ORIGINAL ARTICLE

OPEN ACCESS

Performance of the Multi Attributed Decision-Making Process with Interval-Valued Spherical Fuzzy Dombi Aggregation Operators

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Article History:

Submitted: February 13, 2022
Revised: May 26, 2022
Accepted: June 19, 2022
Published online: June 30, 2022

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ABSTRACT

This article aims to explore the notion of interval-valued spherical fuzzy (IVSF) sets (IVSFS), a modified version of intuitionistic and picture fuzzy sets. Furthermore, we also developed some appropriate operations of Dombi aggregation tools in the light of IVSF information. These aggregation operators are used aggregate several values in a single term. To mitigate the impact of insufficient information during the decision-making process, we developed a class of new approaches under the system of IVSF information, including IVSF Dombi weighted average (IVSFDWA) and IVSF Dombi weighted geometric (IVSFDWG) operators. To reveal effectiveness and intensity of developed approaches, some notable characteristics are also demonstrated. To resolve complicated real-life situations, an algorithm of the MADM problem is discussed under consideration of IVSF information. We establish a numerical example to select a more realistic optimal option available to individuals. To show consistency and applicability of developed methodologies, influence study and comparative analysis verify by contrasting the results of existing approaches with currently developed AOs.

Keywords: Interval-valued spherical fuzzy values, Dombi aggregation tools, Aggregation operators, and decision-making process.

1 | INTRODUCTION

In most of the events of the world, if this was a rule, it would draw a certain amount of unpredictability and imprecision; events involving human opinion are no exception Zadeh [1] proposed the use of fuzzy sets and membership grades (MG) of elements and objects to eliminate these mistakes and inexactness from data impacted by human judgment. Furthermore, Atanassov [2] linked the mechanisms and objects by using two grades, non-membership grade (NMG) and membership grade (MG), in his pioneer idea of intuitionistic FS (IFS). In Atanassov's idea of IFS, the sum of NMD and MG for an object must be contained in [0,1]. The chances of information loss decreased as compared to FS, but still exist. To remove this drawback of IFS, Yager [3] proposed a new idea of the Pythagorean fuzzy set (PyFS) in which he enlarged the range of MG and NMG by proposing that the sum of squares of MG and MNG contains in [0,1]. Yager [4] also enlarged the range of MD and NMG to be chosen by taking power q instead of square and named the idea as q-Rung Ortho Pair FS (q-ROFS). Several mathematicians extended the concepts of fuzzy systems to the circumstances.

In the case of human thoughts, the opinions cannot be stated only in two grades, i.e., MG and NMG. Hence Cuong [5] proposed a new approach, in which he added a new grade named abstinence grade (AG) in his pioneer idea of picture FS (PFS). In PFS, the sum of all three grades lies in [0,1]. After the addition of AG, the loss of information decreased. However, the range of these grades was short due to the condition that the sum of grades should be in [0,1]. To overcome the problem, Mahmood et al. [6] enlarged the range in the idea of Spherical FS (SFS) and T-Spherical FS (TSFS) by taking the sum of grades with power q to contain in [0,1]. After the development of the idea of TSFS, the range of information has enlarged to the maximum level, and decision-makers are independent in choosing the value of the grade of their interest. The Several authors also worked on the framework of PFSs [7]–[9] by including more information compared to IFSs, PyFSs and q-ROFSs.

A class of new AOs developed based on Dombi t-norms (DTNs) and Dombi t-conorms (DTCNs) by the [10]. The idea of DTN and DTCN is used by the authors in [11] and [12] in the framework of IFSs and PyFSs, respectively, for the expansion of DAOs. [13] utilized a novel theory of the MADM problem to resolve an application related to electric supply. The problems in DAOs in an environment of q-ROFSs are also highlighted in [14] by Jana et al. Wang [15] generalized the theory of 2-touple linguistic fuzzy sets with applications. Liu and Jiang [16] explored the theory of distance measures to handle vague information about human opinions. Liu et al. [17] proposed a class of new approaches based on neutrosophic sets. Hussain et al. [13] worked on the decision-making process by using the concepts of graphical fuzzy system. Several aggregation operators based on different aggregation tools under the system of fuzzy situations were proposed by Senapati et al. [18]–[20]. In [21] DAOs are extended to the concept of bipolar FS. Shi and Ye developed DAOs in the framework of neutrosophic FS [22]. For the concept of hesitant FS, the DAOs are developed by He [23]. Zhou et al. [24] investigated some new mathematical approaches by using frank aggregation tools with a decision-making process. Liu et al. [25] extended the theory of Archimedean triangular norms with Hesitant fuzzy circumstances. The remarkable work on the DAOs can be observed in [26]–[28].

The preceding discussion demonstrated the limited reach of the operators produced in the settings of IFSs, PyFSs, and q-ROFSs as well as the likelihood of some information loss. Furthermore, it has revealed the framework of PFSs and SFSs' limitations in handling particular triplets. Liu and Gao [29] presented a strong mechanism for the selection process of a suitable optimal option with some appropriate characteristics of the Maclaurin symmetric mean aggregation models. They used a numerical example to establish this conclusion. This study attempts to resolve these concerns by attempting to construct DAOs in the context of IVSFSs. The IVSF DAOs introduced in this paper further generalize previously developed DAOs. The objective of this article is characterized as follows:

1. To expose the notion of IVSFSs with fundamental rules of IVSF information.

2. Some robust operations of Dombi aggregation tools are presented under the system of IVSF information.
3. We developed a class of new approaches, including IVSFDWA and IVSFDWG operators with notable characteristics and special cases.
4. A novel study of the MADM problem is used to solve complex real-life situations. We also gave a numerical example to choose an appropriate programming bundle considering our developed approaches.
5. Averaging and geometric DAOs have been developed in the framework of IVSFSs, and their basic properties have been explored.
6. To analyze the reliability and applicability of proposed methodologies, we discussed sensitive analysis to show the influence of different parametric values of the Dombi aggregation tools.

The structure of this article is maintained as follows: Section 2 explores basic notions of IVSFSs with some fundamental rules. Some appropriate operations of Dombi aggregation tools are presented in section 3. We established a class of new approaches, including the IVSFDWA operator with some robust characteristics and special cases in section 4. Furthermore, we also presented a series of new approaches based on IVSF information, such as the IVSFDWG operator in section 5. An algorithm of the MADM problem under the system of IVSF information is presented in section 6. The advantages of developed aggregation operators are verified by comparing the outcomes of existing approaches with currently developed aggregation methodologies in section 7. Some valuable remarks related to our research work are established in section 8.

2 | PRELIMANRIES

The goal of this section is to review some basic notions and present the possible research gap. In our work, X represents some universe of discourse, and the terms NMG, MG, AG, and RG are denoted by ḑ, š̉, İ̉, and r, respectively.

Definition 1: [30] A PFS is of the form $\aleph = \left\{\left(\varsigma, (\check{s}, \dot{I}, ḑ)\right) \forall\, \varsigma \in X\right\}$ provided that $0 \leq sum(\check{s}(\varsigma), \dot{I}(\varsigma), ḑ(\varsigma)) \leq 1$ and $r(\varsigma) = 1 - (\check{s}(\varsigma) + \dot{I}(\varsigma) + ḑ(\varsigma))$ is known as the refusal grade (RG) of ς in ℵ. Further, $(\check{s}, \dot{I}, ḑ)$ is known as a picture fuzzy number (PFN).

