1}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left((-\log(1-\text{Ơ}_{Z_2}))^{\text{ℷ}} + (-\log(1-\text{Ơ}_{Z_1}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right), \\ \left(\left(e^{-\left((-\log(\text{ư}_{Z_2}))^{\text{ℷ}} + (-\log(\text{ư}_{Z_1}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left((-\log(1-\text{ȸ}_{Z_2}))^{\text{ℷ}} + (-\log(1-\text{ȸ}_{Z_1}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right) \end{matrix} \right\}$$

$$= Z_2 \otimes Z_1$$

Proof 3.

$$\text{ƀ}(Z_1 \oplus Z_2) = \text{ƀ}\left\{\begin{matrix}\left(\left(1-e^{-\left(\left(-\log\left(1-ø_{Z_1}\right)\right)^{ℷ}+\left(-\log\left(1-ø_{Z_2}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\left(-\log\left(\text{Ơ}_{Z_1}\right)\right)^{ℷ}+\left(-\log\left(\text{Ơ}_{Z_2}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right),\\ \left(\left(1-e^{-\left(\left(-\log\left(1-\text{ư}_{Z_1}\right)\right)^{ℷ}+\left(-\log\left(1-\text{ư}_{Z_2}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\left(-\log\left(\text{ȸ}_{Z_1}\right)\right)^{ℷ}+\left(-\log\left(\text{ȸ}_{Z_2}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right)\end{matrix}\right\}$$

$$= \left\{\begin{matrix}\left(\left(1-e^{-\left(\text{ƀ}\left(-\log\left(1-ø_{Z_1}\right)\right)^{ℷ}+\left(-\log\left(1-ø_{Z_2}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\text{ƀ}\left(-\log\left(\text{Ơ}_{Z_1}\right)\right)^{ℷ}+\left(-\log\left(\text{Ơ}_{Z_2}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right),\\ \left(\left(1-e^{-\left(\text{ƀ}\left(-\log\left(1-\text{ư}_{Z_1}\right)\right)^{ℷ}+\left(-\log\left(1-\text{ư}_{Z_2}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\text{ƀ}\left(-\log\left(\text{ȸ}_{Z_1}\right)\right)^{ℷ}+\left(-\log\left(\text{ȸ}_{Z_2}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right)\end{matrix}\right\}$$

$$= \left\{\begin{matrix}\left(\left(1-e^{-\left(\text{ƀ}\left(-\log\left(1-ø_{Z_1}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\text{ƀ}\left(-\log\left(\text{Ơ}_{Z_1}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right),\\ \left(\left(1-e^{-\left(\text{ƀ}\left(-\log\left(1-\text{ư}_{Z_1}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\text{ƀ}\left(-\log\left(\text{ȸ}_{Z_1}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right)\end{matrix}\right\}$$

$$\oplus \left\{\begin{matrix}\left(\left(1-e^{-\left(\text{ƀ}\left(-\log\left(1-ø_{Z_2}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\text{ƀ}\left(-\log\left(\text{Ơ}_{Z_2}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right),\\ \left(\left(1-e^{-\left(\text{ƀ}\left(-\log\left(1-\text{ư}_{Z_2}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\text{ƀ}\left(-\log\left(\text{ȸ}_{Z_2}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right)\end{matrix}\right\}$$

$$= \text{ƀ}Z_1 \oplus \text{ƀ}Z_2$$

Proof 4.

$$\text{ƀ}_1 Z + \text{ƀ}_2 Z = \left\{\begin{matrix}\left(\left(1-e^{-\left(\text{ƀ}_1\left(-\log\left(1-ø_{Z}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\text{ƀ}_1\left(-\log\left(\text{Ơ}_{Z}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right),\\ \left(\left(1-e^{-\left(\text{ƀ}_1\left(-\log\left(1-\text{ư}_{Z}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\text{ƀ}_1\left(-\log\left(\text{ȸ}_{Z}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right)\end{matrix}\right\}$$

$$\oplus \left\{\begin{matrix}\left(\left(1-e^{-\left(\text{ƀ}_2\left(-\log\left(1-ø_{Z}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\text{ƀ}_2\left(-\log\left(\text{Ơ}_{Z}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right),\\ \left(\left(1-e^{-\left(\text{ƀ}_2\left(-\log\left(1-\text{ư}_{Z}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\text{ƀ}_2\left(-\log\left(\text{ȸ}_{Z}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right)\end{matrix}\right\}$$

$$= \left\{\begin{matrix}\left(\left(1-e^{-\left((\text{ƀ}_1+\text{ƀ}_2)\left(-\log\left(1-ø_{Z}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left((\text{ƀ}_1+\text{ƀ}_2)\left(-\log\left(\text{Ơ}_{Z}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right),\\ \left(\left(1-e^{-\left((\text{ƀ}_1+\text{ƀ}_2)\left(-\log\left(1-\text{ư}_{Z}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left((\text{ƀ}_1+\text{ƀ}_2)\left(-\log\left(\text{ȸ}_{Z}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right)\end{matrix}\right\}$$

$$= (\text{ђ}_1 + \text{ђ}_2)Z$$

Proof 5.

$$(Z_1 \otimes Z_2)^{\text{ђ}} = \left\{ \begin{array}{l} \left(\left(e^{-\left((-\log(ø_{Z_1}))^{\text{ℷ}} + (-\log(ø_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left((-\log(1-\text{Ơ}_{Z_1}))^{\text{ℷ}} + (-\log(1-\text{Ơ}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right), \\ \left(\left(e^{-\left((-\log(\text{ư}_{Z_1}))^{\text{ℷ}} + (-\log(\text{ư}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left((-\log(1-\text{ȸ}_{Z_1}))^{\text{ℷ}} + (-\log(1-\text{ȸ}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right) \end{array} \right\}^{\text{ђ}}$$

$$= \left\{ \begin{array}{l} \left(\left(e^{-\left(\text{ђ}(-\log(ø_{Z_1}))^{\text{ℷ}} + (-\log(ø_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left(\text{ђ}(-\log(1-\text{Ơ}_{Z_1}))^{\text{ℷ}} + (-\log(1-\text{Ơ}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right), \\ \left(\left(e^{-\left(\text{ђ}(-\log(\text{ư}_{Z_1}))^{\text{ℷ}} + (-\log(\text{ư}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left(\text{ђ}(-\log(1-\text{ȸ}_{Z_1}))^{\text{ℷ}} + (-\log(1-\text{ȸ}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right) \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \left(\left(e^{-\left(\text{ђ}(-\log(ø_{Z_1}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left(\text{ђ}(-\log(1-\text{Ơ}_{Z_1}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right), \\ \left(\left(e^{-\left(\text{ђ}(-\log(\text{ư}_{Z_1}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left(\text{ђ}(-\log(1-\text{ȸ}_{Z_1}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right) \end{array} \right\}$$

$$\otimes \left\{ \begin{array}{l} \left(\left(e^{-\left(\text{ђ}(-\log(ø_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left(\text{ђ}(-\log(1-\text{Ơ}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right), \\ \left(\left(e^{-\left(\text{ђ}(-\log(\text{ư}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left(\text{ђ}(-\log(1-\text{ȸ}_{Z_2}))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right) \end{array} \right\}$$

$$= Z_1^{\text{ђ}} \otimes Z_2^{\text{ђ}}$$

Proof 6.

$$Z^{\text{ђ}_1} \otimes Z^{\text{ђ}_2} = \left\{ \begin{array}{l} \left(\left(e^{-\left(\text{ђ}_1(-\log(ø_Z))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left(\text{ђ}_1(-\log(1-\text{Ơ}_Z))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right), \\ \left(\left(e^{-\left(\text{ђ}_1(-\log(\text{ư}_Z))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left(\text{ђ}_1(-\log(1-\text{ȸ}_Z))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right) \end{array} \right\}$$

$$\otimes \left\{ \begin{array}{l} \left(\left(e^{-\left(\text{ђ}_2(-\log(ø_Z))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left(\text{ђ}_2(-\log(1-\text{Ơ}_Z))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right), \\ \left(\left(e^{-\left(\text{ђ}_2(-\log(\text{ư}_Z))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left(\text{ђ}_2(-\log(1-\text{ȸ}_Z))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right) \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \left(\left(e^{-\left((\text{ђ}_1+\text{ђ}_2)(-\log(ø_Z))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left((\text{ђ}_1+\text{ђ}_2)(-\log(1-\text{Ơ}_Z))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right), \\ \left(\left(e^{-\left((\text{ђ}_1+\text{ђ}_2)(-\log(\text{ư}_Z))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right), \left(1 - e^{-\left((\text{ђ}_1+\text{ђ}_2)(-\log(1-\text{ȸ}_Z))^{\text{ℷ}} \right)^{\frac{1}{\text{ℷ}}}} \right) \right) \end{array} \right\}$$

$$= Z^{(\text{ђ}_1+\text{ђ}_2)}.$$

Definition 7. [23], [24] Let $Z = \{(ø_Z, \text{Ơ}_Z), (\text{ư}_Z, \text{ȸ}_Z)\}$ be the IFRVs. To find the best alternative ranked the aggregation outcome by using the following formula of score sunction (SF):

$$SF(Z) = \left(\frac{2 + ø_Z + ɰ_Z - Ơ_Z - ȸ_Z}{4}\right), SF(Z) = [0, 1] \quad (1)$$

The formula of accuracy function (AF) is defined as follows:

$$AF(Z) = \left(\frac{2 + ø_Z + ɰ_Z + Ơ_Z + ȸ_Z}{4}\right), AF(Z) = [0, 1] \quad (2)$$

Definition 8. Let$Z_1 = \{(ø_{Z_1}, Ơ_{Z_1}), (ɰ_{Z_1}, ȸ_{Z_1})\}$ and $Z_2 = \{(ø_{Z_2}, Ơ_{Z_2}), (ɰ_{Z_2}, ȸ_{Z_2})\}$ are pair of IFRVs, So:

If $SV(Z_1) > SV(Z_2)$, so $Z_1 > Z_2$,

If $SV(Z_1) < SV(Z_2)$, so $Z_1 < Z_2$,

If $SV(Z_1) = SV(Z_2)$, so

If $AV(Z_1) > AV(Z_2)$, so $Z_1 > Z_2$,0

If $AV(Z_1) < AV(Z_2)$, so $Z_1 < Z_2$

If $AV(Z_1) = AV(Z_2)$, so $Z_1 = Z_2$

3 | IFR ACZEL-ALSINA PRIORITIZED AVERAGING OPERATOR

In this segment, we define the IFR Aczel-Alsina prioritized aggregation (IFRAAPA) operator based on IFR and AA operational laws and provide a numerical example for better understanding.

