ORIGINAL ARTICLE

OPEN ACCESS

Selection of Database Management System by Using Multi-Attribute Decision-Making Approach Based on Probability Complex Fuzzy Aggregation Operators

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Article History:

Submitted: December 11, 2022
Revised: May 21, 2023
Accepted: June 03, 2023
Published online: June 30, 2023

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ABSTRACT

A database management system (DBMS) is a piece of software that makes it easier to create, organize, store, retrieve, and manage structured data. It functions as a central system for effectively managing data storage and access, ensuring data integrity and security, and offering tools for querying and reporting. Because of its multi-criteria nature, choosing an optimal DBMS is a multi-attribute decision-making (MADM) dilemma. Thus, in this script, we devise some elementary operations and interpret probability AOs within the cartesian form of complex fuzzy set (CFS) such as probability complex fuzzy weighted averaging (P-CFWA), probability complex fuzzy ordered weighted averaging (P-CFOWA), immediate P-CFOWA (IP-CFOWA), probability complex fuzzy weighted geometric (P-CFWG), probability complex fuzzy ordered weighted geometric (P-CFOWG), immediate P-CFOWG (IP-CFOWG) operators. Further, we devise a technique of MADM within the structure of CFS to tackle complicated and awkward genuine life MADM dilemmas that contain extra fuzzy information. After that, in this article, we investigate a numerical example "selection of the optimal database management system" to cope with a MADM dilemma within CFS. In the last of this article, we compare the invented theory with certain other prevailing theories to reveal supremacy and dominance.

Keywords: Database management system; probability averaging/geometric aggregation operators; complex fuzzy set; MADM approach.

1 | INTRODUCTION

A DBMS is a piece of software that enables the systematic and effective generation, maintenance, organization, and retrieval of data. It acts as a link between the users and the data storage itself by providing an abstraction layer that allows users and programmers to interact with the data without having to be aware of the underlying technical details. DBMS can be beneficial in many ways where users can store, update, design, and retrieve data. By using DBMS, users can perform complex searches and protect the integrity and security of data. The significance of a DBMS is its capacity to produce a structured and centralized approach to data management. Data has become a crucial resource for individuals, businesses, and companies in the digital era. To ensure data integrity, DBMS imposes referential integrity conditions, standards for the coherence of data, and verification standards. DBMS provides a shield while managing data manually against errors and differences that might take place. In addition, simultaneous access by multiple users and applications is a characteristic of a well-designed DBMS which makes sure that the data will remain consistent and accessible in case of huge demand. Ogli [1] investigated the difference between the concept of DBSM and database. Fichte et al.[2] devised DBMS and treewidth for counting. Huang et al.[3] analyzed probabilistic DBMS.

DBMSs provide a substantial contribution to data privacy and security. They offer tools for access control, enabling administrators to specify which data pieces may be accessed by which people. This restricted access stops unauthorized people from altering or obtaining sensitive data. To ensure that data can be restored to a prior state in the event of unintentional data loss or system failure, DBMSs frequently provide backup and recovery procedures. Business-wise, DBMSs provide improved decision-making skills. They allow for the storing of enormous volumes of historical and real-time data, which can then be examined to glean insightful information. DBMSs provide reporting, data analysis, and business intelligence by enabling complicated queries and aggregations. This enables firms to make defensible decisions based on accurate and current information. Fry and Sibley [4] discussed the evolution of DBMSs. Selinger et al. [5] interpreted an access path selection in a relational DBMS. Pavlo et al. [6] devised self-driving DBMS.

In circumstances when category or membership boundaries are ambiguous, classical set theory, which deals with definite, well-defined differences between components, is frequently insufficient. The Zadeh [7] theory's fuzzy set (FS) comes into play in this situation. To cope with uncertainty, ambiguity, and imprecision in diverse fields, the theory of FS offers a more practical and flexible framework. Decision experts may use it to integrate a human-like knowledge of uncertainty into their procedures, resulting in more informed and reliable judgments. Each element within the structure of FS has a degree of support that placed in $[0, 1]$. Gholamizadeh et al. [8] discussed the applications and contributions of FS theory. Stefanakis and Sellis [9] diagnosed DBMS for GIS with FS approaches. The fuzziness in DBMSs was discussed by Kacprzyk et al. [10]. Touzi and Hassine [11] interpreted the architecture of fuzzy DBMSs. A complex fuzzy set (CFS) invented by Ramot et al. [12] is a kind of mathematical structure that modifies the notion of FS by allowing elements to have a degree of support in the unit disc of the complex plane rather than $[0, 1]$. Within the structure of CFS invented by Ramot et al. [12] the elements have the degrees of support in polar form. In 2011, Tamir et al. [13] discussed a novel interpretation of the degree of support within the structure of CFS by transforming the range of CFS from unit circle to unit square of a complex plane. The degree of support of each element within the structure of CFS devised by Tamir et al. [13] is in the cartesian form. In several areas, such as decision-making (DM), pattern recognition, artificial intelligence, and control systems, CFS may be utilized to simulate increasingly complex interactions and uncertainties. It's important to keep in mind that compared to FS, working with CFS can involve more complex mathematics and computation, and applying them may call for a thorough grasp of fuzzy logic and related ideas. Sobhi and Dick [14] presented an investigation of CFS for large learning. Bi et al. [15] devised entropy measures for CFS and Luqman et al. [16] diagnosed hypergraph for CFS. Imtiaz et al. [17] discussed CF morphisms.

In data analysis and database administration, aggregate operators (AOs) are mathematical functions that integrate and summarize various data values into a single output. In processes like producing reports, summarizing information, and extracting valuable data from huge databases, aggregation is crucial. It simplifies complicated data into relevant metrics that may be used for analysis and DM. Thus, various scholars developed numerous AOs such as Bi et al. [18] diagnosed arithmetic AOs for CFS, Bi et al. [19] interpreted geometric AOs for CFS, and Hu et al. [20] devised power AOs for CFS. Liaqat et al. [21] devised Aczel-Alsina AOs under the setting of an interval-valued complex single-valued neutrosophic set. Ozer [22] discussed prioritized AOs for complex picture FS by employing Hamacher t-norm and t-conorm.

A notion of probabilistic information dominates the inquiry to manage the subjective and objective information more evidently. Thus, Merigo et al. [23], [24] devised the theory of probability into the AOs to handle the probabilistic information in DM dilemmas. Yager et al. [25] discussed immediate probabilities and Engemann et al. [26] invented a DM approach employing immediate probabilities. Probability intuitionistic fuzzy AOs were diagnosed by Sirbiladze and Sikharulidze [27]. Garg [28] devised immediate probabilities under the setting of Pythagorean FS. Further, in the prevailing literature, we observed that there are a few AOs within the structure of CFS that cope merely with the complex fuzzy (CF) information in polar form and there is no operator in the cartesian structure of CFS in the literature. Moreover, for tackling genuine life dilemmas, probabilistic information with uncertainty and ambiguity plays a critical role. Therefore, there is a requirement to construct some AOs which can cope with probabilistic information, while aggregating the given CF information. Motivated by the requirement of AOs in the setting of CFS and the need for AOs that can handle the probabilistic information, in this article, we devise probabilistic AOs in the setting of CFS. These operators are P-CFWA, P-CFOWA, IP-CFOWA, P-CFWG, P-CFOWG, and IP-CFOWG. We also devise the elementary operation for CFS and discuss the proposition of the invented operators. Additionally, in this script, we demonstrate a technique of MADM dilemma and then we investigate a MADM dilemma "Selection of the optimal database management system".

The other article is constructed as: In section 2, we investigate the theory of FS, CFS, and its operations. In section 3, we inaugurate fundamental operations, score, and accuracy functions under CF information. We also inaugurate the average and geometric AOs under the CF setting based on probability. In Section 4, we devise an approach of MADM within CFS and explain a numerical example "selection of optimal database management system". The comparative study of the proposed theory is also part of Section 4. In Section 5, the conclusion of this article is interpreted.

2 | PRELIAMINARIES

In this part, we investigate the theory of FS, CFS, and its operations.

Definition 1: [7] The below given model

$$\mathcal{C}_{FS} = \left\{\left(\ell, \mu_{\mathcal{C}_{FS}}(\ell)\right) \mid \ell \in \mathcal{L}\right\}$$

Interprets the model of FS, where $\mu_{\mathcal{C}_{CFS}}(\ell)$ is demonstrated as the degree of support placed in $[0,1]$.

Definition 2: [13] The below-given model.

$$\mathcal{C}_{CFS} = \left\{\left(\ell, \mu_{\mathcal{C}_{CFS}}(\ell)\right) \mid \ell \in \mathcal{L}\right\} = \left\{\left(\ell, \mu^{R}_{\mathcal{C}_{CFS}}(\ell) + \iota\, \mu^{I}_{\mathcal{C}_{CFS}}(\ell)\right) \mid \ell \in \mathcal{L}\right\},$$

Interprets the model of CFS, where $\mu_{\mathcal{C}_{CFS}}(\ell) = \left(\mu^{R}_{\mathcal{C}_{CFS}}(\ell) + \iota\, \mu^{I}_{\mathcal{C}_{CFS}}(\ell)\right)$ is demonstrated as the degree of support placed in a unit square of a complex plane and $\mu^{R}_{\mathcal{C}_{CFS}}(\ell), \mu^{I}_{\mathcal{C}_{CFS}}(\ell) \in [0,1]$. $\mathcal{C}_{CFS} = \left(\mu_{\mathcal{C}_{CFS}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS}} + \iota\, \mu^{I}_{\mathcal{C}_{CFS}}\right)$ would be revealed as CF number (CFN)