Definition 2: [6] A SFS is of the form $\aleph = \left\{\left(\varsigma, (\check{s}, \dot{I}, ḑ)\right) \forall\, \varsigma \in X\right\}$ provided that $0 \leq sum(\check{s}^2(\varsigma), \dot{I}^2(\varsigma), ḑ^2(\varsigma)) \leq 1$ and $r(\varsigma) = \sqrt{1 - (\check{s}^2(\varsigma) + \dot{I}^2(\varsigma) + ḑ^2(\varsigma))}$ is known as the RG of ς in ℵ. Further, $(\check{s}, \dot{I}, ḑ)$ is known as a spherical fuzzy number (SFN).

Definition 3: An IVSFS on X is given below:

$$\aleph = \left\{ \begin{matrix} \left(\varsigma, \left(\dot{s}, \dot{I}, \d{d}\right)\right) : \dot{s}, \dot{I}, \d{d}: X \rightarrow some\ closed\ subinterval\ of\ [0,1]\ \forall\ \varsigma \in X, \\ 0 \leq (\dot{s}^{u^2}(\varsigma) + \dot{I}^{u^2}(\varsigma) + \d{d}^{u^2}(\varsigma) \leq 1 \\ r(\varsigma) = [r^l(\varsigma), r^u(\varsigma)] = \left(\begin{matrix} \left[\left(1 - (\dot{s}^u)^2(\varsigma) - (\dot{I}^u)^2(\varsigma) + (\d{d}^u)^2\right)^{\frac{1}{2}} \right], \\ \left[\left(1 - (\dot{s}^l)^2(\varsigma) - (\dot{I}^l)^2(\varsigma) + (\d{d}^l)^2\right)^{\frac{1}{2}} \right] \end{matrix} \right) \end{matrix} \right\}$$

Where $\dot{s} = [\dot{s}^l(\varsigma), \dot{s}^u(\varsigma)]$, denote the MG, $\d{d} = [\d{d}^l(\varsigma), \d{d}^u(\varsigma)]$ denote the NMG, and $\dot{I} = [\dot{I}^l(\varsigma), \dot{I}^u(\varsigma)]$ denote the AG. While $r = [r^l(\varsigma), r^u(\varsigma)]$ denote the RG respectively. Furthermore, the triplet $\left(\dot{s}\,(\varsigma), \dot{I}(\varsigma), \d{d}(\varsigma)\right) = \left([\dot{s}^l(\varsigma), \dot{I}^l(\varsigma), \d{d}^l(\varsigma)], [\dot{s}^u(\varsigma), \dot{I}^u(\varsigma), \d{d}^u(\varsigma)]\right)$ is known as an IVSF value (IVSFV).

Now, we propose the ranking function of IVSFSs as follows:

Definition 4: Let $\aleph = \left(\dot{s}(x), \dot{I}(x), \d{d}(x)\right) = \left([\dot{s}^l, \dot{s}^u], [\dot{I}^l, \dot{I}^u], [\d{d}^l, \d{d}^u]\right)$ represent an IVSFV. Then the score function is given by:

$$SC(\aleph) = \frac{\left(\dot{s}^l\right)^2 \left(1 - \left(\dot{I}^l\right)^2 - \left(\d{d}^l\right)^2\right) + (\dot{s}^u)^2 \left(1 - \left(\dot{I}^u\right)^2 - (\d{d}^u)^2\right)}{3}$$

Where $SC(\aleph) \in [0,1]$.

Definition 5: [31] Let $\aleph_{\mathcal{S}} = \left(\dot{s}_{\mathcal{S}}, \d{d}_{\mathcal{S}}\right)$ be some IFNs. Then the weighted averaging (WA) and weighted geometric (WG) DAOs for IFNs are functions from $\aleph^n$ to $\aleph$ and defined as:

$$IFDWA(\aleph_1, \aleph_2, \aleph_3 \dots \aleph_n) = \left(1 - \frac{1}{1 + \left\{\sum_{\mathcal{S}=1}^{n} \Phi_{\mathcal{S}} \left(\frac{\dot{s}_{\mathcal{S}}}{1 - \dot{s}_{\mathcal{S}}}\right)^{F}\right\}^{\frac{1}{F}}}, \frac{1}{1 + \left\{\sum_{\mathcal{S}=1}^{n} \Phi_{\mathcal{S}} \left(\frac{1 - \d{d}_{\mathcal{S}}}{\d{d}_{\mathcal{S}}}\right)^{F}\right\}^{\frac{1}{F}}} \right)$$

$$IFDWG(\aleph_1, \aleph_2, \aleph_3 \dots \aleph_n) = \left(\frac{1}{1 + \left\{\sum_{\mathcal{S}=1}^{n} \Phi_{\mathcal{S}} \left(\frac{1 - \dot{s}_{\mathcal{S}}}{\dot{s}_{\mathcal{S}}}\right)^{F}\right\}^{\frac{1}{F}}}, 1 - \frac{1}{1 + \left\{\sum_{\mathcal{S}=1}^{n} \Phi_{\mathcal{S}} \left(\frac{\d{d}_{\mathcal{S}}}{1 - \d{d}_{\mathcal{S}}}\right)^{F}\right\}^{\frac{1}{F}}} \right)$$

3 | BASIC OPERATIONS OF DOMBI AGGREGATION TOOLS

DTN and DTCN are among the famous t-norms and t-conorms in the theory of fuzzy algebra. Here we discussed the theory of DTNs and DTCNs and a few previously defined DAOs in previous fuzzy extensions. It is to be noted that throughout this thesis, $\Phi = (\Phi_1, \Phi_2, \Phi_3 \dots \Phi_n)^T$ is termed as the weight vector of IVSFVs $\aleph_{\mathcal{S}}(\mathcal{S} = 1,2,3 \dots n)$ with $\Phi_{\mathcal{S}} > 0$ and $\sum_{\mathcal{S}=1}^{n} \Phi_{\mathcal{S}} = 1$. Further $(\mathcal{S} = 1,2, \dots n)$ denote the indexing terms.

Definition 6: [10] Let $f, g \in \mathbb{R}$. Then, DTN and DTCN are defined as

$$Dom^{tn}(f,g)=\frac{1}{1+\left\{\left(\frac{1-\dot{f}}{\dot{f}}\right)^{₣}+\left(\frac{1-g}{g}\right)^{₣}\right\}^{\frac{1}{₣}}}$$

$$Dom^{tcn}(f,g)=1-\frac{1}{1+\left\{\left(\frac{\dot{f}}{1-\dot{f}}\right)^{₣}+\left(\frac{g}{1-g}\right)^{₣}\right\}^{\frac{1}{₣}}}$$

Definition 7: Let $\aleph=\left([\dot{s}^l,\dot{s}^u],[\dot{i}^l,\dot{i}^u],[d^l,d^u]\right)$, $\aleph_1=\left([\dot{s}_1^l,\dot{s}_1^u],[\dot{i}_1^l,\dot{i}_1^u],[d_1^l,d_1^u]\right)$ and $\aleph_2=\left([\dot{s}_2^l,\dot{s}_2^u],[\dot{i}_2^l,\dot{i}_2^u],[d_2^l,d_2^u]\right)$ denote IVSFVs. For $\lambda>0$, The Dombi operations for IVSFSs are given by:

1. $$\aleph_1\oplus\aleph_2=\left(\begin{array}{c}\left[\sqrt{1-\frac{1}{1+\left\{\left(\frac{\dot{s}_1^{l2}}{1-\dot{s}_1^{l2}}\right)^{₣}+\left(\frac{\dot{s}_2^{l2}}{1-\dot{s}_2^{l2}}\right)^{₣}\right\}^{\frac{1}{₣}}}},\sqrt{1-\frac{1}{1+\left\{\left(\frac{\dot{s}_1^{u2}}{1-\dot{s}_1^{u2}}\right)^{₣}+\left(\frac{\dot{s}_2^{u2}}{1-\dot{s}_2^{u2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\left(\frac{1-\dot{i}_1^{l2}}{\dot{i}_1^{l2}}\right)^{₣}+\left(\frac{1-\dot{i}_2^{l2}}{\dot{i}_2^{l2}}\right)^{₣}\right\}^{\frac{1}{₣}}}},\sqrt{\frac{1}{1+\left\{\left(\frac{1-\dot{i}_1^{u2}}{\dot{i}_1^{u2}}\right)^{₣}+\left(\frac{1-\dot{i}_2^{u2}}{\dot{i}_2^{u2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\left(\frac{1-d_1^{l2}}{d_1^{l2}}\right)^{₣}+\left(\frac{1-d_2^{l2}}{d_2^{l2}}\right)^{₣}\right\}^{\frac{1}{₣}}}},\sqrt{\frac{1}{1+\left\{\left(\frac{1-d_1^{u2}}{d_1^{u2}}\right)^{₣}+\left(\frac{1-d_2^{u2}}{d_2^{u2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right]\end{array}\right)$$

2. $$\aleph_1\otimes\aleph_2=\left(\begin{array}{c}\left[\sqrt{\frac{1}{1+\left\{\left(\frac{1-\dot{s}_1^{l2}}{\dot{s}_1^{l2}}\right)^{₣}+\left(\frac{1-\dot{s}_2^{l2}}{\dot{s}_2^{l2}}\right)^{₣}\right\}^{\frac{1}{₣}}}},\sqrt{\frac{1}{1+\left\{\left(\frac{1-\dot{s}_1^{u2}}{\dot{s}_1^{u2}}\right)^{₣}+\left(\frac{1-\dot{s}_2^{u2}}{\dot{s}_2^{u2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right],\\ \left[\sqrt{1-\frac{1}{1+\left\{\left(\frac{\dot{i}_1^{l2}}{1-\dot{i}_1^{l2}}\right)^{₣}+\left(\frac{\dot{i}_2^{l2}}{1-\dot{i}_2^{l2}}\right)^{₣}\right\}^{\frac{1}{₣}}}},\sqrt{1-\frac{1}{1+\left\{\left(\frac{\dot{i}_1^{u2}}{1-\dot{i}_1^{u2}}\right)^{₣}+\left(\frac{\dot{i}_2^{u2}}{1-\dot{i}_2^{u2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right],\\ \left[\sqrt{1-\frac{1}{1+\left\{\left(\frac{d_1^{l2}}{1-d_1^{l2}}\right)^{₣}+\left(\frac{d_2^{l2}}{1-d_2^{l2}}\right)^{₣}\right\}^{\frac{1}{₣}}}},\sqrt{1-\frac{1}{1+\left\{\left(\frac{d_1^{u2}}{1-d_1^{u2}}\right)^{₣}+\left(\frac{d_2^{u2}}{1-d_2^{u2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right]\end{array}\right)$$

3. $$\lambda\aleph=\left(\begin{array}{c}\left[\sqrt{1-\frac{1}{1+\left\{\lambda\left(\frac{ş^{l^2}}{1-ş^{l^2}}\right)^{₣}\right\}^{\frac{1}{₣}}}},\sqrt{1-\frac{1}{1+\left\{\lambda\left(\frac{ş^{u^2}}{1-ş^{u^2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right],\left[\sqrt{\frac{1}{1+\left\{\lambda\left(\frac{1-ị^{l^2}}{ị^{l^2}}\right)^{₣}\right\}^{\frac{1}{₣}}}},\sqrt{\frac{1}{1+\left\{\lambda\left(\frac{1-ị^{u^2}}{ị^{u^2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\lambda\left(\frac{1-ḑ^{l^2}}{ḑ^{l^2}}\right)^{₣}\right\}^{\frac{1}{₣}}}},\sqrt{\frac{1}{1+\left\{\lambda\left(\frac{1-ḑ^{u^2}}{ḑ^{u^2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right]\end{array}\right)$$

4. $$\aleph^{\lambda}=\left(\begin{array}{c}\left[\sqrt{\frac{1}{1+\left\{\lambda\left(\frac{1-ş^{l^2}}{ş^{l^2}}\right)^{₣}\right\}^{\frac{1}{₣}}}},\sqrt{\frac{1}{1+\left\{\lambda\left(\frac{1-ş^{u^2}}{ş^{u^2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right],\left[\sqrt{1-\frac{1}{1+\left\{\lambda\left(\frac{ị^{l^2}}{1-ị^{l^2}}\right)^{₣}\right\}^{\frac{1}{₣}}}},\sqrt{1-\frac{1}{1+\left\{\lambda\left(\frac{ị^{u^2}}{1-ị^{u^2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right],\\ \left[\sqrt{1-\frac{1}{1+\left\{\lambda\left(\frac{ḑ^{l^2}}{1-ḑ^{l^2}}\right)^{₣}\right\}^{\frac{1}{₣}}}},\sqrt{1-\frac{1}{1+\left\{\lambda\left(\frac{ḑ^{u^2}}{1-ḑ^{u^2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right]\end{array}\right)$$

With the below example, we show the working of the above-discussed principles.

Example 1: Let $\aleph=\begin{pmatrix}[0.65,0.83],\\ [0.4,0.54],\\ [0.26,0.41]\end{pmatrix}$, $\aleph_1=\begin{pmatrix}[0.43,0.53],\\ [0.29,0.43],\\ [0.39,0.47]\end{pmatrix}$ and $\aleph_2=\begin{pmatrix}[0.77,0.89],\\ [0.34,0.49],\\ [0.59,0.68]\end{pmatrix}$ be three IVSFVs and $\lambda=2$ and $₣=1$. Then:

$$\aleph_1\oplus\aleph_2=\left(\begin{array}{c}\left[\begin{array}{c}\sqrt[4]{1-\frac{1}{1+\left\{\left(\frac{0.43^4}{1-0.43^4}\right)^1+\left(\frac{0.53^4}{1-0.53^4}\right)^1\right\}^{\frac{1}{1}}}},\\ \sqrt[4]{1-\frac{1}{1+\left\{\left(\frac{0.77^4}{1-0.77^4}\right)^1+\left(\frac{0.89^4}{1-0.89^4}\right)^1\right\}^{\frac{1}{1}}}}\end{array}\right],\\ \left[\sqrt[4]{\frac{1}{1+\left\{\left(\frac{1-0.29^4}{0.29^4}\right)^1+\left(\frac{1-0.43^4}{0.43^4}\right)^1\right\}^{\frac{1}{1}}}},\sqrt[4]{\frac{1}{1+\left\{\left(\frac{1-0.34^4}{0.34^4}\right)^1+\left(\frac{1-0.49^4}{0.49^4}\right)^1\right\}^{\frac{1}{1}}}}\right],\\ \left[\sqrt[4]{\frac{1}{1+\left\{\left(\frac{1-0.39^4}{0.39^4}\right)^2+\left(\frac{1-0.47^4}{0.47^4}\right)^2\right\}^{\frac{1}{2}}}},\sqrt[4]{\frac{1}{1+\left\{\left(\frac{1-0.59^4}{0.59^4}\right)^1+\left(\frac{1-0.68^4}{0.68^4}\right)^1\right\}^{\frac{1}{1}}}}\right]\end{array}\right)$$