Assume $Z_b = \{(ø_b, Ơ_b), (ɰ_b, ȸ_b)\}$ is the set of the IFRAAPA have accurate prioritization follow $\mathrm{Z}_1 >_{V_1} \mathrm{Z}_2 >_{V_2} \ldots >_{V_{n-1}} \mathrm{Z}_b \mathrm{Z}_b$ in this $\mathrm{Z}_b >_{v_b} \mathrm{Z}_{b+1}$, this expression shows that IFRVs Z_b having a $v_b$greater priority than the Z_{b+1}. The priority vector is $v_b = (v_1, v_2, \ldots, v_n)$satisfied $0 \leq v_b < \infty$. We can explain these values such as IFRAAPA has priority accurate with priority degree as ∂_v.

Definition 9. Suppose the IFRAAPA operator is plan $IFRAAPA$: $\partial_v^y \longrightarrow \partial_v$ clarified as below:

$$IFRAAPA(\mathrm{Z}_1, \mathrm{Z}_2, \ldots, \mathrm{Z}_\mathrm{y}) = \omega_1^v \mathrm{Z}_1 \oplus \omega_2^v \mathrm{Z}_2 \oplus \ldots \omega_y^v \mathrm{Z}_\mathrm{b} \quad (3)$$

In this section, $\omega_b^v = \frac{T_b^v}{\sum_{b=1}^{y} T_b^v}, T_b^v = \prod_{k=1}^{b-1} S(\mathrm{Z}_\mathrm{k})^v$ and $T_1 = 1$. So, the IFRAAPA operator is also called the IFRAAPA operator priority degree.

Theorem 2. Suppose$Z_b = \{(ø_b, Ơ_b), (ɰ_b, ȸ_b)\}$ can be IFRVs, then using IFRAAPA operators, the resulting values are also IFRVs.

$$IFRAAPA(\mathrm{Z}_1, \mathrm{Z}_2, \ldots, \mathrm{Z}_\mathrm{b}) = \left\{ \begin{array}{l} \left(\left(1 - e^{-\left(\sum_{y=1}^{b} \omega_y^v \left(-\log\left(1 - ø_{Z_y}\right)\right)^{ר}\right)^{\frac{1}{ר}}} \right), \left(e^{-\left(\sum_{y=1}^{b} \omega_y^v \left(-\log\left(Ơ_{Z_y}\right)\right)^{ר}\right)^{\frac{1}{ר}}} \right) \right), \\ \left(\left(1 - e^{-\left(\sum_{y=1}^{b} \omega_y^v \left(-\log\left(1 - ɰ_{Z_y}\right)\right)^{ר}\right)^{\frac{1}{ר}}} \right), \left(e^{-\left(\sum_{y=1}^{b} \omega_y^v \left(-\log\left(ȸ_{Z_y}\right)\right)^{ר}\right)^{\frac{1}{ר}}} \right) \right) \end{array} \right\} \quad (4)$$

Proof: Using Definition 6. We can easily prove this theorem. This theorem also satisfies idempotency, boundedness, and monotonic property under IFRVs.

Example 2. Let the four sets of the IFRVs M_1, M_2, M_3 and M_4. Then, suppose that $M_1 = ((0.2,0.4), (0.6,0.2))$, $M_2 = ((0.5,0.3), (0.4,0.5))$, $M_3 = ((0.4,0.5), (0.7,0.1))$ and $M_4 = ((0.6, 0.3), (0.4, 0.4))$.

Using Definition 9. to calculate the SV $S(\mathrm{M}_1) = 0.55$, $S(\grave{\mathrm{M}}_2) = 0.525$, $S(\mathrm{M}_3) = 0.625$ and $S(\mathrm{M}_4) = 0.575$. Next, we obtain the value $Z_1(b = 1, 2, 3, 4)$ for distinct four priority vectors $v_b = (v_1, v_2, v_3)$ with order v_2, v_3 will be constant but m_1 can not be constant. Next,

Condition 1. $v_1 = (1, 1, 1)$, then we calculate $T_2^v = \left(S(\mathrm{M}_1)\right)^{v_1} = 0.55$, $T_3^v = \left(S(\mathrm{M}_1)\right)^{v_1} \times \left(S(\mathrm{M}_2)\right)^{v_2} = 0.2888 T_4^v = \left(S(\mathrm{M}_1)\right)^{v_1} \times \left(S(\mathrm{M}_2)\right)^{v_2} \times \left(S(\mathrm{M}_3)\right)^{v_3} = 0.1805$ and $T_1^v = 1$. Then $\sum_{k=1}^{4} T_k = 1 + 0.55 + 0.2888 + 0.1805 = 2.019$. Next using $\omega_1^v = 0.4952$, $\omega_2^v = 0.2724$, $\omega_3^v = 0.143$ and $\omega_4^v = 0.0894$, we use the following formula:

$$IFRAAPA(\mathrm{M}_1, \mathrm{M}_2, \mathrm{M}_3, \mathrm{M}_4) = \left\{ \begin{array}{l} \left(\left(1 - e^{-\left(\sum_{y=1}^{4} \omega_y^v \left(-\log\left(1 - ø_{Z_y}\right)\right)^{\gimel}\right)^{\frac{1}{\gimel}}} \right), \left(e^{-\left(\sum_{y=1}^{4} \omega_y^v \left(-\log\left(ơ_{Z_y}\right)\right)^{\gimel}\right)^{\frac{1}{\gimel}}} \right) \right), \\ \left(\left(1 - e^{-\left(\sum_{y=1}^{4} \omega_y^v \left(-\log\left(1 - ư_{Z_y}\right)\right)^{\gimel}\right)^{\frac{1}{\gimel}}} \right), \left(e^{-\left(\sum_{y=1}^{4} \omega_y^v \left(-\log\left(ȸ_{Z_y}\right)\right)^{\gimel}\right)^{\frac{1}{\gimel}}} \right) \right) \end{array} \right\}$$

$$= \left(1 - e^{-\left(\sum_{y=1}^{4} \omega_y^v \left(-\log\left(1 - ø_{Z_y}\right)\right)^{\gimel}\right)^{\frac{1}{\gimel}}} \right)$$

$$= \left(1 - e^{-\left(0.4952(-\log(1-0.2))^5 + 0.2724(-\log(1-0.5))^5 + 0.143(-\log(1-0.4))^5 + 0.0894(-\log(1-0.6))^5\right)^{\frac{1}{5}}} \right)$$

$$= 0.2423$$

Same as we can calculate for

$$\left(e^{-\left(\sum_{y=1}^{4} \omega_y^v \left(-\log\left(ơ_{Z_y}\right)\right)^{\gimel}\right)^{\frac{1}{\gimel}}} \right) = \left(e^{-\left(0.4952(-\log(0.4))^5 + 0.2724(-\log(0.3))^5 + 0.143(-\log(0.5))^5 + 0.0894(-\log(0.3))^5\right)^{\frac{1}{5}}} \right)$$

$$= 0.6346$$

and

$$\left(1 - e^{-\left(\sum_{y=1}^{4} \omega_y^v \left(-\log\left(1 - ư_{Z_y}\right)\right)^{\gimel}\right)^{\frac{1}{\gimel}}} \right)$$

$$= \left(1 - e^{-\left(0.4952(-\log(1-0.6))^5 + 0.2724(-\log(1-0.4))^5 + 0.143(-\log(1-0.7))^5 + 0.0894(-\log(1-0.4))^5\right)^{\frac{1}{5}}} \right)$$

$$= 0.3322$$

and

$$\left(e^{-\left(\sum_{y=1}^{4} \omega_y^v \left(-\log\left(ȸ_{Z_y}\right)\right)^{\gimel}\right)^{\frac{1}{\gimel}}} \right) = \left(e^{-\left(0.4952(-\log(0.2))^5 + 0.2724(-\log(0.5))^5 + 0.143(-\log(0.1))^5 + 0.0894(-\log(0.4))^5\right)^{\frac{1}{5}}} \right)$$

$$= 0.3661$$

$$IFRAAPA(\mathrm{M}_1, \mathrm{M}_2, \ldots, \mathrm{M}_4) = \left((0.2423, 0.6346), (0.3322, 0.3661)\right)$$

Condition 2: $v_1 = (2, 1, 1)$, then keep in mind the steps of Condition 1. Then find

$$IFRAAPA(\mathrm{M}_1, \mathrm{M}_2, \ldots, \mathrm{M}_4) = \left((0.2285, 0.6433), (0.3311, 0.4811)\right)$$

Condition 3: $v_1 = (4, 1, 1)$

$$IFRAAPA(\text{Ɱ}_1,\text{Ɱ}_2,\dots,\text{Ɱ}_4)=\big((0.1939,0.6587),(0.3294,0.4903)\big)$$

Condition 4: $v_1=(8,1,1)$

$$IFRAAPA(\text{Ɱ}_1,\text{Ɱ}_2,\dots,\text{Ɱ}_4)=\big((0.1325,0.6702),(0.3284,0.4963)\big)$$

In all environments, we can calculate the answer of IFRAAPA based on the degree vector.