Definition 3: [13] For two CFSs $\mathcal{C}_{CFS-1} = \left\{\left(\ell, \mu_{\mathcal{C}_{CFS-1}}(\ell)\right) \mid \ell \in \mathcal{L}\right\} = \left\{\left(\ell, \mu^{R}_{\mathcal{C}_{CFS-1}}(\ell) + \iota\, \mu^{I}_{\mathcal{C}_{CFS-1}}(\ell)\right) \mid \ell \in \mathcal{L}\right\}$ and $\mathcal{C}_{CFS-2} = \left\{\left(\ell, \mu_{\mathcal{C}_{CFS-2}}(\ell)\right) \mid \ell \in \mathcal{L}\right\} = \left\{\left(\ell, \mu^{R}_{\mathcal{C}_{CFS-2}}(\ell) + \iota\, \mu^{I}_{\mathcal{C}_{CFS-2}}(\ell)\right) \mid \ell \in \mathcal{L}\right\}$, we have

$$(\mathcal{C}_{CFS-1})^{c} = \left\{\left(\ell, 1 - \mu^{R}_{\mathcal{C}_{CFS-1}}(\ell) + \iota\left(1 - \mu^{I}_{\mathcal{C}_{CFS-1}}(\ell)\right)\right) \mid \ell \in \mathcal{L}\right\}$$

$$\mathcal{C}_{CFS-1} \cup \mathcal{C}_{CFS-2} = \left\{\begin{pmatrix} \ell, \max\left(\mu^{R}_{\mathcal{C}_{CFS-1}}(\ell), \mu^{R}_{\mathcal{C}_{CFS-2}}(\ell)\right) \\ +\iota\ \max\left(\mu^{I}_{\mathcal{C}_{CFS-1}}(\ell), \mu^{I}_{\mathcal{C}_{CFS-2}}(\ell)\right) \end{pmatrix} \mid \ell \in \mathcal{L}\right\}$$

$$\mathcal{C}_{CFS-1} \cap \mathcal{C}_{CFS-2} = \left\{\begin{pmatrix} \ell, \min\left(\mu^{R}_{\mathcal{C}_{CFS-1}}(\ell), \mu^{R}_{\mathcal{C}_{CFS-2}}(\ell)\right) \\ +\iota\ \min\left(\mu^{I}_{\mathcal{C}_{CFS-1}}(\ell), \mu^{I}_{\mathcal{C}_{CFS-2}}(\ell)\right) \end{pmatrix} \mid \ell \in \mathcal{L}\right\}$$

3 | PROBABILISTIC AOS FOR COMPLEX FUZZY INFORMATION

Here, we inaugurate the average and geometric AOs under the CF setting based on probability. These AOs are P-CFWA, P-CFOWA, IP-CFOWA, P-CFWG, P-CFOWG, and IP-CFOWG operators. But first, we need to develop fundamental operations, score, and accuracy functions under CF information.

Definition 4: For CFNs, $\mathcal{C}_{CFS-1} = \left(\mu_{\mathcal{C}_{CFS-1}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS-1}} + \iota\, \mu^{I}_{\mathcal{C}_{CFS-1}}\right)$ and $\mathcal{C}_{CFS-2} = \left(\mu_{\mathcal{C}_{CFS-2}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS-2}} + \iota\, \mu^{I}_{\mathcal{C}_{CFS-2}}\right)$ and $\varpi \geq 0$, then

$$\mathcal{C}_{CFS-1} \oplus \mathcal{C}_{CFS-2} = \left(\mu^{R}_{\mathcal{C}_{CFS-1}} + \mu^{R}_{\mathcal{C}_{CFS-2}} - \mu^{R}_{\mathcal{C}_{CFS-1}}\mu^{R}_{\mathcal{C}_{CFS-2}} + \iota\left(\mu^{I}_{\mathcal{C}_{CFS-1}} + \mu^{I}_{\mathcal{C}_{CFS-2}} - \mu^{I}_{\mathcal{C}_{CFS-1}}\mu^{I}_{\mathcal{C}_{CFS-2}}\right)\right)$$

$$\mathcal{C}_{CFS-1} \otimes \mathcal{C}_{CFS-2} = \left(\mu^{R}_{\mathcal{C}_{CFS-1}}\mu^{R}_{\mathcal{C}_{CFS-2}} + \iota\, \mu^{I}_{\mathcal{C}_{CFS-1}}\mu^{I}_{\mathcal{C}_{CFS-2}}\right)$$

$$\varpi\mathcal{C}_{CFS-1} = \left(1 - \left(1 - \mu^{R}_{\mathcal{C}_{CFS-1}}\right)^{\varpi} + \iota\left(1 - \left(1 - \mu^{I}_{\mathcal{C}_{CFS-1}}\right)^{\varpi}\right)\right)$$

$$\mathcal{C}^{\varpi}_{CFS-1} = \left(\left(\mu^{R}_{\mathcal{C}_{CFS-1}}\right)^{\varpi} + \iota\left(\mu^{I}_{\mathcal{C}_{CFS-1}}\right)^{\varpi}\right)$$

Definition 5: For a CFN $\mathcal{C}_{CFS} = \left(\mu_{\mathcal{C}_{CFS}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS}} + \iota\, \mu^{I}_{\mathcal{C}_{CFS}}\right)$ the score and accuracy functions are devised as

$$\dot{\check{S}}(\mathcal{C}_{CFS}) = \mu^{R}_{\mathcal{C}_{CFS}} - \mu^{I}_{\mathcal{C}_{CFS}}, \quad \dot{\check{S}}_{\mathcal{B}}(\mathcal{C}_{CFS}) \in [-1, 1] \qquad (1)$$

$$\acute{H}(\mathcal{C}_{CFS}) = \mu^{R}_{\mathcal{C}_{CFS}} + \mu^{I}_{\mathcal{C}_{CFS}}, \quad \acute{H}(\mathcal{C}_{CFS}) \in [0, 1] \qquad (2)$$

From Eq. (1) and (2), we have

$$\text{If } \mathring{\check{S}}(\mathcal{C}_{CFS-1}) < \mathring{\check{S}}(\mathcal{C}_{CFS-2}) \text{ then } \mathcal{C}_{CFS-1} < \mathcal{C}_{CFS-2}$$

$$\text{If } \mathring{\check{S}}(\mathcal{C}_{CFS-1}) > \mathring{\check{S}}(\mathcal{C}_{CFS-2}) \text{ then } \mathcal{C}_{CFS-1} > \mathcal{C}_{CFS-2}$$

$$\text{If } \mathring{\check{S}}(\mathcal{C}_{CFS-1}) = \mathring{\check{S}}(\mathcal{C}_{CFS-2}) \text{ then we have}$$

$$\text{If } {}'\mathrm{H}(\mathcal{C}_{CFS-1}) < {}'\mathrm{H}(\mathcal{C}_{CFS-2}) \text{ then } \mathcal{C}_{CFS-1} < \mathcal{C}_{CFS-2}$$

$$\text{If } {}'\mathrm{H}(\mathcal{C}_{CFS-1}) > {}'\mathrm{H}(\mathcal{C}_{CFS-2}) \text{ then } \mathcal{C}_{CFS-1} > \mathcal{C}_{CFS-2}$$

$$\text{If } {}'\mathrm{H}(\mathcal{C}_{CFS-1}) = {}'\mathrm{H}(\mathcal{C}_{CFS-2}) \text{ then } \mathcal{C}_{CFS-1} = \mathcal{C}_{CFS-2}$$

3.1 | PROBABILITY CF AVERAGING OPERATOR

In this subsection, we will devise P-CFWA, IP-CFOWA, and P-CFOWA operators and will interpret related propositions.

Definition 6: For the group of CFNs $\mathcal{C}_{CFS-ע} = \left(\mu_{\mathcal{C}_{CFS-ע}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS-ע}} + \iota\, \mu^{I}_{\mathcal{C}_{CFS-ע}}\right)$, $ע = 1, 2, \dots, \mathring{y}$, the P-CFWA operator is devised as

$$P - CFWA(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \dots, \mathcal{C}_{CFS-\check{Y}}) = \bigoplus_{ע=1}^{\mathring{y}} \Xi_{\mathcal{P}\mathcal{W}-ע}\mathcal{C}_{CFS-ע}$$

Where $\Xi_{\mathcal{P}\mathcal{W}-ע} = \xi \mathfrak{B}_{\mathcal{P}\mathcal{B}-ע} + (1-\xi)\omega_{\mathcal{W}-ע}$ with $\xi \in [0,1]$ would be a weight vector that combines the weight $\omega_{\mathcal{W}} = \left(\omega_{\mathcal{W}-1}, \omega_{\mathcal{W}-2}, \dots, \omega_{\mathcal{W}-\mathring{y}}\right)$ with $0 \le \omega_{\mathcal{W}-ע} \le 1$, $\sum_{ע=1}^{\mathring{y}} \omega_{\mathcal{W}-ע} = 1$ and probabilities $\mathfrak{B}_{\mathcal{P}\mathcal{B}-ע} > 0$ with $\sum_{ע=1}^{\mathring{y}} \mathfrak{B}_{\mathcal{P}\mathcal{B}-ע} = 1$ in the one formula.