$$\aleph_1 \oplus \aleph_2 = \begin{pmatrix} [0.092,0.16], \\ [0.0012,0.0055], \\ [0.0050,0.0101] \end{pmatrix}.$$

$$\aleph_1 \otimes \aleph_2 = \begin{pmatrix} \left[\sqrt[4]{\frac{1}{1+\left\{\left(\frac{1-0.43^4}{0.43^4}\right)^1+\left(\frac{1-0.53^4}{0.53^4}\right)^1\right\}^{\frac{1}{1}}}}, \sqrt[4]{\frac{1}{1+\left\{\left(\frac{1-0.77^4}{0.77^4}\right)^1+\left(\frac{1-0.89^4}{0.89^4}\right)^1\right\}^{\frac{1}{1}}}} \right], \\ \left[\sqrt[4]{1-\frac{1}{1+\left\{\left(\frac{0.29^4}{1-0.29^4}\right)^1+\left(\frac{0.43^4}{1-0.43^4}\right)^1\right\}^{\frac{1}{1}}}}, \sqrt[4]{1-\frac{1}{1+\left\{\left(\frac{0.34^4}{1-0.34^4}\right)^1+\left(\frac{0.49^4}{1-0.49^4}\right)^1\right\}^{\frac{1}{1}}}} \right], \\ \left[\sqrt[4]{1-\frac{1}{1+\left\{\left(\frac{0.39^4}{1-0.39^4}\right)^2+\left(\frac{0.47^4}{1-0.47^4}\right)^2\right\}^{\frac{1}{2}}}}, \sqrt[4]{1-\frac{1}{1+\left\{\left(\frac{0.59^4}{1-0.59^4}\right)^1+\left(\frac{0.68^4}{1-0.68^4}\right)^1\right\}^{\frac{1}{1}}}} \right] \end{pmatrix}$$

$$\aleph_1 \otimes \aleph_2 = \begin{pmatrix} [0.008,0.019], \\ [0.0051,0.022], \\ [0.035,0.061] \end{pmatrix}$$

$$2\aleph = \begin{pmatrix} \left[\sqrt[4]{1-\frac{1}{1+\left\{2\left(\frac{0.65^4}{1-0.65^4}\right)^1\right\}^{\frac{1}{1}}}}, \sqrt[4]{1-\frac{1}{1+\left\{2\left(\frac{0.83^4}{1-0.83^4}\right)^1\right\}^{\frac{1}{1}}}} \right], \\ \left[\sqrt[4]{\frac{1}{1+\left\{2\left(\frac{1-0.4^4}{0.4^4}\right)^1\right\}^{\frac{1}{1}}}}, \sqrt[4]{\frac{1}{1+\left\{2\left(\frac{1-0.54^4}{0.54^4}\right)^1\right\}^{\frac{1}{1}}}} \right], \\ \left[\sqrt[4]{\frac{1}{1+\left\{2\left(\frac{1-0.26^4}{0.26^4}\right)^1\right\}^{\frac{1}{1}}}}, \sqrt[4]{\frac{1}{1+\left\{2\left(\frac{1-0.41^4}{0.41^4}\right)^1\right\}^{\frac{1}{1}}}} \right] \end{pmatrix}$$

$$2\aleph=\begin{pmatrix}[0.076,0.161],\\ [0.0032,0.0111],\\ [0,0.004]\end{pmatrix}$$

$$\aleph^2=\begin{pmatrix}\left[\sqrt[4]{\dfrac{1}{1+\left\{2\left(\frac{1-0.65^4}{0.65^4}\right)^1\right\}^{\frac{1}{1}}}},\sqrt[4]{\dfrac{1}{1+\left\{2\left(\frac{1-0.83^4}{0.83^4}\right)^1\right\}^{\frac{1}{1}}}}\right],\\ \left[\sqrt[4]{1-\dfrac{1}{1+\left\{2\left(\frac{0.4^4}{1-0.4^4}\right)^1\right\}^{\frac{1}{1}}}},\sqrt[4]{1-\dfrac{1}{1+\left\{2\left(\frac{0.54^4}{1-0.54^4}\right)^1\right\}^{\frac{1}{1}}}}\right],\\ \left[\sqrt[4]{1-\dfrac{1}{1+\left\{2\left(\frac{0.26^4}{1-0.26^4}\right)^1\right\}^{\frac{1}{1}}}},\sqrt[4]{1-\dfrac{1}{1+\left\{2\left(\frac{0.41^4}{1-0.41^4}\right)^1\right\}^{\frac{1}{1}}}}\right]\end{pmatrix}$$

$$\aleph^2=\begin{pmatrix}[0.026,0.078],\\ [0.0125,0.0392],\\ [0.002,0.014]\end{pmatrix}$$

Theorem 1: Let $\aleph=\left([\check{s}^l,\check{s}^u],[\hat{\imath}^l,\hat{\imath}^u],[\d{d}^l,\d{d}^u]\right)$, $\aleph_1=\left([\check{s}_1^l,\check{s}_1^u],[\hat{\imath}_1^l,\hat{\imath}_1^u],[\d{d}_1^l,\d{d}_1^u]\right)$ and $\aleph_2=\left([\check{s}_2^l,\check{s}_2^u],[\hat{\imath}_2^l,\hat{\imath}_2^u],\right.$ $\left.[\d{d}_2^l,\d{d}_2^u]\right)$ be three IVSFVs and $\lambda\geq 0$. Then

1. $\aleph_1\otimes\aleph_2=\aleph_2\otimes\aleph_1$
2. $\aleph_1\oplus\aleph_2=\aleph_2\oplus\aleph_1$
3. $\lambda(\aleph_1\oplus\aleph_2)=\lambda\aleph_1\oplus\lambda\aleph_2$
4. $(\lambda_1\oplus\lambda_2)\aleph=\lambda_1\aleph\oplus\lambda_2\aleph$
5. $(\aleph_1\otimes\aleph_2)^\lambda=\aleph_1^\lambda\otimes\aleph_2^\lambda$
6. $\aleph^{\lambda_1}\otimes\aleph^{\lambda_2}=\aleph^{\lambda_1+\lambda_2}$

Proof: Straightforward.

4 | INTERVAL-VALUED SPHERICAL FUZZY DOMBI AGGREGATION OPERATORS

In this section, we introduced a class of new aggregation operators based on IVSF information.