4 | INTUITIONISTIC FUZZY ROUGH ACZEL-ALSINA WEIGHTED PRIORITIZED AGGREGATION OPERATORS

In this portion, we define and prove the IFRAAPWA and IFRAAWG operators and discuss some necessary axioms of proposed AOs, such as boundedness, monotonicity, and idempotency.

4.1 | Intuitionistic Fuzzy Rough Aczel-Alsina Weighted Averaging Prioritized Aggregation Operators

Definition 10. Assume $Z_y=\left\{\left(ø_{z_y},Ơ_{z_y}\right),\left(ư_{z_y},ȸ_{z_y}\right)\right\},(y=1,2,3,4,\dots,b)$ be the album of IFRVs and $\omega_y=(\omega_1,\omega_2,\dots,\omega_b)(y=1,2,3,4,\dots,b)$ is the weighted vectors (WVs). The sum of all WV is always 1, such as $\sum_{y=1}^{b}\omega_y=1$. Next, the purpose of IFRAAPWA aggregation is $Ʉ^n\to Ʉ$ state as:

$$IFRAAPWA(Z_1,Z_2,\dots,Z_b)=\oplus_{y=1}^{b}\left(\omega_y Z_y\right)=\omega_1Z_1+\omega_2Z_2+\cdots+\omega_bZ_b$$

Next, this definition is proved, and the solution obtained from IFRAAPWA operators also achieves IFRVs.

Theorem 3. Consider $Z_y=\left\{\left(ø_{z_y},Ơ_{z_y}\right),\left(ư_{z_y},ȸ_{z_y}\right)\right\}(y=1,2,3,\dots,b)$ be the collection of IFRVs, so the aggregation values obtained by operating IFRAAPWA are also allotted IFRVs.

$$IFRAAPWA_{\omega_y}(Z_1,Z_2,\dots,Z_b)=\oplus_{y=1}^{b}\left(\omega_y Z_y\right)$$

$$=\left\{\begin{array}{l}\left(\left(1-e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-ø_{z_y}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(Ơ_{z_y}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right),\\ \left(\left(1-e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-ư_{z_y}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(ȸ_{z_y}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right)\end{array}\right\} \tag{5}$$

In this equation, we use WVs $\omega_y=(\omega_1,\omega_2,\dots,\omega_k)$ for IFRVs $Z_y(y=1,2,\dots,b)$. In this, the sum of WVs is 1, such as $\sum_{y=1}^{b}\omega_y=1$.

Proof. Mathematical induction is used to prove this theorem.

Take $b=2$, now we obtain:

$$\omega_1Z_1=\left\{\begin{array}{l}\left(\left(1-e^{-\left(\omega_1\left(-\log\left(1-ø_{z_1}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\omega_1\left(-\log\left(Ơ_{z_1}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right),\\ \left(\left(1-e^{-\left(\omega_1\left(-\log\left(1-ư_{z_1}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right),\left(e^{-\left(\omega_1\left(-\log\left(ȸ_{z_1}\right)\right)^{ℷ}\right)^{\frac{1}{ℷ}}}\right)\right)\end{array}\right\}$$

$$\omega_2 Z_2 = \left\{\begin{array}{l}\left(\left(\left(1-e^{-\left(\omega_2(-\log(1-ø_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(e^{-\left(\omega_2(-\log(ơ_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\right),\\ \left(\left(\left(1-e^{-\left(\omega_2(-\log(1-ư_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(e^{-\left(\omega_2(-\log(ȸ_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\right)\end{array}\right\}$$

So, to apply Definition 10. we get:

$$IFFAAFWA(Z_1, Z_2) = \oplus_{y=1}^{b} \omega_y Z_y = \omega_1 Z_1 \oplus \omega_2 Z_2$$

$$= \left\{\begin{array}{l}\left(\left(\left(1-e^{-\left(\omega_1(-\log(1-ø_{Z_1}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(e^{-\left(\omega_1(-\log(ơ_{Z_1}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\right),\\ \left(\left(\left(1-e^{-\left(\omega_1(-\log(1-ư_{Z_1}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(e^{-\left(\omega_1(-\log(ȸ_{Z_1}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\right)\end{array}\right\}$$

$$\oplus \left\{\begin{array}{l}\left(\left(\left(1-e^{-\left(\omega_2(-\log(1-ø_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(e^{-\left(\omega_2(-\log(ơ_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\right),\\ \left(\left(\left(1-e^{-\left(\omega_2(-\log(1-ư_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(e^{-\left(\omega_2(-\log(ȸ_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\right)\end{array}\right\}$$

$$= \left\{\begin{array}{l}\left(\left(\left(1-e^{-\left(\omega_1(-\log(1-ø_{Z_1}))^{\text{ঽ}}+\omega_2(-\log(1-ø_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(e^{-\left(\omega_1(-\log(ơ_{Z_1}))^{\text{ঽ}}+\omega_2(-\log(ơ_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\right),\\ \left(\left(\left(1-e^{-\left(\omega_1(-\log(1-ư_{Z_1}))^{\text{ঽ}}+\omega_2(-\log(1-ư_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(e^{-\left(\omega_1(-\log(ȸ_{Z_1}))^{\text{ঽ}}+\omega_2(-\log(ȸ_{Z_2}))^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\right)\end{array}\right\}$$

$$= \left\{\begin{array}{l}\left(\left(\left(1-e^{-\left(\sum_{y=1}^{2}\omega_y\left(-\log\left(1-ø_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(e^{-\left(\sum_{y=1}^{2}\omega_y\left(-\log\left(ơ_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\right),\\ \left(\left(\left(1-e^{-\left(\sum_{y=1}^{2}\omega_y\left(-\log\left(1-ư_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(e^{-\left(\sum_{y=1}^{2}\omega_y\left(-\log\left(ȸ_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\right)\end{array}\right\}$$

Now, the theorem is valid for $b = 2$.

Now, we check for $b = ṇ$.

$$= \left\{\begin{array}{l}\left(\left(\left(1-e^{-\left(\sum_{y=1}^{ṇ}\omega_y\left(-\log\left(1-ø_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(e^{-\left(\sum_{y=1}^{ṇ}\omega_y\left(-\log\left(ơ_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\right),\\ \left(\left(\left(1-e^{-\left(\sum_{y=1}^{ṇ}\omega_y\left(-\log\left(1-ư_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(e^{-\left(\sum_{y=1}^{ṇ}\omega_y\left(-\log\left(ȸ_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\right)\end{array}\right\}$$

So, it is also valid for $b = ṇ$.

Now, we check for $b = ṇ + 1$.

$$IFRAAPWA_{\omega_y}(Z_1, Z_2, \ldots, Z_{ṇ}, Z_{ṇ+1}) = \oplus_{y=1}^{b} (\omega_{ṇ} Z_{ṇ}) \oplus (\omega_{ṇ+1} Z_{ṇ+1})$$

$$= \left\{ \begin{array}{l} \left(\left(1 - e^{-\left(\sum_{y=1}^{\mathrm{n}} \omega_y \left(-\log\left(1-ø_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right), \left(e^{-\left(\sum_{y=1}^{\mathrm{n}} \omega_y \left(-\log\left(\text{Ơ}_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right) \right), \\ \left(\left(1 - e^{-\left(\sum_{y=1}^{\mathrm{n}} \omega_y \left(-\log\left(1-\text{ư}_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right), \left(e^{-\left(\sum_{y=1}^{\mathrm{n}} \omega_y \left(-\log\left(\text{ȸ}_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right) \right) \end{array} \right\}$$

$$\oplus \left\{ \begin{array}{l} \left(\left(1 - e^{-\left(\omega_{\mathrm{n}+1} \left(-\log\left(1-ø_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right), \left(e^{-\left(\omega_{\mathrm{n}+1} \left(-\log\left(\text{Ơ}_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right) \right), \\ \left(\left(1 - e^{-\left(\omega_{\mathrm{n}+1} \left(-\log\left(1-\text{ư}_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right), \left(e^{-\left(\omega_{\mathrm{n}+1} \left(-\log\left(\text{ȸ}_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right) \right) \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \left(\left(1 - e^{-\left(\sum_{y=1}^{\mathrm{n}+1} \omega_y \left(-\log\left(1-ø_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right), \left(e^{-\left(\sum_{y=1}^{\mathrm{n}+1} \omega_y \left(-\log\left(\text{Ơ}_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right) \right), \\ \left(\left(1 - e^{-\left(\sum_{y=1}^{\mathrm{n}+1} \omega_y \left(-\log\left(1-\text{ư}_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right), \left(e^{-\left(\sum_{y=1}^{\mathrm{n}+1} \omega_y \left(-\log\left(\text{ȸ}_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right) \right) \end{array} \right\}$$

So, all results show that it can be valid for all values of b and also, this is valid for all real deals.

Now, we describe the necessary properties for the IFFAAPWA aggregation operator.

Theorem 4. (Idempotency): Assume $Z_y = \left\{\left(ø_{Z_y}, \text{Ơ}_{Z_y}\right), \left(\text{ư}_{Z_y}, \text{ȸ}_{Z_y}\right)\right\} (y = 1, 2, 3, \ldots, b)$ be the collection of IFRVs. So, Z_y are all the same, for example $Z_y = Z$ for all y, therefore:

$IFRAAPWA(Z_1, Z_2, \ldots, Z_k) = Z$.