Theorem 1: The aggregated outcome of a group of CFNs $\mathcal{C}_{CFS-ע} = \left(\mu_{\mathcal{C}_{CFS-ע}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS-ע}} + \iota\, \mu^{I}_{\mathcal{C}_{CFS-ע}}\right)$, $ע = 1, 2, \dots, \mathring{y}$, by utilizing the P-CFWA operator is a CFN that is

$$P - CFWA\left(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \dots \mathcal{C}_{CFS-\mathring{y}}\right) = \left(1 - \prod_{ע=1}^{\mathring{y}} \left(1 - \mu^{R}_{\mathcal{C}_{CFS-ע}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ע}} + \iota \left(1 - \prod_{ע=1}^{\mathring{y}} \left(1 - \mu^{I}_{\mathcal{C}_{CFS-ע}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ע}}\right)\right) \quad (3)$$

Proof: This proof will take place by mathematical induction. Let $\mathring{y} = 2$. From Def (4), we have

$$\Xi_{\mathcal{P}\mathcal{W}-1}\mathcal{C}_{CFS-1} = \left(1 - \left(1 - \mu^{R}_{\mathcal{C}_{CFS-1}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-1}} + \iota \left(1 - \left(1 - \mu^{I}_{\mathcal{C}_{CFS-1}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-1}}\right)\right)$$

$$\Xi_{\mathcal{P}\mathcal{W}-2}\mathcal{C}_{CFS-2} = \left(1 - \left(1 - \mu^{R}_{\mathcal{C}_{CFS-2}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-2}} + \iota \left(1 - \left(1 - \mu^{I}_{\mathcal{C}_{CFS-2}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-2}}\right)\right)$$

Now,

$$P - CFWA(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}) = \Xi_{\mathcal{P}\mathcal{W}-1}\mathcal{C}_{CFS-1} \oplus \Xi_{\mathcal{P}\mathcal{W}-2}\mathcal{C}_{CFS-2}$$

$$= \left(1 - \left(1 - \mu^{R}_{\mathcal{C}_{CFS-1}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-1}} + \iota \left(1 - \left(1 - \mu^{I}_{\mathcal{C}_{CFS-1}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-1}}\right)\right)$$

$$\oplus \left(1 - \left(1 - \mu^{R}_{\mathcal{C}_{CFS-2}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-2}} + \iota \left(1 - \left(1 - \mu^{I}_{\mathcal{C}_{CFS-2}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-2}}\right)\right)$$

$$= \left(1 - \prod_{ע=1}^{2} \left(1 - \mu^{R}_{\mathcal{C}_{CFS-ע}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ע}} + \iota \left(1 - \prod_{ע=1}^{2} \left(1 - \mu^{I}_{\mathcal{C}_{CFS-ע}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ע}}\right)\right)$$

Let Eq. (3) is valid for $\mathring{y} = {}^{\circ}F$. Then

$$P-CFWA(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots\mathcal{C}_{CFS-{}^{\circ}\mathrm{F}})=\left(1-\prod_{ꝑ=1}^{{}^{\circ}\mathrm{F}}\left(1-\mu^{R}_{\mathcal{C}_{CFS-ꝑ}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ꝑ}}+\iota\left(1-\prod_{ꝑ=1}^{{}^{\circ}\mathrm{F}}\left(1-\mu^{I}_{\mathcal{C}_{CFS-ꝑ}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ꝑ}}\right)\right)$$

Next, we have to reveal that Eq. (3) is valid for $\mathring{\mathrm{y}}={}^{\circ}\mathrm{F}+1$. Then,

$$P-CFWA(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots\mathcal{C}_{CFS-{}^{\circ}\mathrm{F}+1})=P-CFWA(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots\mathcal{C}_{CFS-{}^{\circ}\mathrm{F}})\oplus\mathcal{C}_{CFS-{}^{\circ}\mathrm{F}+1}$$
$$=\left(1-\prod_{ꝑ=1}^{{}^{\circ}\mathrm{F}}\left(1-\mu^{R}_{\mathcal{C}_{CFS-ꝑ}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ꝑ}}+\iota\left(1-\prod_{ꝑ=1}^{{}^{\circ}\mathrm{F}}\left(1-\mu^{I}_{\mathcal{C}_{CFS-ꝑ}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ꝑ}}\right)\right)$$
$$\oplus\left(1-\left(1-\mu^{R}_{\mathcal{C}_{CFS-{}^{\circ}\mathrm{F}+1}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-{}^{\circ}\mathrm{F}+1}}+\iota\left(1-\left(1-\mu^{I}_{\mathcal{C}_{CFS-{}^{\circ}\mathrm{F}+1}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-{}^{\circ}\mathrm{F}+1}}\right)\right)$$
$$=\left(1-\prod_{ꝑ=1}^{{}^{\circ}\mathrm{F}+1}\left(1-\mu^{R}_{\mathcal{C}_{CFS-ꝑ}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ꝑ}}+\iota\left(1-\prod_{ꝑ=1}^{{}^{\circ}\mathrm{F}+1}\left(1-\mu^{I}_{\mathcal{C}_{CFS-ꝑ}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ꝑ}}\right)\right)$$
$$=P-CFWA(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots\mathcal{C}_{CFS-{}^{\circ}\mathrm{F}},\mathcal{C}_{CFS-{}^{\circ}\mathrm{F}+1})$$

$\Rightarrow$ Eq. (3) is valid for $\mathring{\mathrm{y}}={}^{\circ}\mathrm{F}+1$. This implies that Eq. (3) is valid for all positive integers.

Proposition 1: (idempotency) For the group of CFNs $\mathcal{C}_{CFS-ꝑ}=\left(\mu_{\mathcal{C}_{CFS-ꝑ}}\right)=\left(\mu^{R}_{\mathcal{C}_{CFS-ꝑ}}+\iota\,\mu^{I}_{\mathcal{C}_{CFS-ꝑ}}\right)$, $ꝑ=1,2,\dots,\mathring{\mathrm{y}}$, if for all $ꝑ$, $\mathcal{C}_{CFS-ꝑ}=\mathcal{C}_{CFS}$, then

$$P-CFWA\left(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots\mathcal{C}_{CFS-\mathring{\mathrm{y}}}\right)=\mathcal{C}_{CFS-ꝑ}$$

Proof: From Theorem (1), we have

$$P-CFWA\left(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots\mathcal{C}_{CFS-\mathring{\mathrm{y}}}\right)=\left(1-\prod_{ꝑ=1}^{\mathring{\mathrm{y}}}\left(1-\mu^{R}_{\mathcal{C}_{CFS-ꝑ}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ꝑ}}+\iota\left(1-\prod_{ꝑ=1}^{\mathring{\mathrm{y}}}\left(1-\mu^{I}_{\mathcal{C}_{CFS-ꝑ}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ꝑ}}\right)\right)$$

As $\mathcal{C}_{CFS-ꝑ}=\mathcal{C}_{CFS}$, then

$$P-CFWA\left(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots\mathcal{C}_{CFS-\mathring{\mathrm{y}}}\right)=\left(1-\prod_{ꝑ=1}^{\mathring{\mathrm{y}}}\left(1-\mu^{R}_{\mathcal{C}_{CFS}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ꝑ}}+\iota\left(1-\prod_{ꝑ=1}^{\mathring{\mathrm{y}}}\left(1-\mu^{I}_{\mathcal{C}_{CFS}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-ꝑ}}\right)\right)$$
$$=\left(\left(\mu^{R}_{\mathcal{C}_{CFS}}\right)^{\sum_{ꝑ=1}^{\mathring{\mathrm{y}}}\Xi_{\mathcal{P}\mathcal{W}-ꝑ}}+\iota\left(\mu^{I}_{\mathcal{C}_{CFS}}\right)^{\sum_{ꝑ=1}^{\mathring{\mathrm{y}}}\Xi_{\mathcal{P}\mathcal{W}-ꝑ}}\right)=\left(\mu^{R}_{\mathcal{C}_{CFS}}+\iota\,\mu^{I}_{\mathcal{C}_{CFS}}\right)$$

Where $\sum_{ꝑ=1}^{\mathring{\mathrm{y}}}\Xi_{\mathcal{P}\mathcal{W}-ꝑ}=\sum_{ꝑ=1}^{\mathring{\mathrm{y}}}\left(\xi\mathfrak{B}_{\mathcal{P}\mathcal{B}-ꝑ}+(1-\xi)\omega_{\mathcal{W}-ꝑ}\right)=\xi\sum_{ꝑ=1}^{\mathring{\mathrm{y}}}\mathfrak{B}_{\mathcal{P}\mathcal{B}-ꝑ}+(1-\xi)\sum_{ꝑ=1}^{\mathring{\mathrm{y}}}\omega_{\mathcal{W}-ꝑ}=\xi+1-\xi=1$.

Proposition 2: (Monotonicity) For two groups of CFNs $\mathcal{C}_{CFS-ꝑ}=\left(\mu_{\mathcal{C}_{CFS-ꝑ}}\right)=\left(\mu^{R}_{\mathcal{C}_{CFS-ꝑ}}+\iota\,\mu^{I}_{\mathcal{C}_{CFS-ꝑ}}\right)$, and $\mathcal{C}^{\star}_{CFS-ꝑ}=\left(\mu_{\mathcal{C}^{\star}_{CFS-ꝑ}}\right)=\left(\mu^{R}_{\mathcal{C}^{\star}_{CFS-ꝑ}}+\iota\,\mu^{I}_{\mathcal{C}^{\star}_{CFS-ꝑ}}\right)$, $ꝑ=1,2,\dots,\mathring{\mathrm{y}}$, if $\mu^{R}_{\mathcal{C}_{CFS-ꝑ}}\le\mu^{R}_{\mathcal{C}^{\star}_{CFS-ꝑ}}$, $\mu^{I}_{\mathcal{C}_{CFS-ꝑ}}\le\mu^{I}_{\mathcal{C}^{\star}_{CFS-ꝑ}}$, then

$$P-CFWA\left(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots\mathcal{C}_{CFS-\mathring{\mathrm{y}}}\right)\le P-CFWA\left(\mathcal{C}^{\star}_{CFS-1},\mathcal{C}^{\star}_{CFS-2},\dots\mathcal{C}^{\star}_{CFS-\mathring{\mathrm{y}}}\right)$$

Proof: As

$$\mu^{R}_{\mathcal{C}_{CFS-ꝑ}}\le\mu^{R}_{\mathcal{C}^{\star}_{CFS-ꝑ}}$$
$$\Rightarrow 1-\mu^{R}_{\mathcal{C}_{CFS-ꝑ}}\ge 1-\mu^{R}_{\mathcal{C}^{\star}_{CFS-ꝑ}}$$

$$\Rightarrow \left(1-\mu^{R}_{\mathcal{C}_{CFS}}\right)^{\Xi_{\mathcal{P}W-ա}} \geq \left(1-\mu^{R}_{\mathcal{C}^{\star}_{CFS}}\right)^{\Xi_{\mathcal{P}W-ա}}$$

$$\Rightarrow \prod_{ա=1}^{ŷ}\left(1-\mu^{R}_{\mathcal{C}_{CFS}}\right)^{\Xi_{\mathcal{P}W-ա}} \geq \prod_{ա=1}^{ŷ}\left(1-\mu^{R}_{\mathcal{C}^{\star}_{CFS}}\right)^{\Xi_{\mathcal{P}W-ա}}$$

$$\Rightarrow 1-\prod_{ա=1}^{ŷ}\left(1-\mu^{R}_{\mathcal{C}_{CFS}}\right)^{\Xi_{\mathcal{P}W-ա}} \leq 1-\prod_{ա=1}^{ŷ}\left(1-\mu^{R}_{\mathcal{C}^{\star}_{CFS}}\right)^{\Xi_{\mathcal{P}W-ա}}$$