Definition 8: Let $\aleph_{\text{ß}}=\left([\check{s}_{\text{ß}}^l,\check{s}_{\text{ß}}^u],[\hat{\imath}_{\text{ß}}^l,\hat{\imath}_{\text{ß}}^u],[\d{d}_{\text{ß}}^l,\d{d}_{\text{ß}}^u]\right)$ be few IVSFVs. Then IVSFDWA operator is given as:

$$IVSFDWA(\aleph_1,\aleph_2\,\ldots.\,\aleph_n)=\bigoplus_{\text{ß}=1}^{n}\left(\Phi_{\text{ß}}\aleph_{\text{ß}}\right)$$

Theorem 2: Let $\aleph_{\text{ß}}=\left([\check{s}_{\text{ß}}^l,\check{s}_{\text{ß}}^u],[\hat{\imath}_{\text{ß}}^l,\hat{\imath}_{\text{ß}}^u],[\d{d}_{\text{ß}}^l,\d{d}_{\text{ß}}^u]\right)$ denote some IVSFVs. Then using the IVSFDWA operator, we get an IVSFV given as:

$$IVSFDWA(\aleph_1,\aleph_2,\aleph_3\dots\aleph_n)=\left(\begin{array}{c}\left[\sqrt{1-\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{\dot{s}^{l\,2}_{\mathfrak{s}}}{1-\dot{s}^{l\,2}_{\mathfrak{s}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{1-\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{\dot{s}^{u\,2}_{\mathfrak{s}}}{1-\dot{s}^{u\,2}_{\mathfrak{s}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{1-\mathring{i}^{l\,2}_{\mathfrak{s}}}{\mathring{i}^{l\,2}_{\mathfrak{s}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{1-\mathring{i}^{u\,2}_{\mathfrak{s}}}{\mathring{i}^{u\,2}_{\mathfrak{s}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{1-\c{d}^{l\,2}_{\mathfrak{s}}}{\c{d}^{l\,2}_{\mathfrak{s}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{1-\c{d}^{u\,2}_{\mathfrak{s}}}{\c{d}^{u\,2}_{\mathfrak{s}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right]\end{array}\right)$$

Proof: To prove this result, we use the induction method.

When $n=2$. Then,

$$IVSFDWA(\aleph_1,\aleph_2)=\aleph_1\oplus\aleph_2=\Phi_1\left([\dot{s}^l_1,\dot{s}^u_1],[\mathring{i}^l_1,\mathring{i}^u_1],[\c{d}^l_1,\c{d}^u_1]\right)\oplus\Phi_2\left([\dot{s}^l_2,\dot{s}^u_2],[\mathring{i}^l_2,\mathring{i}^u_2],[\c{d}^l_2,\c{d}^u_2]\right)$$

$$=\left(\begin{array}{c}\left[\sqrt{1-\dfrac{1}{1+\left\{\Phi_1\left(\dfrac{\dot{s}^{l\,2}_1}{1-\dot{s}^{l\,2}_1}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{1-\dfrac{1}{1+\left\{\Phi_1\left(\dfrac{\dot{s}^{u\,2}_1}{1-\dot{s}^{u\,2}_1}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\right],\\ \left[\sqrt{\dfrac{1}{1+\left\{\Phi_1\left(\dfrac{1-\mathring{i}^{u\,2}_1}{\mathring{i}^{u\,2}_1}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\left[\begin{array}{c}\sqrt{\dfrac{1}{1+\left\{\Phi_1\left(\dfrac{1-\c{d}^{l\,2}_1}{\c{d}^{l\,2}_1}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\\ \sqrt{\dfrac{1}{1+\left\{\Phi_1\left(\dfrac{1-\c{d}^{u\,2}_1}{\c{d}^{u\,2}_1}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\end{array}\right]\end{array}\right)$$

$$\oplus\left(\begin{array}{c}\left[\sqrt{1-\frac{1}{1+\left\{\Phi_2\left(\frac{\dot{\text{ṡ}}^{l}{}_{2}^{2}}{1-\dot{\text{ṡ}}^{l}{}_{2}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{1-\frac{1}{1+\left\{\Phi_2\left(\frac{\dot{\text{ṡ}}^{u}{}_{2}^{2}}{1-\dot{\text{ṡ}}^{u}{}_{2}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\Phi_2\left(\frac{1-\text{ỉ}^{l}{}_{2}^{2}}{\text{ỉ}^{l}{}_{2}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\Phi_2\left(\frac{1-\text{ỉ}^{u}{}_{2}^{2}}{\text{ỉ}^{u}{}_{2}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\Phi_2\left(\frac{1-\text{ḑ}^{l}{}_{2}^{2}}{\text{ḑ}^{l}{}_{2}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\Phi_2\left(\frac{1-\text{ḑ}^{u}{}_{2}^{2}}{\text{ḑ}^{u}{}_{2}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right]\end{array}\right)$$

$$=\left(\begin{array}{c}\left[\sqrt{1-\frac{1}{1+\left\{\sum_{\text{ß}=1}^{2}\Phi_{\text{ß}}\left(\frac{\dot{\text{ṡ}}^{l}{}_{\text{ß}}^{2}}{1-\dot{\text{ṡ}}^{l}{}_{\text{ß}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{\text{ß}=1}^{2}\Phi_{\text{ß}}\left(\frac{\dot{\text{ṡ}}^{u}{}_{\text{ß}}^{2}}{1-\dot{\text{ṡ}}^{u}{}_{\text{ß}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\text{ß}=1}^{2}\Phi_{\text{ß}}\left(\frac{1-\text{ỉ}^{l}{}_{\text{ß}}^{2}}{\text{ỉ}^{l}{}_{\text{ß}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\sum_{\text{ß}=1}^{2}\Phi_{\text{ß}}\left(\frac{1-\text{ỉ}^{u}{}_{\text{ß}}^{2}}{\text{ỉ}^{u}{}_{\text{ß}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\text{ß}=1}^{2}\Phi_{\text{ß}}\left(\frac{1-\text{ḑ}^{l}{}_{\text{ß}}^{2}}{\text{ḑ}^{l}{}_{\text{ß}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\sum_{\text{ß}=1}^{2}\Phi_{\text{ß}}\left(\frac{1-\text{ḑ}^{u}{}_{\text{ß}}^{2}}{\text{ḑ}^{u}{}_{\text{ß}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right]\end{array}\right)$$

Hence for $n=2$ the result is true. Suppose that the above aggregation operator is true for $n=k$. Then:

$$IVSFDWA(\aleph_1,\aleph_2,\aleph_3\ldots\aleph_k)=\left(\begin{array}{c}\left[\sqrt{1-\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{k}\Phi_{\mathfrak{s}}\left(\frac{\dot{s}l_{\mathfrak{s}}^{2}}{1-\dot{s}l_{\mathfrak{s}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{k}\Phi_{\mathfrak{s}}\left(\frac{\dot{s}u_{\mathfrak{s}}^{2}}{1-\dot{s}u_{\mathfrak{s}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{k}\Phi_{\mathfrak{s}}\left(\frac{1-\mathrm{I}l_{\mathfrak{s}}^{2}}{\mathrm{I}l_{\mathfrak{s}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{k}\Phi_{\mathfrak{s}}\left(\frac{1-\mathrm{I}u_{\mathfrak{s}}^{2}}{\mathrm{I}u_{\mathfrak{s}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{k}\Phi_{\mathfrak{s}}\left(\frac{1-\d{d}l_{\mathfrak{s}}^{2}}{\d{d}l_{\mathfrak{s}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{k}\Phi_{\mathfrak{s}}\left(\frac{1-\d{d}u_{\mathfrak{s}}^{2}}{\d{d}u_{\mathfrak{s}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right]\end{array}\right)$$