Proof: Assume $Z_y = \left\{\left(ø_{Z_y}, \text{Ơ}_{Z_y}\right), \left(\text{ư}_{Z_y}, \text{ȸ}_{Z_y}\right)\right\} (y = 1, 2, 3, \ldots, b)$ so by utilizing Eq. 5. we obtain:

$$IFRAAPWA_{\omega_y}(Z_1, Z_2, \ldots, Z_b) = \oplus_{y=1}^{b} \left(\omega_y Z_y\right)$$

$$= \left\{ \begin{array}{l} \left(\left(1 - e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(1-ø_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right), \left(e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(\text{Ơ}_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right) \right), \\ \left(\left(1 - e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(1-\text{ư}_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right), \left(e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(\text{ȸ}_{Z_y}\right)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right) \right) \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \left(\left(1 - e^{-\left(\left(-\log(1-ø_Z)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right), \left(e^{-\left(\left(-\log(\text{Ơ}_Z)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right) \right), \\ \left(\left(1 - e^{-\left(\left(-\log(1-\text{ư}_Z)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right), \left(e^{-\left(\left(-\log(\text{ȸ}_Z)\right)^{ঀ}\right)^{\frac{1}{ঀ}}} \right) \right) \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \left(\left(1 - e^{\log(1-ø_Z)}\right), \left(e^{\log((\text{Ơ}_Z))}\right) \right), \\ \left(\left(1 - e^{\log(1-\text{ư}_Z)}\right), \left(e^{\log(\text{ȸ}_Z)}\right) \right) \end{array} \right\}$$

$$= \{(ø_Z, \text{Ơ}_Z), (\text{ư}_Z, \text{ȸ}_Z)\}$$

$$= Z$$

Theorem 5. (Boundedness) Assume $Z_y = \{(ø_{z_y}, Ơ_{z_y}), (ư_{z_y}, ȸ_{z_y})\}(y = 1,2,3,\dots,b)$ be the collection of IFRVs. Consider $Z^- = \min(Z_1, Z_2, \dots, Z_b)$ and $Z^+ = \max(Z_1, Z_2, \dots, Z_b)$, therefore

$$Z^- \le IFRAAPWA_{\omega_y}(Z_1, Z_2, \dots, Z_b) \le Z^+$$

Proof: Assume $Z_y = \{(ø_{z_y}, Ơ_{z_y}), (ư_{z_y}, ȸ_{z_y})\}(y = 1,2,3,\dots,b)$ be the collection of IFRVs and consider $Z^- = \min(Z_1, Z_2, \dots, Z_b) = \{(ø_z^-, Ơ_z^-), (ư_z^-, ȸ_z^-)\}$ and $Z^+ = \max(Z_1, Z_2, \dots, Z_b) = \{(ø_z^+, Ơ_z^+), (ư_z^+, ȸ_z^+)\}$. So, $ø_z^- = \min(ø_{z_y})$, $Ơ_z^- = \max(Ơ_{z_y})$,$ø_z^+ = \max(ø_{z_y})$, $Ơ_z^+ = \min(Ơ_{z_y})$ and$ư_z^- = \min(ư_{z_y})$, $ȸ_z^- = \max(\partial_{\text{Ʀ}_z})$, $ư_z^+ = \max(ư_{z_y})$, $ȸ_z^+ = \min(ȸ_{z_y})$. So, we can get the following.

$$\left(1 - e^{-(\sum_{y=1}^{b}\omega_y(-\log(1-ø_z^-))^{ǡ})^{\frac{1}{ǡ}}}\right) \le \left(1 - e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-ø_{z_y}\right)\right)^{ǡ}\right)^{\frac{1}{ǡ}}}\right) \le \left(1 - e^{-(\sum_{y=1}^{b}\omega_y(-\log(1-ø_z^+))^{ǡ})^{\frac{1}{ǡ}}}\right)$$

and

$$\left(1 - e^{-(\sum_{y=1}^{b}\omega_y(-\log(1-ư_z^-))^{ǡ})^{\frac{1}{ǡ}}}\right) \le \left(1 - e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-ư_{z_y}\right)\right)^{ǡ}\right)^{\frac{1}{ǡ}}}\right) \le \left(1 - e^{-(\sum_{y=1}^{b}\omega_y(-\log(1-ư_z^+))^{ǡ})^{\frac{1}{ǡ}}}\right)$$

and

$$\left(e^{-(\sum_{y=1}^{b}\omega_y(-\log(Ơ_z^-))^{ǡ})^{\frac{1}{ǡ}}}\right) \le \left(e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(Ơ_{z_y}\right)\right)^{ǡ}\right)^{\frac{1}{ǡ}}}\right) \le \left(e^{-(\sum_{y=1}^{b}\omega_y(-\log(Ơ_z^+))^{ǡ})^{\frac{1}{ǡ}}}\right)$$

and

$$\left(e^{-(\sum_{y=1}^{b}\omega_y(-\log(ȸ_z^-))^{ǡ})^{\frac{1}{ǡ}}}\right) \le \left(e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(ȸ_{z_y}\right)\right)^{ǡ}\right)^{\frac{1}{ǡ}}}\right) \le \left(e^{-(\sum_{y=1}^{b}\omega_y(-\log(ȸ_z^+))^{ǡ})^{\frac{1}{ǡ}}}\right)$$

In the end,

$$Z^- \le IFRAAPWA_{\omega_y}(Z_1, Z_2, \dots, Z_b) \le Z^+$$

Theorem 5 (Monotonicity) Assume Z_y and $\breve{¥}_y(y = 1,2,\dots,b)$ be a pair set of IFRVs, so $Z_y \le \breve{¥}_y$ for very y, so,

$$IFRAAPWA_{\omega_y}(Z_1, Z_2, \dots, Z_b) \le IFRAAPWA_{\omega_z}(\breve{¥}_1, \breve{¥}_2, \dots, \breve{¥}_b)$$

Proof: Straightforward

4.2 | Intuitionistic Fuzzy Rough Aczel-Alsina Prioritized Weighted Geometric Aggregation Operators

In this portion, we clarify the concept of IFRAA prioritized weighted geometric.

Definition 11. Assume$Z_y = \{(ø_{z_y}, Ơ_{z_y}), (ư_{z_y}, ȸ_{z_y})\}(y = 1,2,3,4,\dots,b)$ be the album of IFRVs and $\omega_y = (\omega_1, \omega_2, \dots, \omega_b)(y = 1,2,3,4,\dots,b)$is the weighted vectors (WVs). The sum of all WV is always 1, such as$\sum_{y=1}^{b}\omega_y = 1$. When IFRAAPWG aggregation are $Ụ^n \rightarrow Ụ$ state as:

$$IFRAAPWG(Z_1, Z_2, \dots, Z_b) = \otimes_{y=1}^{b}\left({Z_y}^{\omega_y}\right)$$

Next, this definition is proved, and the solution obtained from IFRAAPWG operators also achieves IFRVs.

Theorem 6. Consider$Z_y = \left\{\left(\text{ø}_{Z_y}, \text{Ơ}_{Z_y}\right), \left(\text{ư}_{Z_y}, \text{ȸ}_{Z_y}\right)\right\} (y = 1, 2, 3, \dots, b)$ be the album of IFRVs, so the aggregation values obtained by operating IFRAAPWG are also allotted IFRVs.

$$IFRAAPWG_{\omega_y}(Z_1, Z_2, \dots, Z_b) = \otimes_{y=1}^{b} \left({Z_y}^{\omega_y}\right) =$$

$$\left\{\begin{array}{l} \left(\left(e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(\text{ø}_{Z_y}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right), \left(1 - e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-\text{Ơ}_{Z_y}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right)\right), \\ \left(\left(e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(\text{ư}_{Z_y}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right), \left(1 - e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-\text{ȸ}_{Z_y}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right)\right) \end{array}\right\} \quad (6)$$

In this equation, we use WVs $\omega_y = (\omega_1, \omega_2, \dots, \omega_k)$ for IFRVs $Z_y(y = 1, 2, \dots, b)$. In this, the sum of WVs is 1, such as $\sum_{y=1}^{b}\omega_y = 1$.

Proof. The mathematical induction technique is used to prove this theorem.

Take $b = 2$. Now we get:

$${Z_1}^{\omega_1} = \left\{\begin{array}{l} \left(\left(e^{-\left(\omega_1\left(-\log\left(\text{ø}_{Z_1}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right), \left(1 - e^{-\left(\omega_1\left(-\log\left(1-\text{Ơ}_{Z_1}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right)\right), \\ \left(\left(e^{-\left(\omega_1\left(-\log\left(\text{ư}_{Z_1}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right), \left(1 - e^{-\left(\omega_1\left(-\log\left(1-\text{ȸ}_{Z_1}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right)\right) \end{array}\right\}$$

$${Z_2}^{\omega_2} = \left\{\begin{array}{l} \left(\left(e^{-\left(\omega_2\left(-\log\left(\text{ø}_{Z_2}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right), \left(1 - e^{-\left(\omega_2\left(-\log\left(1-\text{Ơ}_{Z_2}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right)\right), \\ \left(\left(e^{-\left(\omega_2\left(-\log\left(\text{ư}_{Z_2}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right), \left(1 - e^{-\left(\omega_2\left(-\log\left(1-\text{ȸ}_{Z_2}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right)\right) \end{array}\right\}$$