Similarly,

$$1-\prod_{ա=1}^{ŷ}\left(1-\mu^{I}_{\mathcal{C}_{CFS}}\right)^{\Xi_{\mathcal{P}W-ա}} \leq 1-\prod_{ա=1}^{ŷ}\left(1-\mu^{I}_{\mathcal{C}^{\star}_{CFS}}\right)^{\Xi_{\mathcal{P}W-ա}}$$

This implies that

$$P-CFWA\left(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\ldots\mathcal{C}_{CFS-ŷ}\right) \leq P-CFWA\left(\mathcal{C}^{\star}_{CFS-1},\mathcal{C}^{\star}_{CFS-2},\ldots\mathcal{C}^{\star}_{CFS-ŷ}\right)$$

Proposition 3: (boundedness) For the group of CFNs $\mathcal{C}_{CFS-ա} = \left(\mu_{\mathcal{C}_{CFS-ա}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS-ա}} + \iota\,\mu^{I}_{\mathcal{C}_{CFS-ա}}\right)$, $ա = 1,2,\ldots,ŷ$, if $\mathcal{C}^{-}_{CFS} = \left(\min_{ա}\left\{\mu^{R}_{\mathcal{C}_{CFS-ա}}\right\} + \iota\min_{ա}\left\{\mu^{I}_{\mathcal{C}_{CFS-ա}}\right\}\right)$ and $\mathcal{C}^{+}_{CFS} = \left(\max_{ա}\left\{\mu^{R}_{\mathcal{C}_{CFS-ա}}\right\} + \iota\max_{ա}\left\{\mu^{I}_{\mathcal{C}_{CFS-ա}}\right\}\right)$, then

$$\mathcal{C}^{-}_{CFS} \leq P-CFWA\left(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\ldots\mathcal{C}_{CFS-ŷ}\right) \leq \mathcal{C}^{+}_{CFS}$$

Proof: This can be proved by employing Propositions (1) and (2)

Definition 7: For the group of CFNs $\mathcal{C}_{CFS-ա} = \left(\mu_{\mathcal{C}_{CFS-ա}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS-ա}} + \iota\,\mu^{I}_{\mathcal{C}_{CFS-ա}}\right)$, $ա = 1,2,\ldots,ŷ$, the P-CFOWA operator is devised as

$$P-CFOWA(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\ldots,\mathcal{C}_{CFS-Ÿ}) = \bigoplus_{ա=1}^{ŷ} \Xi_{\mathcal{P}W-ա}\mathcal{C}_{CFS-\psi(ա)}$$

Where $\Xi_{\mathcal{P}W-ա} = \xi\mathfrak{B}_{\mathcal{P}\mathcal{B}-ա} + (1-\xi)\omega_{W-ա}$ with $\xi \in [0,1]$ would be a weight vector that combines the weight $\omega_{W} = \left(\omega_{W-1},\omega_{W-2},\ldots,\omega_{W-ŷ}\right)$ with $0 \leq \omega_{W-ա} \leq 1$, $\sum_{ա=1}^{ŷ}\omega_{W-ա} = 1$ and probabilities $\mathfrak{B}_{\mathcal{P}\mathcal{B}-ա} > 0$ with $\sum_{ա=1}^{ŷ}\mathfrak{B}_{\mathcal{P}\mathcal{B}-ա} = 1$ in the one formula. Further, $\left(\psi(1),\psi(2),\ldots,\psi(ŷ)\right)$ is signified as a permutation of $(1,2,\ldots,ŷ)$ such that for $ա = 2,3,\ldots,ŷ$ $\psi(ա-1) \geq \psi(ա)$.

Theorem 2: The aggregated outcome of a group of CFNs $\mathcal{C}_{CFS-ա} = \left(\mu_{\mathcal{C}_{CFS-ա}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS-ա}} + \iota\,\mu^{I}_{\mathcal{C}_{CFS-ա}}\right)$, $ա = 1,2,\ldots,ŷ$, by utilizing the P-CFOWA operator is a CFN that is

$$P-CFOWA\left(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\ldots\mathcal{C}_{CFS-ŷ}\right)$$
$$= \left(1-\prod_{ա=1}^{ŷ}\left(1-\mu^{R}_{\mathcal{C}_{CFS-\psi(ա)}}\right)^{\Xi_{\mathcal{P}W-ա}} + \iota\left(1-\prod_{ա=1}^{ŷ}\left(1-\mu^{I}_{\mathcal{C}_{CFS-\psi(ա)}}\right)^{\Xi_{\mathcal{P}W-ա}}\right)\right)$$

Proof: Omitted.

The notion of immediate probability is employed for adding the decision expert's attitude in a probabilistic DM dilemma. The rationale for doing so is that since probabilistic information is objective but unreliable, it cannot be based on anticipated outcomes. To get this in this article, we invent immediate probability CF-weighted AOs.

Definition 8: For the group of CFNs $\mathcal{C}_{CFS-ự} = \left(\mu_{\mathcal{C}_{CFS-ự}}\right) = \left(\mu^R_{\mathcal{C}_{CFS-ự}} + \iota\, \mu^I_{\mathcal{C}_{CFS-ự}}\right)$, $ự = 1, 2, \ldots, \mathring{y}$, the P-CFWA operator is devised as

$$IP - CFOWA(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \ldots, \mathcal{C}_{CFS-\check{Y}}) = \bigoplus_{ự=1}^{\mathring{y}} X_{\mathcal{JP}-ự}\mathcal{C}_{CFS-\psi(ự)}$$

Where $X_{\mathcal{JP}-ự} = \frac{\omega_{\mathcal{W}-ự}\mathfrak{B}_{\mathcal{PB}-ự}}{\sum_{ự=1}^{\mathring{y}} \omega_{\mathcal{W}-ự}\mathfrak{B}_{\mathcal{PB}-ự}}$ is identified as IP provided to CFN, $\omega_{\mathcal{W}} = \left(\omega_{\mathcal{W}-1}, \omega_{\mathcal{W}-2}, \ldots, \omega_{\mathcal{W}-\mathring{y}}\right)$ with $0 \leq \omega_{\mathcal{W}-ự} \leq 1$, $\sum_{ự=1}^{\mathring{y}} \omega_{\mathcal{W}-ự} = 1$ is the linked weight, $\mathfrak{B}_{\mathcal{PB}-ự} > 0$ with $\sum_{ự=1}^{\mathring{y}} \mathfrak{B}_{\mathcal{PB}-ự} = 1$ is linked probability. Further, $\left(\psi(1), \psi(2), \ldots, \psi(\mathring{y})\right)$ is signified as a permutation of $(1, 2, \ldots, \mathring{y})$ such that for $ự = 2, 3, \ldots, \mathring{y}$ $\psi(ự - 1) \geq \psi(ự)$.

Theorem 3: The aggregated outcome of a group of CFNs $\mathcal{C}_{CFS-ự} = \left(\mu_{\mathcal{C}_{CFS-ự}}\right) = \left(\mu^R_{\mathcal{C}_{CFS-ự}} + \iota\, \mu^I_{\mathcal{C}_{CFS-ự}}\right)$, $ự = 1, 2, \ldots, \mathring{y}$, by utilizing IP-CFOWA operator is a CFN that is

$$IP - CFOWA\left(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \ldots \mathcal{C}_{CFS-\mathring{y}}\right) = \left(1 - \prod_{ự=1}^{\mathring{y}} \left(1 - \mu^R_{\mathcal{C}_{CFS-\psi(ự)}}\right)^{X_{\mathcal{JP}-ự}} + \iota \left(1 - \prod_{ự=1}^{\mathring{y}} \left(1 - \mu^I_{\mathcal{C}_{CFS-\psi(ự)}}\right)^{X_{\mathcal{JP}-ự}}\right)\right)$$

Proof: Omitted.

Proposition 4: (idempotency) For the group of CFNs $\mathcal{C}_{CFS-ự} = \left(\mu_{\mathcal{C}_{CFS-ự}}\right) = \left(\mu^R_{\mathcal{C}_{CFS-ự}} + \iota\, \mu^I_{\mathcal{C}_{CFS-ự}}\right)$, $ự = 1, 2, \ldots, \mathring{y}$, if for all $ự$, $\mathcal{C}_{CFS-ự} = \mathcal{C}_{CFS}$, then

$$IP - CFOWA\left(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \ldots \mathcal{C}_{CFS-\mathring{y}}\right) = \mathcal{C}_{CFS-ự}$$

Proposition 5: (Monotonicity) For two groups of CFNs $\mathcal{C}_{CFS-ự} = \left(\mu_{\mathcal{C}_{CFS-ự}}\right) = \left(\mu^R_{\mathcal{C}_{CFS-ự}} + \iota\, \mu^I_{\mathcal{C}_{CFS-ự}}\right)$, and $\mathcal{C}^\star_{CFS-ự} = \left(\mu_{\mathcal{C}^\star_{CFS-ự}}\right) = \left(\mu^R_{\mathcal{C}^\star_{CFS-ự}} + \iota\, \mu^I_{\mathcal{C}^\star_{CFS-ự}}\right)$, $ự = 1, 2, \ldots, \mathring{y}$, if $\mu^R_{\mathcal{C}_{CFS-ự}} \leq \mu^R_{\mathcal{C}^\star_{CFS-ự}}$, $\mu^I_{\mathcal{C}_{CFS-ự}} \leq \mu^I_{\mathcal{C}^\star_{CFS-ự}}$, then

$$IP - CFOWA\left(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \ldots \mathcal{C}_{CFS-\mathring{y}}\right) \leq IP - CFOWA\left(\mathcal{C}^\star_{CFS-1}, \mathcal{C}^\star_{CFS-2}, \ldots \mathcal{C}^\star_{CFS-\mathring{y}}\right)$$