Now for $n = k+1$, then

$IVSFDWA(\aleph_1,\aleph_2,\aleph_3 \dots \aleph_k,\aleph_{k+1})$

$$= \left(\left(\begin{array}{c}\left[\sqrt{1-\frac{1}{1+\left\{\sum_{\text{ꞩ}=1}^{k}\Phi_{\text{ꞩ}}\left(\frac{\text{ṡ}l_{\text{ꞩ}}^{2}}{1-\text{ṡ}l_{\text{ꞩ}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{\text{ꞩ}=1}^{k}\Phi_{\text{ꞩ}}\left(\frac{\text{ṡ}u_{\text{ꞩ}}^{2}}{1-\text{ṡ}u_{\text{ꞩ}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\text{ꞩ}=1}^{k}\Phi_{\text{ꞩ}}\left(\frac{1-\text{ỉ}l_{\text{ꞩ}}^{2}}{\text{ỉ}l_{\text{ꞩ}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\sum_{\text{ꞩ}=1}^{k}\Phi_{\text{ꞩ}}\left(\frac{1-\text{ỉ}u_{\text{ꞩ}}^{2}}{\text{ỉ}u_{\text{ꞩ}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\text{ꞩ}=1}^{k}\Phi_{\text{ꞩ}}\left(\frac{1-\text{ḑ}l_{\text{ꞩ}}^{2}}{\text{ḑ}l_{\text{ꞩ}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\sum_{\text{ꞩ}=1}^{k}\Phi_{\text{ꞩ}}\left(\frac{1-\text{ḑ}u_{\text{ꞩ}}^{2}}{\text{ḑ}u_{\text{ꞩ}}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right]\end{array}\right)\oplus\left(\begin{array}{c}\left[\sqrt{1-\frac{1}{1+\left\{\sum_{\text{ꞩ}=1}^{k+1}\Phi_{\text{ꞩ}}\left(\frac{\text{ṡ}l_{k+1}^{2}}{1-\text{ṡ}l_{k+1}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{\text{ꞩ}=1}^{k+1}\Phi_{\text{ꞩ}}\left(\frac{\text{ṡ}u_{k+1}^{2}}{1-\text{ṡ}u_{k+1}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\text{ꞩ}=1}^{k+1}\Phi_{\text{ꞩ}}\left(\frac{1-\text{ỉ}l_{k+1}^{2}}{\text{ỉ}l_{k+1}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\sum_{\text{ꞩ}=1}^{k+1}\Phi_{\text{ꞩ}}\left(\frac{1-\text{ỉ}u_{k+1}^{2}}{\text{ỉ}u_{k+1}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\text{ꞩ}=1}^{k+1}\Phi_{\text{ꞩ}}\left(\frac{1-\text{ḑ}l_{k+1}^{2}}{\text{ḑ}l_{k+1}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\sum_{\text{ꞩ}=1}^{k+1}\Phi_{\text{ꞩ}}\left(\frac{1-\text{ḑ}u_{k+1}^{2}}{\text{ḑ}u_{k+1}^{2}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right]\end{array}\right)\right)$$

$$= \left(\begin{array}{c} \left[\sqrt{1 - \frac{1}{1 + \left\{ \sum_{\mathcal{S}=1}^{k+1} \Phi_{\mathcal{S}} \left(\frac{\dot{s} l_{\mathcal{S}}^{2}}{1 - \dot{s} l_{\mathcal{S}}^{2}} \right)^{\text{₣}} \right\}^{\frac{1}{\text{₣}}}}}, \sqrt{1 - \frac{1}{1 + \left\{ \sum_{\mathcal{S}=1}^{k+1} \Phi_{\mathcal{S}} \left(\frac{\dot{s} u_{\mathcal{S}}^{2}}{1 - \dot{s} u_{\mathcal{S}}^{2}} \right)^{\text{₣}} \right\}^{\frac{1}{\text{₣}}}}} \right], \\ \left[\sqrt{\frac{1}{1 + \left\{ \sum_{\mathcal{S}=1}^{k+1} \Phi_{\mathcal{S}} \left(\frac{1 - \text{ỉ} l_{\mathcal{S}}^{2}}{\text{ỉ} l_{\mathcal{S}}^{2}} \right)^{\text{₣}} \right\}^{\frac{1}{\text{₣}}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{\mathcal{S}=1}^{k+1} \Phi_{\mathcal{S}} \left(\frac{1 - \text{ỉ} u_{\mathcal{S}}^{2}}{\text{ỉ} u_{\mathcal{S}}^{2}} \right)^{\text{₣}} \right\}^{\frac{1}{\text{₣}}}}} \right], \\ \left[\sqrt{\frac{1}{1 + \left\{ \sum_{\mathcal{S}=1}^{k+1} \Phi_{\mathcal{S}} \left(\frac{1 - \text{ḑ} l_{\mathcal{S}}^{2}}{\text{ḑ} l_{\mathcal{S}}^{2}} \right)^{\text{₣}} \right\}^{\frac{1}{\text{₣}}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{\mathcal{S}=1}^{k+1} \Phi_{\mathcal{S}} \left(\frac{1 - \text{ḑ} u_{\mathcal{S}}^{2}}{\text{ḑ} u_{\mathcal{S}}^{2}} \right)^{\text{₣}} \right\}^{\frac{1}{\text{₣}}}}} \right] \end{array} \right)$$

Along these lines, the outcome is valid for $n = k + 1$ which shows that this result is valid for n. The accompanying example shows the applicability of the IVSFDWA operator.

Example 2: Let $\aleph_1 = \begin{pmatrix} [0.65,0.83], \\ [0.4,0.54], \\ [0.26,0.41] \end{pmatrix}$, $\aleph_2 = \begin{pmatrix} [0.43,0.53], \\ [0.29,\ 0.43], \\ [0.39,0.47] \end{pmatrix}$, $\aleph_3 = \begin{pmatrix} [0.77,0.89], \\ [0.34,0.49], \\ [0.59,0.68] \end{pmatrix}$ and $\aleph_4 = \begin{pmatrix} [0.71,0.81], \\ [0.56,0.65], \\ [0.64,0.65] \end{pmatrix}$ be four IVSFVs and let $\text{₣} = 1$ and $\Phi = (0.15, 0.25, 0.41, 0.19)^T$. Then