So, to apply Definition 11. we obtain:

$$IFRAAPWG(Z_1, Z_2) = \otimes_{y=1}^{b} {Z_y}^{\omega_y} = {Z_1}^{\omega_1} \otimes {Z_1}^{\omega_1}$$

$$= \left\{\begin{array}{l} \left(\left(e^{-\left(\omega_1\left(-\log\left(\text{ø}_{Z_1}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right), \left(1 - e^{-\left(\omega_1\left(-\log\left(1-\text{Ơ}_{Z_1}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right)\right), \\ \left(\left(e^{-\left(\omega_1\left(-\log\left(\text{ư}_{Z_1}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right), \left(1 - e^{-\left(\omega_1\left(-\log\left(1-\text{ȸ}_{Z_1}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right)\right) \end{array}\right\}$$

$$\otimes \left\{\begin{array}{l} \left(\left(e^{-\left(\omega_2\left(-\log\left(\text{ø}_{Z_2}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right), \left(1 - e^{-\left(\omega_2\left(-\log\left(1-\text{Ơ}_{Z_2}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right)\right), \\ \left(\left(e^{-\left(\omega_2\left(-\log\left(\text{ư}_{Z_2}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right), \left(1 - e^{-\left(\omega_2\left(-\log\left(1-\text{ȸ}_{Z_2}\right)\right)^{\daleth}\right)^{\frac{1}{\daleth}}}\right)\right) \end{array}\right\}$$

$$=\left\{\begin{array}{l}\left(\left(e^{-\left(\omega_1\left(-\log\left(ø_{Z_1}\right)\right)^{\text{ঽ}}+\omega_2\left(-\log\left(ø_{Z_2}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(1-e^{-\left(\omega_1\left(-\log\left(1-\text{Ơ}_{Z_1}\right)\right)^{\text{ঽ}}+\omega_2\left(-\log\left(1-\text{Ơ}_{Z_2}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right),\\ \left(\left(e^{-\left(\omega_1\left(-\log\left(\text{ư}_{Z_1}\right)\right)^{\text{ঽ}}+\omega_2\left(-\log\left(\text{ư}_{Z_2}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(1-e^{-\left(\omega_1\left(-\log\left(1-\text{ȸ}_{Z_1}\right)\right)^{\text{ঽ}}+\omega_2\left(-\log\left(1-\text{ȸ}_{Z_2}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\end{array}\right\}$$

$$=\left\{\begin{array}{l}\left(\left(e^{-\left(\sum_{y=1}^{2}\omega_y\left(-\log\left(ø_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(1-e^{-\left(\sum_{y=1}^{2}\omega_y\left(-\log\left(1-\text{Ơ}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right),\\ \left(\left(e^{-\left(\sum_{y=1}^{2}\omega_y\left(-\log\left(\text{ư}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(1-e^{-\left(\sum_{y=1}^{2}\omega_y\left(-\log\left(1-\text{ȸ}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\end{array}\right\}$$

Now, the theorem is valid for $b=2$.

Now, we check to use $b=\text{ṇ}$.

$$=\left\{\begin{array}{l}\left(\left(e^{-\left(\sum_{y=1}^{\text{ṇ}}\omega_y\left(-\log\left(ø_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(1-e^{-\left(\sum_{y=1}^{\text{ṇ}}\omega_y\left(-\log\left(1-\text{Ơ}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right),\\ \left(\left(e^{-\left(\sum_{y=1}^{\text{ṇ}}\omega_y\left(-\log\left(\text{ư}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(1-e^{-\left(\sum_{y=1}^{\text{ṇ}}\omega_y\left(-\log\left(1-\text{ȸ}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\end{array}\right\}$$

So, it is also valid for $b=\text{ṇ}$.

Now, we check for$b=\text{ṇ}+1$.

$$IFRAAPWG_{\omega_y}\left(Z_1,Z_2,\ldots,Z_{\text{ṇ}},Z_{\text{ṇ}+1}\right)=\otimes_{y=1}^{b}\left({Z_{\text{ṇ}}}^{\omega_{\text{ṇ}}}\right)\otimes\left({Z_{\text{ṇ}+1}}^{\omega_{\text{ṇ}+1}}\right)$$

$$=\left\{\begin{array}{l}\left(\left(e^{-\left(\sum_{y=1}^{\text{ṇ}}\omega_y\left(-\log\left(ø_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(1-e^{-\left(\sum_{y=1}^{\text{ṇ}}\omega_y\left(-\log\left(1-\text{Ơ}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right),\\ \left(\left(e^{-\left(\sum_{y=1}^{\text{ṇ}}\omega_y\left(-\log\left(\text{ư}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(1-e^{-\left(\sum_{y=1}^{\text{ṇ}}\omega_y\left(-\log\left(1-\text{ȸ}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\end{array}\right\}$$

$$\otimes\left\{\begin{array}{l}\left(\left(e^{-\left(\omega_{\text{ṇ}+1}\left(-\log\left(ø_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(1-e^{-\left(\omega_{\text{ṇ}+1}\left(-\log\left(1-\text{Ơ}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right),\\ \left(\left(e^{-\left(\omega_{\text{ṇ}+1}\left(-\log\left(\text{ư}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(1-e^{-\left(\omega_{\text{ṇ}+1}\left(-\log\left(1-\text{ȸ}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\end{array}\right\}$$

$$=\left\{\begin{array}{l}\left(\left(e^{-\left(\sum_{y=1}^{\text{ṇ}+1}\omega_y\left(-\log\left(ø_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(1-e^{-\left(\sum_{y=1}^{\text{ṇ}+1}\omega_y\left(-\log\left(1-\text{Ơ}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right),\\ \left(\left(e^{-\left(\sum_{y=1}^{\text{ṇ}+1}\omega_y\left(-\log\left(\text{ư}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right),\left(1-e^{-\left(\sum_{y=1}^{\text{ṇ}+1}\omega_y\left(-\log\left(1-\text{ȸ}_{Z_y}\right)\right)^{\text{ঽ}}\right)^{\frac{1}{\text{ঽ}}}}\right)\right)\end{array}\right\}$$

So, all results show that it can be valid for all values of b and also, this is valid for all real deals.

Now, we describe the necessary properties for the IFRAAPWG aggregation operator.

Theorem 7. (Idempotency): Assume$Z_y = \left\{\left(ø_{z_y}, Ơ_{z_y}\right), \left(ư_{z_y}, ȸ_{z_y}\right)\right\} (y = 1, 2, 3, \dots, b)$ be the album of IFRVs. So, Z_y are all the same, for example $Z_y = Z$ for all y, therefore:

$IFRAAPWG(Z_1, Z_2, \dots, Z_k) = Z$.

Proof: Assume$Z_y = \left\{\left(ø_{z_y}, Ơ_{z_y}\right), \left(ư_{z_y}, ȸ_{z_y}\right)\right\} (y = 1, 2, 3, \dots, b)$ so by utilizing Eq. 6. we obtain:

$$IFRAAPWG_{\omega_y}(Z_1, Z_2, \dots, Z_b) = \otimes_{y=1}^{b} \left(Z_y{}^{\omega_y}\right)$$

$$= \left\{ \begin{array}{c} \left(\left(e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(ø_{Z_y}\right)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right), \left(1 - e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(1 - Ơ_{Z_y}\right)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right) \right), \\ \left(\left(e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(ư_{Z_y}\right)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right), \left(1 - e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(1 - ȸ_{Z_y}\right)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right) \right) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(\left(e^{-\left(\left(-\log(ø_z)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right), \left(1 - e^{-\left(\left(-\log(1 - Ơ_z)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right) \right), \\ \left(\left(e^{-\left(\left(-\log(ư_z)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right), \left(1 - e^{-\left(\left(-\log(1 - ȸ_z)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right) \right) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(\left(e^{\log(ø_z)} \right), \left(1 - e^{\log((1 - Ơ_z))} \right) \right), \\ \left(\left(e^{\log(ư_z)} \right), \left(1 - e^{\log(1 - ȸ_z)} \right) \right) \end{array} \right\}$$

$$= \{(ø_z, Ơ_z), (ư_z, ȸ_z)\}$$

$$= Z$$

Theorem 8. (Boundedness) Assume $Z_y = \left\{\left(ø_{z_y}, Ơ_{z_y}\right), \left(ư_{z_y}, ȸ_{z_y}\right)\right\} (y = 1, 2, 3, \dots, b)$ be the collection of IFRVs. consider$Z^- = \min(Z_1, Z_2, \dots, Z_b)$ and $Z^+ = \max(Z_1, Z_2, \dots, Z_b)$, therefore

$$Z^- \leq IFRAAPWG_{\omega_y}(Z_1, Z_2, \dots, Z_b) \leq Z^+$$

Proof: Assume $Z_y = \left\{\left(ø_{z_y}, Ơ_{z_y}\right), \left(ư_{z_y}, ȸ_{z_y}\right)\right\} (y = 1, 2, 3, \dots, b)$ be the collection of IFRVs and consider$Z^- = \min(Z_1, Z_2, \dots, Z_b) = \{(ø_z^-, Ơ_z^-), (ư_z^-, ȸ_z^-)\}$ and $Z^+ = \max(Z_1, Z_2, \dots, Z_b) = \{(ø_z^+, Ơ_z^+), (ư_z^+, ȸ_z^+)\}$.So $ø_z^- = \min\left(ø_{z_y}\right)$, $Ơ_z^- = \max\left(Ơ_{z_y}\right)$,$ø_z^+ = \max\left(ø_{z_y}\right)$, $Ơ_z^+ = \min\left(Ơ_{z_y}\right)$ and$ư_z^- = \min\left(ư_{z_y}\right)$, $ȸ_z^- = \max\left(\partial_{R_z}\right)$, $ư_z^+ = \max\left(ư_{z_y}\right)$, $ȸ_z^+ = \min\left(ȸ_{z_y}\right)$. So, we obtain:

$$\left(e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log(ø_z^-)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right) \leq \left(e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(ø_{Z_y}\right)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right) \leq \left(e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log(ø_z^+)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right)$$

and

$$\left(e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log(ư_z^-)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right) \leq \left(e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(ư_{Z_y}\right)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right) \leq \left(e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log(ư_z^+)\right)^{ℸ}\right)^{\frac{1}{ℸ}}} \right)$$