Proposition 6: (boundedness) For the group of CFNs $\mathcal{C}_{CFS-ự} = \left(\mu_{\mathcal{C}_{CFS-ự}}\right) = \left(\mu^R_{\mathcal{C}_{CFS-ự}} + \iota\, \mu^I_{\mathcal{C}_{CFS-ự}}\right)$, $ự = 1, 2, \ldots, \mathring{y}$, if $\mathcal{C}^-_{CFS} = \left(\min_{ự}\left\{\mu^R_{\mathcal{C}_{CFS-ự}}\right\} + \iota \min_{ự}\left\{\mu^I_{\mathcal{C}_{CFS-ự}}\right\}\right)$ and $\mathcal{C}^+_{CFS} = \left(\max_{ự}\left\{\mu^R_{\mathcal{C}_{CFS-ự}}\right\} + \iota \max_{ự}\left\{\mu^I_{\mathcal{C}_{CFS-ự}}\right\}\right)$, then

$$\mathcal{C}^-_{CFS} \leq IP - CFOWA\left(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \ldots \mathcal{C}_{CFS-\mathring{y}}\right) \leq \mathcal{C}^+_{CFS}$$

3.2| PROBABILISTIC GEOMETRIC AOS FOR CFNS

In this subsection, we will devise P-CFWG, IP-CFOWG, and P-CFOWG operators and will interpret related propositions.

Definition 9: For the group of CFNs $\mathcal{C}_{CFS-ự} = \left(\mu_{\mathcal{C}_{CFS-ự}}\right) = \left(\mu^R_{\mathcal{C}_{CFS-ự}} + \iota\, \mu^I_{\mathcal{C}_{CFS-ự}}\right)$, $ự = 1, 2, \ldots, \mathring{y}$, the P-CFWG operator is devised as

$$P - CFWG(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \ldots, \mathcal{C}_{CFS-\check{Y}}) = \bigotimes_{ự=1}^{\mathring{y}} \left(\mathcal{C}_{CFS-ự}\right)^{\Xi_{\mathcal{PW}-ự}}$$

Where $\Xi_{\mathcal{PW}-ᶙ} = \xi \mathfrak{B}_{\mathcal{PB}-ᶙ} + (1-\xi)\omega_{\mathcal{W}-ᶙ}$ with $\xi \in [0,1]$ would be a weight vector that combines the weight $\omega_{\mathcal{W}} = (\omega_{\mathcal{W}-1}, \omega_{\mathcal{W}-2}, \dots, \omega_{\mathcal{W}-\mathring{y}})$ with $0 \le \omega_{\mathcal{W}-ᶙ} \le 1$, $\sum_{ᶙ=1}^{\mathring{y}} \omega_{\mathcal{W}-ᶙ} = 1$ and probabilities $\mathfrak{B}_{\mathcal{PB}-ᶙ} > 0$ with $\sum_{ᶙ=1}^{\mathring{y}} \mathfrak{B}_{\mathcal{PB}-ᶙ} = 1$ in the one formula.

Theorem 4: The aggregated outcome of a group of CFNs $\mathcal{C}_{CFS-ᶙ} = \left(\mu_{\mathcal{C}_{CFS-ᶙ}}\right) = \left(\mu^R_{\mathcal{C}_{CFS-ᶙ}} + \iota\, \mu^I_{\mathcal{C}_{CFS-ᶙ}}\right)$, $ᶙ = 1, 2, \dots, \mathring{y}$, by utilizing the P-CFWG operator is a CFN that is

$$P - CFWG(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \dots \mathcal{C}_{CFS-\mathring{y}}) = \left(\prod_{ᶙ=1}^{\mathring{y}} \left(\mu^R_{\mathcal{C}_{CFS-ᶙ}}\right)^{\Xi_{\mathcal{PW}-ᶙ}} + \iota \prod_{ᶙ=1}^{\mathring{y}} \left(\mu^I_{\mathcal{C}_{CFS-ᶙ}}\right)^{\Xi_{\mathcal{PW}-ᶙ}} \right) \quad (4)$$

Proof: This proof will take place by mathematical induction. Let $\mathring{y} = 2$. From Def (4), we have

$$(\mathcal{C}_{CFS-1})^{\Xi_{\mathcal{PW}-1}} = \left(\left(\mu^R_{\mathcal{C}_{CFS-1}}\right)^{\Xi_{\mathcal{PW}-1}} + \iota \left(\mu^I_{\mathcal{C}_{CFS-1}}\right)^{\Xi_{\mathcal{PW}-1}} \right)$$

$$(\mathcal{C}_{CFS-2})^{\Xi_{\mathcal{PW}-2}} = \left(\left(\mu^R_{\mathcal{C}_{CFS-2}}\right)^{\Xi_{\mathcal{PW}-2}} + \iota \left(\mu^I_{\mathcal{C}_{CFS-2}}\right)^{\Xi_{\mathcal{PW}-2}} \right)$$

Now,

$$P - CFWG(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}) = (\mathcal{C}_{CFS-1})^{\Xi_{\mathcal{PW}-1}} \otimes (\mathcal{C}_{CFS-2})^{\Xi_{\mathcal{PW}-2}}$$

$$= \left(\left(\mu^R_{\mathcal{C}_{CFS-1}}\right)^{\Xi_{\mathcal{PW}-1}} + \iota \left(\mu^I_{\mathcal{C}_{CFS-1}}\right)^{\Xi_{\mathcal{PW}-1}} \right) \otimes \left(\left(\mu^R_{\mathcal{C}_{CFS-2}}\right)^{\Xi_{\mathcal{PW}-2}} + \iota \left(\mu^I_{\mathcal{C}_{CFS-2}}\right)^{\Xi_{\mathcal{PW}-2}} \right)$$

$$= \left(\prod_{ᶙ=1}^{2} \left(\mu^R_{\mathcal{C}_{CFS-ᶙ}}\right)^{\Xi_{\mathcal{PW}-ᶙ}} + \iota \prod_{ᶙ=1}^{2} \left(\mu^I_{\mathcal{C}_{CFS-ᶙ}}\right)^{\Xi_{\mathcal{PW}-ᶙ}} \right)$$

Let Eq. (4) is valid for $\mathring{y} = {}^\circ\mathrm{F}$. Then

$$P - CFWG(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \dots \mathcal{C}_{CFS-{}^\circ\mathrm{F}}) = \left(\prod_{ᶙ=1}^{{}^\circ\mathrm{F}} \left(\mu^R_{\mathcal{C}_{CFS-ᶙ}}\right)^{\Xi_{\mathcal{PW}-ᶙ}} + \iota \prod_{ᶙ=1}^{{}^\circ\mathrm{F}} \left(\mu^I_{\mathcal{C}_{CFS-ᶙ}}\right)^{\Xi_{\mathcal{PW}-ᶙ}} \right)$$

Next, we have to reveal that Eq. (4) is valid for $\mathring{y} = {}^\circ\mathrm{F} + 1$. Then,

$$P - CFWG(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \dots \mathcal{C}_{CFS-{}^\circ\mathrm{F}+1}) = P - CFWG(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \dots \mathcal{C}_{CFS-{}^\circ\mathrm{F}}) \otimes \mathcal{C}_{CFS-{}^\circ\mathrm{F}+1}$$

$$= \left(\prod_{ᶙ=1}^{{}^\circ\mathrm{F}} \left(\mu^R_{\mathcal{C}_{CFS-ᶙ}}\right)^{\Xi_{\mathcal{PW}-ᶙ}} + \iota \prod_{ᶙ=1}^{{}^\circ\mathrm{F}} \left(\mu^I_{\mathcal{C}_{CFS-ᶙ}}\right)^{\Xi_{\mathcal{PW}-ᶙ}} \right) \otimes \left(\left(\mu^R_{\mathcal{C}_{CFS-{}^\circ\mathrm{F}+1}}\right)^{\Xi_{\mathcal{PW}-{}^\circ\mathrm{F}+1}} + \iota \left(\mu^I_{\mathcal{C}_{CFS-{}^\circ\mathrm{F}+1}}\right)^{\Xi_{\mathcal{PW}-{}^\circ\mathrm{F}+1}} \right)$$

$$= \left(\prod_{ᶙ=1}^{{}^\circ\mathrm{F}+1} \left(\mu^R_{\mathcal{C}_{CFS-ᶙ}}\right)^{\Xi_{\mathcal{PW}-ᶙ}} + \iota \prod_{ᶙ=1}^{{}^\circ\mathrm{F}+1} \left(\mu^I_{\mathcal{C}_{CFS-ᶙ}}\right)^{\Xi_{\mathcal{PW}-ᶙ}} \right)$$

$$= P - CFWG(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \dots \mathcal{C}_{CFS-{}^\circ\mathrm{F}}, \mathcal{C}_{CFS-{}^\circ\mathrm{F}+1})$$

$\Rightarrow$ Eq. (4) is valid for $\mathring{y} = {}^\circ\mathrm{F} + 1$. This implies that Eq. (4) is valid for all positive integers.