$$IVSFDWA(\aleph_1,\aleph_2,\aleph_3,\aleph_4)=\left(\begin{array}{c}\left[\sqrt[4]{1-\frac{1}{1+\left\{\begin{array}{c}0.15\left(\frac{0.65^4}{1-0.65^4}\right)^1+0.25\left(\frac{0.43^4}{1-0.43^4}\right)^1+\\0.41\left(\frac{0.77^4}{1-0.77^4}\right)^1+0.19\left(\frac{0.71^4}{1-0.71^4}\right)^1\end{array}\right\}^{\frac{1}{1}}}},\sqrt[4]{1-\frac{1}{1+\left\{\begin{array}{c}0.15\left(\frac{0.83^4}{1-0.83^4}\right)^1+0.25\left(\frac{0.53^4}{1-0.53^4}\right)^1+\\0.41\left(\frac{0.89^4}{1-0.89^4}\right)^1+0.19\left(\frac{0.81^4}{1-0.81^4}\right)^1\end{array}\right\}^{\frac{1}{1}}}}\right],\\ \left[\sqrt[4]{\frac{1}{1+\left\{\begin{array}{c}0.15\left(\frac{1-0.4^4}{0.4^4}\right)^1+0.25\left(\frac{1-0.29^4}{0.29^4}\right)^1+\\0.41\left(\frac{1-0.34^4}{0.34^4}\right)^1+0.19\left(\frac{1-0.56^4}{0.56^4}\right)^1\end{array}\right\}^{\frac{1}{1}}}},\sqrt[4]{\frac{1}{1+\left\{\begin{array}{c}0.15\left(\frac{1-0.54^4}{0.54^4}\right)^1+0.25\left(\frac{1-0.43^4}{0.43^4}\right)^1+\\0.41\left(\frac{1-0.49^4}{0.49^4}\right)^1+0.19\left(\frac{1-0.65^4}{0.65^4}\right)^1\end{array}\right\}^{\frac{1}{1}}}}\right],\\ \left[\sqrt[4]{\frac{1}{1+\left\{\begin{array}{c}0.15\left(\frac{1-0.26^4}{0.26^4}\right)^1+0.25\left(\frac{1-0.39^4}{0.39^4}\right)^1+\\0.41\left(\frac{1-0.59^4}{0.59^4}\right)^1+0.19\left(\frac{1-0.64^4}{0.64^4}\right)^1\end{array}\right\}^{\frac{1}{1}}}},\sqrt[4]{\frac{1}{1+\left\{\begin{array}{c}0.15\left(\frac{1-0.41^4}{0.41^4}\right)^1+0.25\left(\frac{1-0.47^4}{0.47^4}\right)^1+\\0.41\left(\frac{1-0.68^4}{0.68^4}\right)^2+0.19\left(\frac{1-0.65^4}{0.65^4}\right)^2\end{array}\right\}^{\frac{1}{2}}}}\right]\end{array}\right)$$

$$IVSFDWA(\aleph_1,\aleph_2,\aleph_3,\aleph_4)=\left(\begin{array}{c}[0.705,0.84],\\ [0.3412,0.4907],\\ [0.38,0.523]\end{array}\right)$$

The next three theorems give us some properties, including Idempotency, bounded, and monotonicity of the IVSFDWA operator as follows:

Theorem 3: If $\aleph_{\mathfrak{z}}=\left([\dot{s}^l_{\mathfrak{z}},\dot{s}^u_{\mathfrak{z}}],[\mathring{\imath}^l_{\mathfrak{z}},\mathring{\imath}^u_{\mathfrak{z}}],[\d{d}^l_{\mathfrak{z}},\d{d}^u_{\mathfrak{z}}]\right)$ denote like IVSFVs. i.e., $\aleph_{\mathfrak{z}}=\aleph$ for all $\mathfrak{z}$, then $IVSFDWA(\aleph_1,\aleph_2,\aleph_3,\dots\aleph_n)=\aleph$

Proof: Since $\aleph_{\mathfrak{z}}=\left([\dot{s}^l_{\mathfrak{z}},\dot{s}^u_{\mathfrak{z}}],[\mathring{\imath}^l_{\mathfrak{z}},\mathring{\imath}^u_{\mathfrak{z}}],[\d{d}^l_{\mathfrak{z}},\d{d}^u_{\mathfrak{z}}]\right)=\aleph$

Then, we have

$$IVSFDWA(\aleph_1,\aleph_2,\aleph_3\ldots\aleph_n,)=\left(\begin{array}{c}\left[\sqrt{1-\frac{1}{1+\left\{\sum_{\text{ֆ}=1}^{n}\Phi_{\text{ֆ}}\left(\frac{\dot{s}{l_{\text{ֆ}}^{2}}}{1-\dot{s}{l_{\text{ֆ}}^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{\text{ֆ}=1}^{n}\Phi_{\text{ֆ}}\left(\frac{\dot{s}{u_{\text{ֆ}}^{2}}}{1-\dot{s}{u_{\text{ֆ}}^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\text{ֆ}=1}^{n}\Phi_{\text{ֆ}}\left(\frac{1-\mathring{I}{l_{\text{ֆ}}^{2}}}{\mathring{I}{l_{\text{ֆ}}^{q}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\sum_{\text{ֆ}=1}^{n}\Phi_{\text{ֆ}}\left(\frac{1-\mathring{I}{u_{\text{ֆ}}^{q}}}{\mathring{I}{u_{\text{ֆ}}^{q}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\text{ֆ}=1}^{n}\Phi_{\text{ֆ}}\left(\frac{1-\text{ḑ}{l_{\text{ֆ}}^{q}}}{\text{ḑ}{l_{\text{ֆ}}^{q}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\sum_{\text{ֆ}=1}^{n}\Phi_{\text{ֆ}}\left(\frac{1-\text{ḑ}{u_{\text{ֆ}}^{q}}}{\text{ḑ}{u_{\text{ֆ}}^{q}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right]\end{array}\right)$$

$$=\left(\begin{array}{c}\left[\sqrt{1-\frac{1}{1+\left\{\left(\frac{\dot{s}{l_{\text{ֆ}}^{2}}}{1-\dot{s}{l_{\text{ֆ}}^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{1-\frac{1}{1+\left\{\left(\frac{\dot{s}{u_{\text{ֆ}}^{2}}}{1-\dot{s}{u_{\text{ֆ}}^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\left(\frac{1-\mathring{I}{l_{\text{ֆ}}^{2}}}{\mathring{I}{l_{\text{ֆ}}^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\left(\frac{1-\mathring{I}{u_{\text{ֆ}}^{2}}}{\mathring{I}{u_{\text{ֆ}}^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{\frac{1}{1+\left\{\left(\frac{1-\text{ḑ}{l_{\text{ֆ}}^{2}}}{\text{ḑ}{l_{\text{ֆ}}^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\frac{1}{1+\left\{\left(\frac{1-\text{ḑ}{u_{\text{ֆ}}^{2}}}{\text{ḑ}{u_{\text{ֆ}}^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right]\end{array}\right)$$

$$=\left(\begin{array}{c}\left[\sqrt{1-\frac{1}{1+\frac{\dot{s}{l}^{2}}{1-\dot{s}{l}^{2}}}},\sqrt{1-\frac{1}{1+\frac{\dot{s}{u}^{2}}{1-\dot{s}{u}^{2}}}}\right],\left[\sqrt{\frac{1}{1+\frac{1-\mathring{I}{l}^{2}}{\mathring{I}{l}^{2}}}},\sqrt{\frac{1}{1+\frac{1-\mathring{I}{u}^{2}}{\mathring{I}{u}^{2}}}}\right],\\ \left[\sqrt{\frac{1}{1+\frac{1-\text{ḑ}{l}^{2}}{\text{ḑ}{l}^{2}}}},\sqrt{\frac{1}{1+\frac{1-\text{ḑ}{u}^{2}}{\text{ḑ}{u}^{2}}}}\right]\end{array}\right)$$

$$=\left([\dot{s}^{l},\dot{s}^{u}],[\mathring{I}^{l},\mathring{I}^{u}],[\text{ḑ}^{l},\text{ḑ}^{u}]\right)=\aleph$$