and

$$\left(1-e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-\text{ơ}_z^{-}\right)\right)^{ঋ}\right)^{\frac{1}{ঋ}}}\right)\leq\left(1-e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-\text{ơ}_{z_y}\right)\right)^{ঋ}\right)^{\frac{1}{ঋ}}}\right)\leq\left(1-e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-\text{ơ}_z^{+}\right)\right)^{ঋ}\right)^{\frac{1}{ঋ}}}\right)$$

and

$$\left(1-e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-\text{ȸ}_z^{-}\right)\right)^{ঋ}\right)^{\frac{1}{ঋ}}}\right)\leq\left(1-e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-\text{ȸ}_{z_y}\right)\right)^{ঋ}\right)^{\frac{1}{ঋ}}}\right)\leq\left(1-e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-\text{ȸ}_z^{+}\right)\right)^{ঋ}\right)^{\frac{1}{ঋ}}}\right)$$

then,

$$Z^{-}\leq IFRAAPWG_{\omega_y}(Z_1,Z_2,\dots,Z_b)\leq Z^{+}$$

Theorem 9. (Monotonicity) Assume Z_y and $\breve{¥}_y (y=1,2,\dots,b)$ be a pair set of IFRVs, so $Z_y\leq \breve{¥}_y$ for very y, so,

$$IFRAAPWG_{\omega_y}(Z_1,Z_2,\dots,Z_b)\leq IFRAAPWG_{\omega_z}\left(\breve{¥}_1,\breve{¥}_2,\dots,\breve{¥}_b\right)$$

Proof: Straightforward.

5 | MADM ALGORITHM BASED ON IFRAAPWA AND IFRAAPWG OPERATORS

In this portion, we can define the problem of utilizing the proposed work based on IFRAAPWA and IFRAAPWG operators for IFRVs.

Assume $\text{ჶ}=(\text{ჶ}_1,\text{ჶ}_2,\dots,\text{ჶ}_z)$ are the selection of b different options for decision. Assume $=(b_1,b_2,\dots,b_k)$ are the selection of k attributes. Assume $\omega=(\omega_1,\omega_2,\dots,\omega_b)$ are the set of the weight vector (WV) for attributes with $\sum_{y=1}^{b}=1$. Assume the set of decision selectors offer the IFRVs $Z_y=\left\{\left(\text{ø}_{z_y},\text{ơ}_{z_y}\right),\left(\text{ư}_{z_y},\text{ȸ}_{z_y}\right)\right\}$. Next, we utilized the IFRVs decision matrix.

$$\text{Ӂ}=(Z_{xb})_{m\times n}=\begin{bmatrix}(\text{ø}_{11},\text{ơ}_{11}),(\text{ư}_{11},\text{ȸ}_{11}) & (\text{ø}_{12},\text{ơ}_{12}),(\text{ư}_{12},\text{ȸ}_{12}) & \cdots & (\text{ø}_{1m},\text{ơ}_{1m}),(\text{ư}_{1m},\text{ȸ}_{1m})\\(\text{ø}_{21},\text{ơ}_{21}),(\text{ư}_{21},\text{ȸ}_{21}) & (\text{ø}_{22},\text{ơ}_{22}),(\text{ư}_{22},\text{ȸ}_{22}) & \dots & (\text{ø}_{2m},\text{ơ}_{2m}),(\text{ư}_{2m},\text{ȸ}_{2m})\\\vdots & \vdots & \ddots & \vdots\\(\text{ø}_{n1},\text{ơ}_{n1}),(\text{ư}_{n1},\text{ȸ}_{n1}) & (\text{ø}_{n2},\text{ơ}_{n2}),(\text{ư}_{n2},\text{ȸ}_{n2}) & \cdots & (\text{ø}_{nm},\text{ơ}_{nm}),(\text{ư}_{nm},\text{ȸ}_{nm})\end{bmatrix}$$

We utilized IFRAAPWA and IFRAAPWG operators on this matrix for the best option. And illuminate the technique of MADM in the IFR system. The movement of my work is expressed:

Step 1. In the first movement, we create the IFR decision matrix, and we can utilize the data into a standard functional matrix.

Step 2. IFRAAPWA operator and IFRAAPWG operator are used on this matrix Z_{xb} and aggregate the IFR information.

$$IFRAAPWA_{\omega_y}(Z_1,Z_2,\dots,Z_b)=\oplus_{y=1}^{b}\left(\omega_y Z_y\right)$$

$$=\left\{\begin{array}{l}\left(\left(1-e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-\text{ø}_{z_y}\right)\right)^{ঋ}\right)^{\frac{1}{ঋ}}}\right),\left(e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(\text{ơ}_{z_y}\right)\right)^{ঋ}\right)^{\frac{1}{ঋ}}}\right)\right),\\\left(\left(1-e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(1-\text{ư}_{z_y}\right)\right)^{ঋ}\right)^{\frac{1}{ঋ}}}\right),\left(e^{-\left(\sum_{y=1}^{b}\omega_y\left(-\log\left(\text{ȸ}_{z_y}\right)\right)^{ঋ}\right)^{\frac{1}{ঋ}}}\right)\right)\end{array}\right\}$$

and

$$IFRAAPWG_{\omega_y}(Z_1, Z_2, \dots, Z_b) = \otimes_{y=1}^{b} \left(Z_y{}^{\omega_y}\right)$$

$$= \left\{ \begin{array}{l} \left(\left(e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(\text{ø}_{Z_y}\right)\right)^{\gimel}\right)^{\frac{1}{\gimel}}} \right), \left(1 - e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(1-\text{Ơ}_{Z_y}\right)\right)^{\gimel}\right)^{\frac{1}{\gimel}}} \right) \right), \\ \left(\left(e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(\text{ư}_{Z_y}\right)\right)^{\gimel}\right)^{\frac{1}{\gimel}}} \right), \left(1 - e^{-\left(\sum_{y=1}^{b} \omega_y \left(-\log\left(1-\text{ȸ}_{Z_y}\right)\right)^{\gimel}\right)^{\frac{1}{\gimel}}} \right) \right) \end{array} \right\}$$

Step 3. We apply the SF formula defined in Definition 7. to choose the best option.

$$SF(Z) = \left(\frac{2 + \text{ø}_Z + \text{ư}_Z - \text{Ơ}_Z - \text{ȸ}_Z}{4}\right), SF(Z) = [0, 1]$$

Step 4. To arrange the score value, take from Step 3. and find the most suitable alternative.

Step 5. End.

6 | APPLICATIONS

In this portion, we explained our proposed work with the help of numerical examples based on the MADM problem and medical diagnosis problem.

6.1 | Medical Diagnosis

In this section, we elaborate on the importance of robotic technology or AOs based on fuzzy logic in the medical field and how we use artificial intelligence by applying developed AOs in fuzzy environments.

Robotic technology plays a vital role in MD, prescribes medicians, performs operations in operation theatres (OT), and many robot designs for physiotherapy (PT). The importance of robotic technology is described as follows:

1. MD through artificial intelligence (AI) gives more preciseness and accuracy to investigate problems in the human body.
2. Robotic technology gives more precision with better ethnicity in real-time imaging (X-rays and ultrasound) through AI.
3. Using AI, we diagnose many decisions within very short periods with preciseness.
4. In many PT clinics, robots are used to perform therapy and minimize the workload of premedical staff.

While robotic technology has many advantages for diagnosing diseases, it is crucial to maintain a balance between human skill and automation. Soon, worldwide will be filled with robotic technology, and these robots will perform multiple tasks. These robotic technologies perform tasks using programs and algorithms based on fuzzy logic that mathematicians construct. In this numerical problem, we will discuss the MD challenging problem through AI technology based on proposed IFRAAPWA and IFRAAPWG operators.

Figure 1: Graphical representation of MD and treatment through artificial intelligence.

We use the MD method to detect a disease based on signs and symptoms. To see the illness, DM tries to find the condition based on the patient's symptoms. Based on a person's symptoms, DM chooses the right disease. FS theory (FST) gives way to decision-makers who decide the proper condition for the patients. In this process, we utilize IFRVs for MD. Also, a better understanding of of the MD problem is shown in Flowchart 1.

Flowchart 2: Graphical representation of MD procedure.

Example 2. Suppose $֍ = \{֍_1 = Viral\ hepatitis, ֍_2 = poliomyelitis, ֍_3 = Rubella, ֍_4 = Tetanus, ֍_5 = Tuberculosis\}$ is the set of five disease-like alternatives, and $ƀ = \{ƀ_1 = Fever, ƀ_2 = Diarrhea, ƀ_3 = Fatigue, ƀ_4 = Muscle\ aches\}$ be the set of attributes. Assume DM experts Ŗ that give all the collection in IFRVs form and $\omega = (0.53, 0.24, 0.16, 0.07)$ are the WV of this attribute. So, the sum of all WV is one as

$\sum_{z=1}^{k} \omega_z = 1$. Constructing decision matrices Z_{xb} in Table 1. It is necessary to find the disease of Mr. A. So, to see the condition, we can use all the algorithms step by step.

Step 1. In this step, we construct the decision matrix as IFRVs.