Proposition 7: (idempotency) For the group of CFNs $\mathcal{C}_{CFS-ᶙ} = \left(\mu_{\mathcal{C}_{CFS-ᶙ}}\right) = \left(\mu^R_{\mathcal{C}_{CFS-ᶙ}} + \iota\, \mu^I_{\mathcal{C}_{CFS-ᶙ}}\right)$, $ᶙ = 1, 2, \dots, \mathring{y}$, if for all $ᶙ$, $\mathcal{C}_{CFS-ᶙ} = \mathcal{C}_{CFS}$, then

$$P - CFWG(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \dots \mathcal{C}_{CFS-\mathring{y}}) = \mathcal{C}_{CFS-ᶙ}$$

Proposition 8: (Monotonicity) For two groups of CFNs $\mathcal{C}_{CFS-ᶙ} = \left(\mu_{\mathcal{C}_{CFS-ᶙ}}\right) = \left(\mu^R_{\mathcal{C}_{CFS-ᶙ}} + \iota\, \mu^I_{\mathcal{C}_{CFS-ᶙ}}\right)$, and $\mathcal{C}^\star_{CFS-ᶙ} = \left(\mu_{\mathcal{C}^\star_{CFS-ᶙ}}\right) = \left(\mu^R_{\mathcal{C}^\star_{CFS-ᶙ}} + \iota\, \mu^I_{\mathcal{C}^\star_{CFS-ᶙ}}\right)$, $ᶙ = 1, 2, \dots, \mathring{y}$, if $\mu^R_{\mathcal{C}_{CFS-ᶙ}} \le \mu^R_{\mathcal{C}^\star_{CFS-ᶙ}}$, $\mu^I_{\mathcal{C}_{CFS-ᶙ}} \le \mu^I_{\mathcal{C}^\star_{CFS-ᶙ}}$, then

$$P-CFWG\left(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots\mathcal{C}_{CFS-\mathring{y}}\right)\le P-CFWG\left(\mathcal{C}^{\star}_{CFS-1},\mathcal{C}^{\star}_{CFS-2},\dots\mathcal{C}^{\star}_{CFS-\mathring{y}}\right)$$

Proposition 9: (boundedness) For the group of CFNs $\mathcal{C}_{CFS-\text{ụ}}=\left(\mu_{\mathcal{C}_{CFS-\text{ụ}}}\right)=\left(\mu^{R}_{\mathcal{C}_{CFS-\text{ụ}}}+\iota\,\mu^{I}_{\mathcal{C}_{CFS-\text{ụ}}}\right)$, $\text{ụ}=1,2,\dots,\mathring{y}$, if $\mathcal{C}^{-}_{CFS}=\left(\min_{\text{ụ}}\left\{\mu^{R}_{\mathcal{C}_{CFS-\text{ụ}}}\right\}+\iota\min_{\text{ụ}}\left\{\mu^{I}_{\mathcal{C}_{CFS-\text{ụ}}}\right\}\right)$ and $\mathcal{C}^{+}_{CFS}=\left(\max_{\text{ụ}}\left\{\mu^{R}_{\mathcal{C}_{CFS-\text{ụ}}}\right\}+\iota\max_{\text{ụ}}\left\{\mu^{I}_{\mathcal{C}_{CFS-\text{ụ}}}\right\}\right)$, then

$$\mathcal{C}^{-}_{CFS}\le P-CFWG\left(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots\mathcal{C}_{CFS-\mathring{y}}\right)\le\mathcal{C}^{+}_{CFS}$$

Definition 10: For the group of CFNs $\mathcal{C}_{CFS-\text{ụ}}=\left(\mu_{\mathcal{C}_{CFS-\text{ụ}}}\right)=\left(\mu^{R}_{\mathcal{C}_{CFS-\text{ụ}}}+\iota\,\mu^{I}_{\mathcal{C}_{CFS-\text{ụ}}}\right)$, $\text{ụ}=1,2,\dots,\mathring{y}$, the P-CFOWG operator is devised as

$$P-CFOWG(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots,\mathcal{C}_{CFS-\check{Y}})=\bigotimes_{\text{ụ}=1}^{\mathring{y}}\left(\mathcal{C}_{CFS-\psi(\text{ụ})}\right)^{\Xi_{\mathcal{P}\mathcal{W}-\text{ụ}}}$$

Where $\Xi_{\mathcal{P}\mathcal{W}-\text{ụ}}=\xi\mathfrak{B}_{\mathcal{P}\mathcal{B}-\text{ụ}}+(1-\xi)\omega_{\mathcal{W}-\text{ụ}}$ with $\xi\in[0,1]$ would be a weight vector that combines the weight $\omega_{\mathcal{W}}=\left(\omega_{\mathcal{W}-1},\omega_{\mathcal{W}-2},\dots,\omega_{\mathcal{W}-\mathring{y}}\right)$ with $0\le\omega_{\mathcal{W}-\text{ụ}}\le1$, $\sum_{\text{ụ}=1}^{\mathring{y}}\omega_{\mathcal{W}-\text{ụ}}=1$ and probabilities $\mathfrak{B}_{\mathcal{P}\mathcal{B}-\text{ụ}}>0$ with $\sum_{\text{ụ}=1}^{\mathring{y}}\mathfrak{B}_{\mathcal{P}\mathcal{B}-\text{ụ}}=1$ in the one formula. Further, $\left(\psi(1),\psi(2),\dots,\psi(\mathring{y})\right)$ is signified as a permutation of $(1,2,\dots,\mathring{y})$ such that for $\text{ụ}=2,3,\dots,\mathring{y}$ $\psi(\text{ụ}-1)\ge\psi(\text{ụ})$.

Theorem 5: The aggregated outcome of a group of CFNs $\mathcal{C}_{CFS-\text{ụ}}=\left(\mu_{\mathcal{C}_{CFS-\text{ụ}}}\right)=\left(\mu^{R}_{\mathcal{C}_{CFS-\text{ụ}}}+\iota\,\mu^{I}_{\mathcal{C}_{CFS-\text{ụ}}}\right)$, $\text{ụ}=1,2,\dots,\mathring{y}$, by utilizing the P-CFOWG operator is a CFN that is

$$P-CFOWG\left(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots\mathcal{C}_{CFS-\mathring{y}}\right)=\left(\prod_{\text{ụ}=1}^{\mathring{y}}\left(\mu^{R}_{\mathcal{C}_{CFS-\psi(\text{ụ})}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-\text{ụ}}}+\iota\prod_{\text{ụ}=1}^{\mathring{y}}\left(\mu^{I}_{\mathcal{C}_{CFS-\psi(\text{ụ})}}\right)^{\Xi_{\mathcal{P}\mathcal{W}-\text{ụ}}}\right)$$

Proof: Omitted.

Definition 11: For the group of CFNs $\mathcal{C}_{CFS-\text{ụ}}=\left(\mu_{\mathcal{C}_{CFS-\text{ụ}}}\right)=\left(\mu^{R}_{\mathcal{C}_{CFS-\text{ụ}}}+\iota\,\mu^{I}_{\mathcal{C}_{CFS-\text{ụ}}}\right)$, $\text{ụ}=1,2,\dots,\mathring{y}$, the IP-CFOWG operator is devised as

$$IP-CFOWG(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots,\mathcal{C}_{CFS-\check{Y}})=\bigotimes_{\text{ụ}=1}^{\mathring{y}}\left(\mathcal{C}_{CFS-\psi(\text{ụ})}\right)^{X_{\mathcal{J}\mathcal{P}-\text{ụ}}}$$

Where $X_{\mathcal{J}\mathcal{P}-\text{ụ}}=\frac{\omega_{\mathcal{W}-\text{ụ}}\mathfrak{B}_{\mathcal{P}\mathcal{B}-\text{ụ}}}{\sum_{\text{ụ}=1}^{\mathring{y}}\omega_{\mathcal{W}-\text{ụ}}\mathfrak{B}_{\mathcal{P}\mathcal{B}-\text{ụ}}}$ is identified as IP provided to CFN, $\omega_{\mathcal{W}}=\left(\omega_{\mathcal{W}-1},\omega_{\mathcal{W}-2},\dots,\omega_{\mathcal{W}-\mathring{y}}\right)$ with $0\le\omega_{\mathcal{W}-\text{ụ}}\le1$, $\sum_{\text{ụ}=1}^{\mathring{y}}\omega_{\mathcal{W}-\text{ụ}}=1$ is the linked weight, $\mathfrak{B}_{\mathcal{P}\mathcal{B}-\text{ụ}}>0$ with $\sum_{\text{ụ}=1}^{\mathring{y}}\mathfrak{B}_{\mathcal{P}\mathcal{B}-\text{ụ}}=1$ is linked probability. Further, $\left(\psi(1),\psi(2),\dots,\psi(\mathring{y})\right)$ is signified as a permutation of $(1,2,\dots,\mathring{y})$ such that for $\text{ụ}=2,3,\dots,\mathring{y}$ $\psi(\text{ụ}-1)\ge\psi(\text{ụ})$.

Theorem 6: The aggregated outcome of a group of CFNs $\mathcal{C}_{CFS-\text{ụ}}=\left(\mu_{\mathcal{C}_{CFS-\text{ụ}}}\right)=\left(\mu^{R}_{\mathcal{C}_{CFS-\text{ụ}}}+\iota\,\mu^{I}_{\mathcal{C}_{CFS-\text{ụ}}}\right)$, $\text{ụ}=1,2,\dots,\mathring{y}$, by utilizing IP-CFOWG operator is a CFN that is

$$IP-CFOWG\left(\mathcal{C}_{CFS-1},\mathcal{C}_{CFS-2},\dots\mathcal{C}_{CFS-\mathring{y}}\right)=\left(\prod_{\text{ụ}=1}^{\mathring{y}}\left(\mu^{R}_{\mathcal{C}_{CFS-\psi(\text{ụ})}}\right)^{X_{\mathcal{J}\mathcal{P}-\text{ụ}}}+\iota\prod_{\text{ụ}=1}^{\mathring{y}}\left(\mu^{I}_{\mathcal{C}_{CFS-\psi(\text{ụ})}}\right)^{X_{\mathcal{J}\mathcal{P}-\text{ụ}}}\right)$$

Proof: Omitted.