Table 1: Decision matrix Z_{xb}

	$ƀ_1 = Fever$	$ƀ_2 = Diarrhea$	$ƀ_3 = Fatigue$	$ƀ_4 = Muscle\ aches$
$֍_1 = Viral\ hepatitis$	$\left\{\begin{matrix}(0.4,0.5),\\(0.3,0.4)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.4,0.3),\\(0.8,0.2)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.4,0.3),\\(0.2,0.6)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.1,0.7),\\(0.5,0.5)\end{matrix}\right\}$
$֍_2 = poliomyelitis$	$\left\{\begin{matrix}(0.6,0.2),\\(0.5,0.2)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.7,0.2),\\(0.6,0.3)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.6,0.2),\\(0.4,0.5)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.6,0.4),\\(0.5,0.4)\end{matrix}\right\}$
$֍_3 = Rubella$	$\left\{\begin{matrix}(0.5,0.3),\\(0.4,0.4)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.6,0.1),\\(0.5,0.4)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.3,0.6),\\(0.5,0.2)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.3,0.2),\\(0.2,0.1)\end{matrix}\right\}$
$֍_4 = Tetanus$	$\left\{\begin{matrix}(0.7,0.1),\\(0.6,0.3)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.8,0.2),\\(0.4,0.6)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.9,0.1),\\(0.6,0.1)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.5,0.4),\\(0.3,0.3)\end{matrix}\right\}$

Step 2. Suppose parameter ঽ = 5 for our calculation. Using IFRAAPWA and IFRAAPWG formulas on this matrix all the aggregating values in Table 2.

Table 2: Aggregation outcomes of decision matrix Z_{xb}

	IFRAAPWA	IFRAAPWG
$ƀ_1$	$\left\{\begin{matrix}(0.1965,0.6424),\\(0.4088,0.5834)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.5487,0.2825),\\(0.5802,0.2521)\end{matrix}\right\}$
$ƀ_2$	$\left\{\begin{matrix}(0.3575,0.5017),\\(0.2816,0.5327)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.8089,0.1263),\\(0.7280,0.1936)\end{matrix}\right\}$
$ƀ_3$	$\left\{\begin{matrix}(0.2787,0.4629),\\(0.2318,0.5299)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.6694,0.2426),\\(0.6399,0.1899)\end{matrix}\right\}$
$ƀ_4$	$\left\{\begin{matrix}(0.5229,0.3908),\\(0.3101,0.4899)\end{matrix}\right\}$	$\left\{\begin{matrix}(0.7897,0.1619),\\(0.7030,0.2593)\end{matrix}\right\}$

Step 3. Using data from Table 3. We find the SF, defined in Definition 7.

Table 3: Scores value of aggregated data

	IFRAAPWA	IFRAAPWG
$ƀ_1$	0.3448	0.6486
$ƀ_2$	0.4011	0.8042
$ƀ_3$	0.3794	0.7192
$ƀ_4$	0.4881	0.7679

The geometrical representation of SF calculated using IFRAAPWA and IFRAAPWG operators is provided in Figure 2.

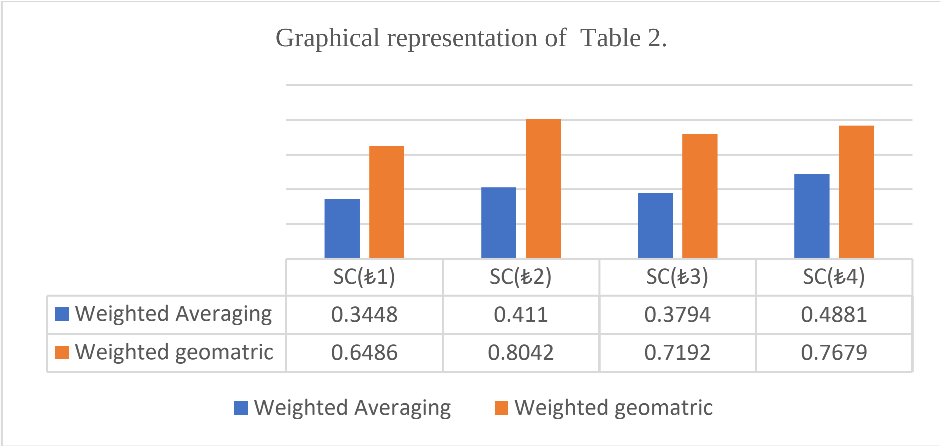


Figure 2: Geometrical representation of SF.

Step 4. From this calculation, we can find Mr. A's tetanus disease using IFRAAPWA. By using IFRAAPWG disease will be poliomyelitis. The arrangement of all alternatives is given in Table 3.

Table 3: Ranking ordering

	Ranking
IFRAAPWA	$ƀ_4 > ƀ_2 > ƀ_3 > ƀ_1$
IFRAAPWG	$ƀ_2 > ƀ_4 > ƀ_3 > ƀ_1$

7 | SENSITIVITY ANALYSIS

In this article segment, we discussed the parametric effects on ranking order of aggregation findings. Different ordering by changing the parameter value in IFRAAPWA and IFRAAPWG operators are discussed in Table 5. and Table 6, along with graphical representations in Figures 3 and 4. respectively.

7.1 | The effect of parameter

In this article, we noticed that all the aggregation operators depend on the parameter ঽ. We observed that the unlike result of ঽ effect on the arrangement of the ranking values of our proposed work.

The effect of ঽ

In this article, we take the value of the parameter as ঽ $= 5$. If we change the value of ঽ, then it can also affect the ordering of alternatives. We observed that the SF of our AOs will change when the parameter ঽ is varied. Also, the variation of parametric value in the proposed IFRAAPWA and IFRAAPWG operators is shown in Table 5. and Table 6.

Table 5: The arrangement SF by varying of ঽ IFRAAPWA

ঽ	Ordering	ঽ	Ordering
1	$ƀ_4 > ƀ_2 > ƀ_3 > ƀ_1$	2	No results
3	$ƀ_4 > ƀ_2 > ƀ_3 > ƀ_1$	4	No results
5	$ƀ_4 > ƀ_2 > ƀ_3 > ƀ_1$	6	No results
7	$ƀ_4 > ƀ_2 > ƀ_3 > ƀ_1$	8	No results
9	$ƀ_4 > ƀ_2 > ƀ_3 > ƀ_1$	10	No results
11	$ƀ_4 > ƀ_3 > ƀ_2 > ƀ_1$	20	No results

13	$ƀ_4 > ƀ_3 > ƀ_2 > ƀ_1$	40	No results
19	$ƀ_4 > ƀ_3 > ƀ_2 > ƀ_1$	80	No results
101	$ƀ_4 > ƀ_3 > ƀ_2 > ƀ_1$	100	No results
199	$ƀ_4 > ƀ_3 > ƀ_2 > ƀ_1$	200	No results
299	$ƀ_4 > ƀ_3 > ƀ_2 > ƀ_1$	300	No results

We can quickly notice that, in the varying in ϡ in the IFRAAPWA operator, if we use the odd number in proposed AOs, then the arrangement of SF also varies. When we change the parameter, the ranking order also varies. When we take ϡ = 11 in IFRAAPWA, all the ranking values will be the same. Conversely, we put the even values in the IFRAAPWA operator; no results and arrangement outcomes can be found.

Table 6: The arrangement score value by varying of ϡ IFRAAPWG

ϡ	Ordering	ϡ	Ordering
1	$ƀ_4 > ƀ_2 > ƀ_3 > ƀ_1$	2	No results
3	$ƀ_2 > ƀ_4 > ƀ_3 > ƀ_1$	4	No results
5	$ƀ_2 > ƀ_4 > ƀ_3 > ƀ_1$	6	No results
7	$ƀ_2 > ƀ_4 > ƀ_3 > ƀ_1$	8	No results
9	$ƀ_2 > ƀ_4 > ƀ_3 > ƀ_1$	10	No results
11	$ƀ_2 > ƀ_4 > ƀ_3 > ƀ_1$	20	No results
13	$ƀ_2 > ƀ_4 > ƀ_3 > ƀ_1$	40	No results
19	$ƀ_2 > ƀ_4 > ƀ_3 > ƀ_1$	80	No results
101	$ƀ_2 > ƀ_4 > ƀ_3 > ƀ_1$	100	No results
199	$ƀ_2 > ƀ_4 > ƀ_3 > ƀ_1$	200	No results
299	$ƀ_2 > ƀ_4 > ƀ_3 > ƀ_1$	300	No results

We observed that when we take a variation of parametric value ϡ in the IFRAAPWG operator and use the odd number in offered AOs, the arrangement of SF also varies, and the ranking order varies. But when we take ϡ = 3 in IFRAAPWG, all the ranking values will remain the same. Conversely, we put the even values in the IFRAAPWG operator; no results and arrangement outcomes will be found.

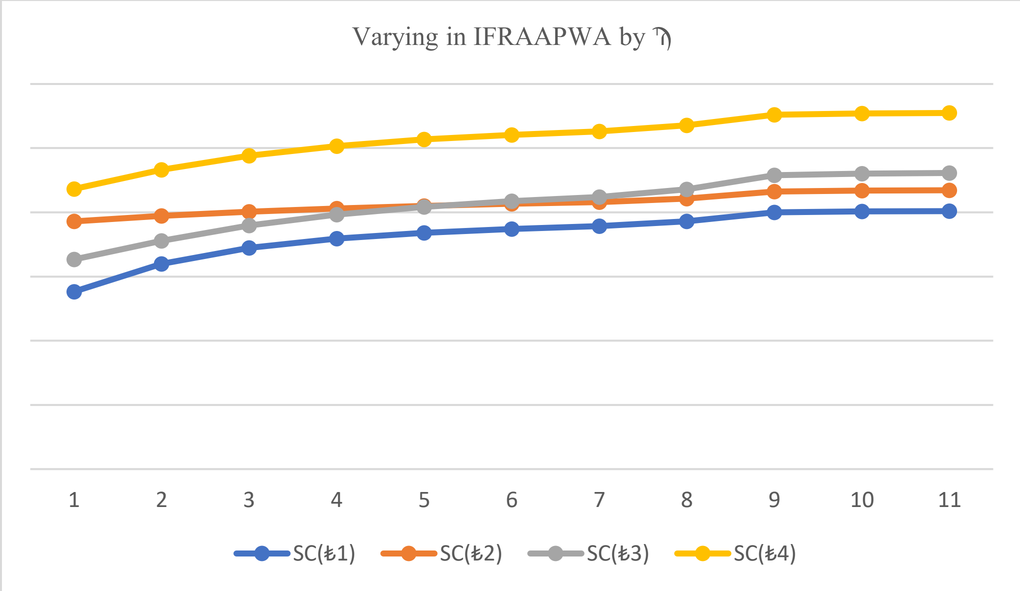


Figure 3: Graphically presentation of SF by variation of ℷ in IFRAAPWA operator.

The aggregation outcomes of Table 5. we are represented in Figure 3. We observe that the arrangements of the SF vary when the value of parameter ℷ is varied. It can be observed that when parameter Ɗ is an even number, our aggregation operator cannot evaluate IFR information.

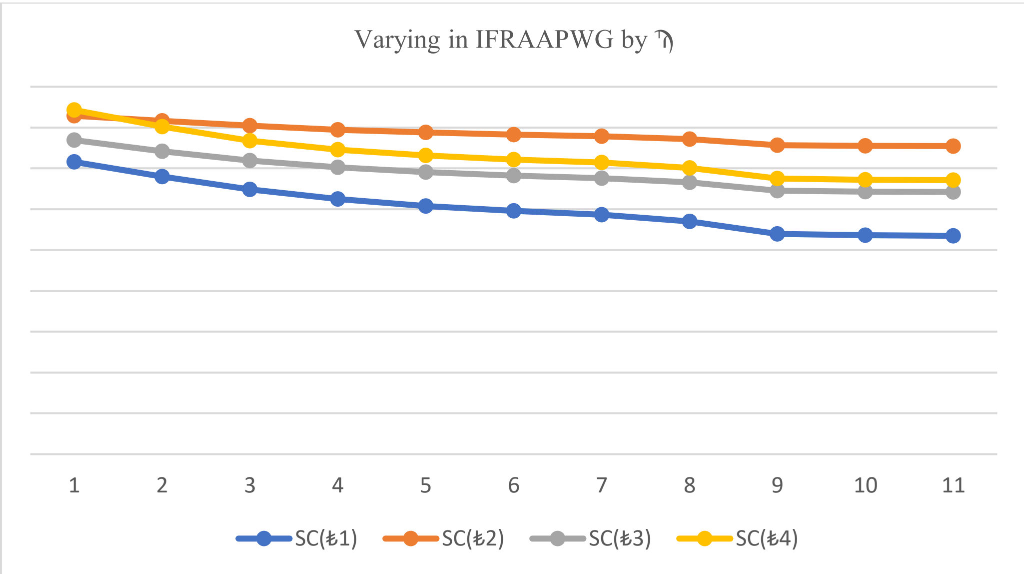


Figure 4: Graphically presentation of SF by variation of parameter ℷ in IFRAAPWG operator.

The aggregation outcomes of Table 6. can be represented in Figure 4. We observed that the arrangements of the SF vary when the value of parameter ℷ changes. It is also noticed that when we take the weight of parameter Ɗ as an even number, our aggregation operator cannot aggregate the IFR information.

8 | COMPARATIVE STUDY

The following portion compares our proposed work on IFRAAPWA and IFRAAPWG operators with other AOs. We corresponded with the existing approach presented by Hussain et al. [25], known as Dombi IFR Weighted averaging (IFRDWA) and Dombi IFR weighted geometric (IFRDWG) operators, and Yahya et al. [26] developed IFR Frank weighted averaging (IFRFWA) and IFR Frank weighted geometric (IFRFWG) operators. IF Einstein Hybrid weighted averaging (IFRRHWA) and IF Einstein Hybrid weighted geometric (IFRRHWG) operators were introduced by Zhang [27].

Furthermore, it is noticed that we understand that all current ideas are difficult for dealing with the IFRVs. But our proposed work can do. So, our proposed work is more reliable. However, when we examined the concept of IFWA proposed by Xu. [28], the thought of IFWG developed by Xu and Yager [12] and the idea of IF Hybrid averaging (IFHA) addressed by Huang et al. [29] operators can deal IFVs effectively. So, there are many decision specialist states where we essential IFRVs. Our proposed work allows decision specialists to apply IFRVs according to their conditions. So, in a comparative study of all properties, we observe that current operators are more general and practical, assisting decision specialists. On the other hand, many different FS cannot handle IFRS information. For example, cubic IFS (CIFS) was developed by Khan et al. [30]. Table 5.

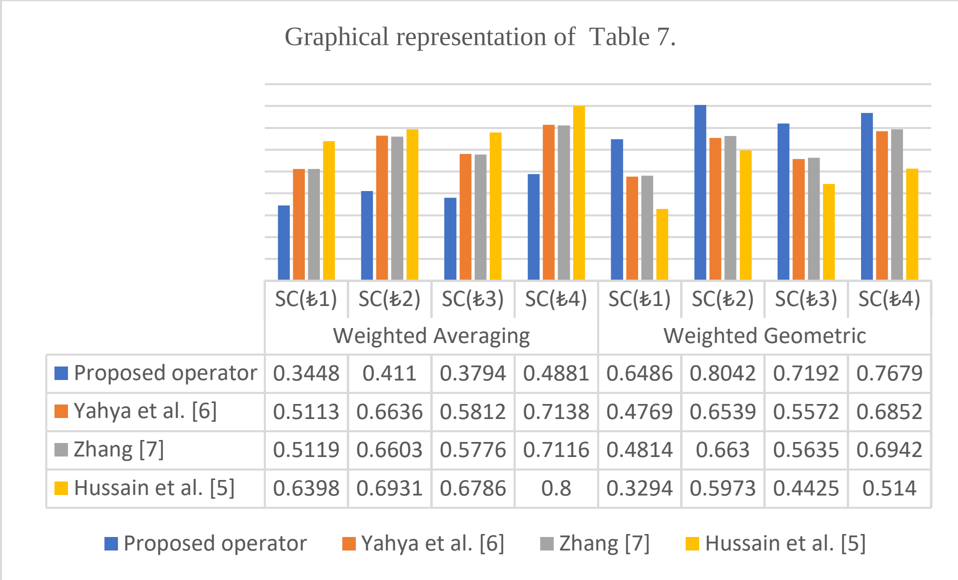
Show all the comparative analyses with existing AOs. In addition, the geometrical representation of information in Table 7. is provided in Figure 5.

Table 7. Comparative study

Method	Operator	Ranking
Proposed work	IFRAAPWA	$Ƚ_4 > Ƚ_2 > Ƚ_3 > Ƚ_1$
	IFRAAPWG	$Ƚ_2 > Ƚ_4 > Ƚ_3 > Ƚ_1$
Yahya et al. [26]	IFRFWA	$Ƚ_4 > Ƚ_2 > Ƚ_3 > Ƚ_1$
	IFRFWG	$Ƚ_4 > Ƚ_2 > Ƚ_3 > Ƚ_1$
Zhang [27]	IFEHWA	$Ƚ_4 > Ƚ_2 > Ƚ_3 > Ƚ_1$
	IFEHWG	$Ƚ_4 > Ƚ_2 > Ƚ_3 > Ƚ_1$
Hussain et al. [25]	IFRDWA	$Ƚ_4 > Ƚ_2 > Ƚ_3 > Ƚ_1$
	IFRDWG	$Ƚ_2 > Ƚ_4 > Ƚ_3 > Ƚ_1$
Xu. [28]	IFWA	No result
Xu and Yager [12]	IFWG	No result
Huang et al. [29]	IFHWA	No result
Khan et al. [30]	CIFS	No result

Table 7. provides a comprehensive analysis of proposed IFRAAPWA and IFRAAPWG operators with existing prevailing AOs approaches. Also discussed is that the simple IFS structure cannot deal with complex and advanced information because there is no Idea for LA and UA spaces in a simple IFS concept. So, our presented MADM approaches are more reliable and suitable for data aggregation than existing ones.

MADM approaches are more reliable and suitable for data aggregation than existing ones.



	Weighted Averaging				Weighted Geometric			
	SC(ŧ1)	SC(ŧ2)	SC(ŧ3)	SC(ŧ4)	SC(ŧ1)	SC(ŧ2)	SC(ŧ3)	SC(ŧ4)
Proposed operator	0.3448	0.411	0.3794	0.4881	0.6486	0.8042	0.7192	0.7679
Yahya et al. [6]	0.5113	0.6636	0.5812	0.7138	0.4769	0.6539	0.5572	0.6852
Zhang [7]	0.5119	0.6603	0.5776	0.7116	0.4814	0.663	0.5635	0.6942
Hussain et al. [5]	0.6398	0.6931	0.6786	0.8	0.3294	0.5973	0.4425	0.514

Figure 5: Geometrical presentation of comparative study

9 | CONCLUSION

Researchers have many attempts to find more reliable and valuable results for new theories in FS structures as IFRS is a more appropriate and advanced approach than IFS, which gives more dedicated space for decision specialists. The MADM is a powerful and crucial technique for selecting the best alternative from the numerous alternatives, and AOs play a vital role in data aggregation. So, by inspiring the IFRS framework and AATNM, AATVNM operational rules, here in this article, we established new AOs called IFRAAPWA and IFRAAPWG operators. Also, we addressed some necessary axioms of AOs. We offered the MADM algorithm based on the proposed IFRAAPWA and IFRAAPWG operators and numerical examples. In addition, a comparative analysis of developed AOs shows the superiority and authenticity of suggested AOs.

In the near future, we will extend our work for the Picture fuzzy (PF) Maclaurin symmetric (PFMS) AOs developed by Ullah [31], the bipolar fuzzy set (BFS) framework given by Mahmood [32], the idea of spherical FS is presented by Mahmood et al. [33]. The thought of T-spherical FS (TSFS) is delivered by Ullah et al. [34].

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