Proposition 10: (idempotency) For the group of CFNs $\mathcal{C}_{CFS-ɥ} = \left(\mu_{\mathcal{C}_{CFS-ɥ}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS-ɥ}} + \iota\, \mu^{I}_{\mathcal{C}_{CFS-ɥ}}\right)$, $ɥ = 1, 2, \dots, \mathring{y}$, if for all $ɥ$, $\mathcal{C}_{CFS-ɥ} = \mathcal{C}_{CFS}$, then

$$IP - CFOWG(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \dots \mathcal{C}_{CFS-\mathring{y}}) = \mathcal{C}_{CFS-ɥ}$$

Proposition 11: (Monotonicity) For two groups of CFNs $\mathcal{C}_{CFS-ɥ} = \left(\mu_{\mathcal{C}_{CFS-ɥ}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS-ɥ}} + \iota\, \mu^{I}_{\mathcal{C}_{CFS-ɥ}}\right)$, and $\mathcal{C}^{\star}_{CFS-ɥ} = \left(\mu_{\mathcal{C}^{\star}_{CFS-ɥ}}\right) = \left(\mu^{R}_{\mathcal{C}^{\star}_{CFS-ɥ}} + \iota\, \mu^{I}_{\mathcal{C}^{\star}_{CFS-ɥ}}\right)$, $ɥ = 1, 2, \dots, \mathring{y}$, if $\mu^{R}_{\mathcal{C}_{CFS-ɥ}} \le \mu^{R}_{\mathcal{C}^{\star}_{CFS-ɥ}}$, $\mu^{I}_{\mathcal{C}_{CFS-ɥ}} \le \mu^{I}_{\mathcal{C}^{\star}_{CFS-ɥ}}$, then

$$IP - CFOWG(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \dots \mathcal{C}_{CFS-\mathring{y}}) \le IP - CFOWG(\mathcal{C}^{\star}_{CFS-1}, \mathcal{C}^{\star}_{CFS-2}, \dots \mathcal{C}^{\star}_{CFS-\mathring{y}})$$

Proposition 12: (boundedness) For the group of CFNs $\mathcal{C}_{CFS-ɥ} = \left(\mu_{\mathcal{C}_{CFS-ɥ}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS-ɥ}} + \iota\, \mu^{I}_{\mathcal{C}_{CFS-ɥ}}\right)$, $ɥ = 1, 2, \dots, \mathring{y}$, if $\mathcal{C}^{-}_{CFS} = \left(\min_{ɥ}\left\{\mu^{R}_{\mathcal{C}_{CFS-ɥ}}\right\} + \iota \min_{ɥ}\left\{\mu^{I}_{\mathcal{C}_{CFS-ɥ}}\right\}\right)$ and $\mathcal{C}^{+}_{CFS} = \left(\max_{ɥ}\left\{\mu^{R}_{\mathcal{C}_{CFS-ɥ}}\right\} + \iota \max_{ɥ}\left\{\mu^{I}_{\mathcal{C}_{CFS-ɥ}}\right\}\right)$, then

$$\mathcal{C}^{-}_{CFS} \le IP - CFOWG(\mathcal{C}_{CFS-1}, \mathcal{C}_{CFS-2}, \dots \mathcal{C}_{CFS-\mathring{y}}) \le \mathcal{C}^{+}_{CFS}$$

4 | APPROACH OF MADM WITHIN CF INFORMATION

Consider there are $\mathring{y}$ number of alternatives $\mathcal{P}_{\mathfrak{AE}} = \{\mathcal{P}_{\mathfrak{AE}-1}, \mathcal{P}_{\mathfrak{AE}-2}, \dots, \mathcal{P}_{\mathfrak{AE}-\mathring{y}}\}$ and $\hat{z}$ number of attributes $\mathcal{R}_{\mathfrak{AT}} = \{\mathcal{R}_{\mathfrak{AT}-1}, \mathcal{R}_{\mathfrak{AT}-2}, \dots, \hat{z}\}$ on which the provided alternatives would be evaluated. Further, there is a weight vector $\omega_{\mathcal{W}} = (\omega_{\mathcal{W}-1}, \omega_{\mathcal{W}-2}, \dots, \omega_{\mathcal{W}-\hat{z}})$ with $0 \le \omega_{\mathcal{W}-\mathrm{y}} \le 1$, $\sum_{\mathrm{y}=1}^{\hat{z}} \omega_{\mathcal{W}-\mathrm{y}} = 1$ for attributes demonstrated by the decision expert by his/her choice. After that, the decision expert would give the evaluation values of these alternatives keeping in mind the interpreted attributes in the model of CFNs $\mathcal{C}_{CFS-ɥ\mathrm{y}} = \left(\mu_{\mathcal{C}_{CFS-ɥ\mathrm{y}}}\right) = \left(\mu^{R}_{\mathcal{C}_{CFS-ɥ\mathrm{y}}} + \iota\, \mu^{I}_{\mathcal{C}_{CFS-ɥ\mathrm{y}}}\right)$, $ɥ = 1, 2, \dots, \mathring{y}$, $\mathrm{y} = 1, 2, \dots, \hat{z}$ which will devise a CF decision matrix $\mathcal{D}_{CFS}$. Underneath are the steps for tackling such sort of MADM dilemmas

Step 1: In such sort of DM dilemmas, the attributes are of two kinds that are benefit kind or cost type. Thus, the decision expert needs to standardize the CF decision matrix which will be done by employing the underlying formula.

$$(\mathcal{D}_{CFS})^{S} = \begin{cases} \left(\mu^{R}_{\mathcal{C}_{CFS-ɥ\mathrm{y}}} + \iota\, \mu^{I}_{\mathcal{C}_{CFS-ɥ\mathrm{y}}}\right) & for\ be\mathring{y}efit\ type \\ \left(1 - \mu^{R}_{\mathcal{C}_{CFS-ɥ\mathrm{y}}} + \iota\left(1 - \mu^{I}_{\mathcal{C}_{CFS-ɥ\mathrm{y}}}\right)\right) & for\ cost\ type \end{cases}$$

Step 2: Compute $\Xi_{\mathcal{PW}-\mathrm{y}} = \xi \mathfrak{B}_{\mathcal{PB}-\mathrm{y}} + (1-\xi)\omega_{\mathcal{W}-\mathrm{y}}$ and $X_{\mathcal{JP}-\mathrm{y}} = \frac{\omega_{\mathcal{W}-\mathrm{y}}\mathfrak{B}_{\mathcal{PB}-\mathrm{y}}}{\sum_{\mathrm{y}=1}^{\hat{z}} \omega_{\mathcal{W}-\mathrm{y}}\mathfrak{B}_{\mathcal{PB}-\mathrm{y}}}$ by utilizing the interpreted weight vector and probabilistic information.

Step 3: To aggregate every alternative, employ any of the invented operators (P-CFWA, P-CFOWA, IP-CFOWA, P-CFWG, P-CFOWG, and IP-CFOWG).

Step 4: To get the optimal alternative, determine the score values of the aggregated outcomes.

Step 5: With the assistance of score values, rank the alternatives and determine the optimal one.

4.1 | NUMERICAL EXAMPLE

The goal of this MADM issue is to choose the optimal database management system among given database management systems $\mathcal{P}_{\mathfrak{AE}-ɥ}$, $ɥ = 1, 2, 3, 4$ which is explained below.

$\mathcal{P}_{\mathfrak{UC}-1}$: A reputable commercial DBMS with a reputation for excellent performance and scalability capabilities. It does, however, cost more to obtain a license and its administration calls for specific knowledge.

$\mathcal{P}_{\mathfrak{UC}-2}$: A DBMS that is open-source and has a good reputation for scalability and community support is option B. Although it saves money, continual maintenance and optimization may take more work.

$\mathcal{P}_{\mathfrak{UC}-3}$: A DBMS that is cloud-native and focuses on automation and simplicity of use. Although it has a pay-as-you-go price structure, there are issues with data security and management.

$\mathcal{P}_{\mathfrak{UC}-4}$: A new player that positions itself as a user-friendly and economical option. Its promises about performance and scalability need to be verified, and there may be hazards involved with employing a less-proven technology.

To make a selection of optimal database management system, the decision expert requires key attributes $\mathcal{R}_{\mathfrak{UT}-\text{y}}, \text{y} = 1, 2, 3, 4$ which are discussed below

$\mathcal{R}_{\mathfrak{UT}-1}$: Performance: Performance measures how quickly and effectively a DBMS can retrieve data, perform queries, and carry out transactions. Faster response times and more system efficiency are indicated by a higher performance score.

$\mathcal{R}_{\mathfrak{UT}-2}$: Scalability: The DBMS's capacity to manage growing workloads, data quantities, and user needs without compromising performance is measured. Greater flexibility to changing requirements is indicated by a higher scalability score.

$\mathcal{R}_{\mathfrak{UT}-3}$: Cost: The cost characteristic includes DBMS-related upfront and continuing expenditures such as license fees, hardware requirements, maintenance fees, and support charges. Better cost-effectiveness is indicated by a lower cost score.

$\mathcal{R}_{\mathfrak{UT}-4}$: Ease of Use: This characteristic evaluates how user-friendly the DBMS is, including how simple it is to install, configure, and manage. An easier-to-use system has a higher ease-of-use score.

Keep in mind the priorities of attributes the decision expert assigns weight $(0.32, 0.2, 0.26, 0.22)$ and the probabilistic information of the decision expert is $(0.1, 0.2, 0.3, 0.4)$. The decision expert evaluated the performance of every DBMS across these attributes and interpreted the evaluation values in Table 1.

Table 1.

The evaluated values of each DBMS.

	$\mathcal{R}_{\mathfrak{UT}-1}$	$\mathcal{R}_{\mathfrak{UT}-2}$	$\mathcal{R}_{\mathfrak{UT}-3}$	$\mathcal{R}_{\mathfrak{UT}-4}$
$\mathcal{P}_{\mathfrak{UC}-1}$	$(0.346 + \iota\, 0.364)$	$(0.356 + \iota\, 0.856)$	$(0.334 + \iota\, 0.33)$	$(0.456 + \iota\, 0.633)$
$\mathcal{P}_{\mathfrak{UC}-2}$	$(0.234 + \iota\, 0.567)$	$(0.574 + \iota\, 0.567)$	$(0.673 + \iota\, 0.563)$	$(0.634 + \iota\, 0.745)$
$\mathcal{P}_{\mathfrak{UC}-3}$	$(0.686 + \iota\, 0.254)$	$(0.364 + \iota\, 0.674)$	$(0.356 + \iota\, 0.563)$	$(0.645 + \iota\, 0.563)$
$\mathcal{P}_{\mathfrak{UC}-4}$	$(0.345 + \iota\, 0.785)$	$(0.234 + \iota\, 0.976)$	$(0.463 + \iota\, 0.563)$	$(0.443 + \iota\, 0.745)$

To handle this MADM, we have the underneath steps

Step 1: All attributes are benefit types, so we are skipping this step.

Step 2: Computed $\Xi_{\mathcal{PW}-\text{ɣ}}$ and $X_{\mathcal{JP}-\text{ɣ}}$ $\text{ɣ} = 1, 2, 3, 4$ as

$\Xi_{\mathcal{PW}} = (0.21, 0.2, 0.28, 0.31)$ $X_{\mathcal{JP}} = (0.13, 0.17, 0.33, 0.37)$

Step 3: The aggregated values of each DBMS after employing P-CFWA, P-CFOWA, IP-CFOWA, P-CFWG, P-CFOWG, and IP-CFOWG operators are displayed in Table 2.

Table 2.

The aggregate values of every DBMS.

Operators	$\mathcal{P}_{\mathfrak{AC}-1}$	$\mathcal{P}_{\mathfrak{AC}-2}$	$\mathcal{P}_{\mathfrak{AC}-3}$	$\mathcal{P}_{\mathfrak{AC}-4}$
P-CFWA	$(0.381 + \iota\, 0.596)$	$(0.573 + \iota\, 0.632)$	$(0.541 + \iota\, 0.539)$	$(0.392 + \iota\, 0.822)$
P-CFOWA	$(0.379 + \iota\, 0.652)$	$(0.537 + \iota\, 0.626)$	$(0.51 + \iota\, 0.554)$	$(0.362 + \iota\, 0.869)$
IP-CFOWA	$(0.386 + \iota\, 0.691)$	$(0.514 + \iota\, 0.636)$	$(0.473 + \iota\, 0.579)$	$(0.342 + \iota\, 0.892)$
P-CFWG	$(0.375 + \iota\, 0.499)$	$(0.513 + \iota\, 0.616)$	$(0.493 + \iota\, 0.494)$	$(0.375 + \iota\, 0.735)$
P-CFOWG	$(0.374 + \iota\, 0.543)$	$(0.462 + \iota\, 0.611)$	$(0.463 + \iota\, 0.504)$	$(0.342 + \iota\, 0.775)$
IP-CFOWG	$(0.381 + \iota\, 0.591)$	$(0.435 + \iota\, 0.619)$	$(0.433 + \iota\, 0.541)$	$(0.324 + \iota\, 0.807)$

Step 4: The score value of each DBMS is demonstrated in Table 3.

Table 3.

The score values of DBMSs.

Operators	$\mathcal{S}(\mathcal{P}_{\mathfrak{AC}-1})$	$\mathcal{S}(\mathcal{P}_{\mathfrak{AC}-2})$	$\mathcal{S}(\mathcal{P}_{\mathfrak{AC}-3})$	$\mathcal{S}(\mathcal{P}_{\mathfrak{AC}-4})$
P-CFWA	-0.21	-0.06	0.002	-0.43
P-CFOWA	-0.27	-0.09	-0.04	-0.51
IP-CFOWA	-0.3	-0.12	-0.11	-0.55
P-CFWG	-0.124	-0.103	-0.0004	-0.361
P-CFOWG	-0.168	-0.149	-0.04	-0.433
IP-CFOWG	-0.21	-0.185	-0.107	-0.482

Step 5: With the assistance of score values, the ranking of various DBMS is portrayed in Table 4.

Table 4.

The ranking among considered DBMSs.

Operators	Ranking
P-CFWA	$\mathcal{P}_{\mathfrak{AC}-3} > \mathcal{P}_{\mathfrak{AC}-2} > \mathcal{P}_{\mathfrak{AC}-1} > \mathcal{P}_{\mathfrak{AC}-4}$
P-CFOWA	$\mathcal{P}_{\mathfrak{AC}-3} > \mathcal{P}_{\mathfrak{AC}-2} > \mathcal{P}_{\mathfrak{AC}-1} > \mathcal{P}_{\mathfrak{AC}-4}$

IP-CFOWA	$\mathcal{P}_{\mathfrak{AE}-3} > \mathcal{P}_{\mathfrak{AE}-2} > \mathcal{P}_{\mathfrak{AE}-1} > \mathcal{P}_{\mathfrak{AE}-4}$
P-CFWG	$\mathcal{P}_{\mathfrak{AE}-3} > \mathcal{P}_{\mathfrak{AE}-2} > \mathcal{P}_{\mathfrak{AE}-1} > \mathcal{P}_{\mathfrak{AE}-4}$
P-CFOWG	$\mathcal{P}_{\mathfrak{AE}-3} > \mathcal{P}_{\mathfrak{AE}-2} > \mathcal{P}_{\mathfrak{AE}-1} > \mathcal{P}_{\mathfrak{AE}-4}$
IP-CFOWG	$\mathcal{P}_{\mathfrak{AE}-3} > \mathcal{P}_{\mathfrak{AE}-2} > \mathcal{P}_{\mathfrak{AE}-1} > \mathcal{P}_{\mathfrak{AE}-4}$

According to all the utilized operators in the MADM technique within CFS, the database management system $\mathcal{P}_{\mathfrak{AE}-3}$ is the optimal choice.

4.2| COMPARATIVE STUDY

To reveal the dominance and supremacy of the devised theory, here in this part of the article, we will compare the devised theory with three prevailing theories which are given below.

The theory of arithmetic AOs for a polar form of CFS was established by Bi et al. [18].

The theory of geometric AOs for a polar form of CFS was devised by Bi et al. [19].

The theory of power AOs under the polar form of CF information was interpreted by Hu et al. [20].

Let us consider the information displayed in Table 1 and apply the devised operators as well as the AOs invented by Bi et al. [18], Bi et al. [19], and Hu et al. [20]. The necessary result is displayed in Table 5.

Table 5.

The comparison among considered prevailing theories and invented theories.

Source	$\mathcal{S}(\mathcal{P}_{\mathfrak{AE}-1})$	$\mathcal{S}(\mathcal{P}_{\mathfrak{AE}-2})$	$\mathcal{S}(\mathcal{P}_{\mathfrak{AE}-3})$	$\mathcal{S}(\mathcal{P}_{\mathfrak{AE}-4})$	Ranking
Bi et al. [18]	×↝×↝×	×↝×↝×	×↝×↝×	×↝×↝×	×↝×↝×
Bi et al. [19]	×↝×↝×	×↝×↝×	×↝×↝×	×↝×↝×	×↝×↝×
Hu et al. [20]	×↝×↝×	×↝×↝×	×↝×↝×	×↝×↝×	×↝×↝×
P-CFWA	-0.21	-0.06	0.002	-0.43	$\mathcal{P}_{\mathfrak{AE}-3} > \mathcal{P}_{\mathfrak{AE}-2} > \mathcal{P}_{\mathfrak{AE}-1} > \mathcal{P}_{\mathfrak{AE}-4}$
P-CFOWA	-0.27	-0.09	-0.04	-0.51	$\mathcal{P}_{\mathfrak{AE}-3} > \mathcal{P}_{\mathfrak{AE}-2} > \mathcal{P}_{\mathfrak{AE}-1} > \mathcal{P}_{\mathfrak{AE}-4}$
IP-CFOWA	-0.3	-0.12	-0.11	-0.55	$\mathcal{P}_{\mathfrak{AE}-3} > \mathcal{P}_{\mathfrak{AE}-2} > \mathcal{P}_{\mathfrak{AE}-1} > \mathcal{P}_{\mathfrak{AE}-4}$
P-CFWG	-0.124	-0.103	-0.0004	-0.361	$\mathcal{P}_{\mathfrak{AE}-3} > \mathcal{P}_{\mathfrak{AE}-2} > \mathcal{P}_{\mathfrak{AE}-1} > \mathcal{P}_{\mathfrak{AE}-4}$
P-CFOWG	-0.168	-0.149	-0.04	-0.433	$\mathcal{P}_{\mathfrak{AE}-3} > \mathcal{P}_{\mathfrak{AE}-2} > \mathcal{P}_{\mathfrak{AE}-1} > \mathcal{P}_{\mathfrak{AE}-4}$
IP-CFOWG	-0.21	-0.185	-0.107	-0.482	$\mathcal{P}_{\mathfrak{AE}-3} > \mathcal{P}_{\mathfrak{AE}-2} > \mathcal{P}_{\mathfrak{AE}-1} > \mathcal{P}_{\mathfrak{AE}-4}$

The Theories invented by Bi et al. [18], Bi et al. [19], and Hu et al. [20] are in the structure of CFS but in polar structure and not in a cartesian form. Thus, these prevailing theories fail to cope with the information that contains extra fuzzy information in the form of an unreal part of the degree of support. More, there are no AOs in the setting of CFS that can handle the information that is in the form of real and imaginary parts. This shows how much the invented AOs and MADM techniques are necessary for coping with such sort of information. The invented theory can also cope with fuzzy information by letting the unreal part equal to zero in the degree of support.

5 | CONCLUSION

The creation, organization, storage, retrieval, and administration of structured data are all made simpler using a database management system (DBMS), a piece of software. In addition to providing tools for querying and reporting, it serves as a central system for efficiently managing data storage and access. Choosing the best DBMS is a MADM challenge due to its multi-criteria nature. As a result, in this script, we developed some fundamental operations and diagnosed probability AOs within the cartesian form of the CFS, including the P-CFWA, P-CFOWA, IP-CFOWA, and P-CFWG and IP-CFOWG operators. Additionally, we developed a MADM method within the CFS framework to deal with challenging and uncomfortable real-world MADM conundrums that involve additional fuzzy information (2nd dimension). After that, we looked into a numerical example of "Selection of the Optimal Database Management System" in this article to deal with a MADM conundrum in CFS. The created theory is compared to a few other widely held theories at the end of this article to show which is more dominant and superior.

In the future, our goal is to investigate the notion of bipolar complex fuzzy sets [29], bipolar complex fuzzy soft sets [30], [31] and complex hesitant fuzzy sets [32] and develop the devised AOs in these models.

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