Thus'

$$IVSFDWA(\aleph_1,\aleph_2,\aleph_3,\dots n)=\aleph\,.$$

Theorem 4: Let $\aleph_{\mathfrak{s}}=\left(\left[\dot{s}^{l}_{\mathfrak{s}},\dot{s}^{u}_{\mathfrak{s}}\right],\left[\dot{I}^{l}_{\mathfrak{s}},\dot{I}^{u}_{\mathfrak{s}}\right],\left[d^{l}_{\mathfrak{s}},d^{u}_{\mathfrak{s}}\right]\right)$ denote few IVSFVs and $\aleph^{-}=min(\aleph_1,\aleph_2,\aleph_3,\dots\aleph_n)$ and $\aleph^{+}=max(\aleph_1,\aleph_2,\aleph_3,\dots\aleph_n)$.Then

$$\aleph^{-}\le IVSFDWA(\aleph_1,\aleph_2,\aleph_3,\dots\aleph_n)\le\aleph^{+}$$

Proof: Let $\aleph_{\mathfrak{s}}=\left(\left[\dot{s}^{l}_{\mathfrak{s}},\dot{s}^{u}_{\mathfrak{s}}\right],\left[\dot{I}^{l}_{\mathfrak{s}},\dot{I}^{u}_{\mathfrak{s}}\right],\left[d^{l}_{\mathfrak{s}},d^{u}_{\mathfrak{s}}\right]\right)$ $(\mathfrak{s}=1,2,3,\dots n)$ be IVSFVs and $\aleph^{-}=min(\aleph_1,\aleph_2,\aleph_3,\dots\aleph_n)=\left(\left[\dot{s}^{l\,-}_{\mathfrak{s}},\dot{s}^{u-}_{\mathfrak{s}}\right],\left[\dot{I}^{l\,-}_{\mathfrak{s}},\dot{I}^{u-}_{\mathfrak{s}}\right],\left[d^{l\,-}_{\mathfrak{s}},d^{u-}_{\mathfrak{s}}\right]\right)$ and $\aleph^{+}=max(\aleph_1,\aleph_2,\aleph_3,\dots\aleph_n)=\left(\left[\dot{s}^{l\,+}_{\mathfrak{s}},\dot{s}^{u+}_{\mathfrak{s}}\right],\left[\dot{I}^{l\,+}_{\mathfrak{s}},\dot{I}^{u+}_{\mathfrak{s}}\right],\left[d^{l\,+}_{\mathfrak{s}},d^{u+}_{\mathfrak{s}}\right]\right)$.

We have $\dot{s}^{-}=min\left\{\left[\dot{s}^{l}_{\mathfrak{s}},\dot{s}^{u}_{\mathfrak{s}}\right]\right\}$, $\dot{I}^{-}=max\left\{\left[\dot{I}^{l}_{\mathfrak{s}},\dot{I}^{u}_{\mathfrak{s}}\right]\right\}$, $d^{-}=max\left\{\left[d^{l}_{\mathfrak{s}},d^{u}_{\mathfrak{s}}\right]\right\}$, $\dot{s}^{+}=max\left\{\left[\dot{s}^{l}_{\mathfrak{s}},\dot{s}^{u}_{\mathfrak{s}}\right]\right\}$, $\dot{I}^{+}=min\left\{\left[\dot{I}^{l}_{\mathfrak{s}},\dot{I}^{u}_{\mathfrak{s}}\right]\right\}$, $d^{+}=min\left\{\left[d^{l}_{\mathfrak{s}},d^{u}_{\mathfrak{s}}\right]\right\}$.

Now consider,

$$\sqrt{1-\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{\dot{s}^{-l^{2}}}{1-\dot{s}^{-l^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\le\sqrt{1-\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{\dot{s}^{l^{2}}}{1-\dot{s}^{l^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\le\sqrt{1-\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{\dot{s}^{+l^{2}}}{1-\dot{s}^{+l^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}$$

$$\sqrt{1-\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{\dot{s}^{-u^{2}}}{1-\dot{s}^{-u^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\le\sqrt{1-\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{\dot{s}^{u^{2}}}{1-\dot{s}^{u^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}$$

$$\le\sqrt{1-\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{\dot{s}^{+u^{2}}}{1-\dot{s}^{+u^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}$$

$$\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{1-\dot{I}^{+l^{2}}}{\dot{I}^{+l^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\le\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{1-\dot{I}^{l^{2}}}{\dot{I}^{l^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\le\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{1-\dot{I}^{-l^{2}}}{\dot{I}^{+l^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}$$

$$\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{1-\dot{I}^{+u^{2}}}{\dot{I}^{+u^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\le\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{1-\dot{I}^{u^{2}}}{\dot{I}^{u^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\le\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{1-\dot{I}^{-u^{2}}}{\dot{I}^{+u^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}$$

$$\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{1-d^{+l^{2}}}{d^{+l^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\le\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{1-d^{l^{2}}}{d^{l^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\le\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{1-d^{-l^{2}}}{d^{-l^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}$$

$$\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{1-d^{+u^{2}}}{d^{+u^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\le\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{1-d^{u^{2}}}{d^{u^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\le\sqrt{\frac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\frac{1-d^{-u^{2}}}{d^{-u^{2}}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}$$

Therefore, $\aleph^{-} \leq IVSFDWA(\aleph_1, \aleph_2, \aleph_3, \dots \aleph_n) \leq \aleph^{+}$.

Theorem 5: For two sets of IVSFVs $\aleph_{\mathfrak{s}}$ and $\aleph'_{\mathfrak{s}}$ $\aleph_{\mathfrak{s}} \leq \aleph'_{\mathfrak{s}}\ \forall\ \mathfrak{s}$. We have

$$IVSFDWA = (\aleph_1, \aleph_2, \aleph_3, \dots \aleph_n) \leq\ IVSFDWA = (\aleph'_1, \aleph'_2, \aleph'_3, \dots \aleph'_n)$$

We introduce the IVSFDOWA operator in the definition, due to the importance of the ordered location of the uncertain information as given below.

Definition 9: Let $\aleph_{\mathfrak{s}} = \left(\left[\dot{\mathrm{s}}^{l}_{\mathfrak{s}}, \dot{\mathrm{s}}^{u}_{\mathfrak{s}}\right], \left[\dot{\mathrm{i}}^{l}_{\mathfrak{s}}, \dot{\mathrm{i}}^{u}_{\mathfrak{s}}\right], \left[\dot{\mathrm{d}}^{l}_{\mathfrak{s}}, \dot{\mathrm{d}}^{u}_{\mathfrak{s}}\right]\right)$ be few IVSFVs. Then IVSFDOWA operator is given as:

$$IVSFDOWA(\aleph_1, \aleph_2, \aleph_3, \dots \aleph_n) = \bigoplus_{\mathfrak{s}=1}^{n} \left(\Phi_{\mathfrak{s}} \aleph_{\sigma(\mathfrak{s})}\right)$$

Where $\sigma(\mathfrak{s})$ $(\mathfrak{s} = 1,2,3, \dots n)$ are permutations such that $\aleph_{\sigma(\mathfrak{s}=1)} \geq \aleph_{\sigma(\mathfrak{s})}$.

We have the following theorem using the four Dombi operations and the above definition.

Theorem 6: Let $\aleph_{\mathfrak{s}} = \left(\left[\dot{\mathrm{s}}^{l}_{\mathfrak{s}}, \dot{\mathrm{s}}^{u}_{\mathfrak{s}}\right], \left[\dot{\mathrm{i}}^{l}_{\mathfrak{s}}, \dot{\mathrm{i}}^{u}_{\mathfrak{s}}\right], \left[\dot{\mathrm{d}}^{l}_{\mathfrak{s}}, \dot{\mathrm{d}}^{u}_{\mathfrak{s}}\right]\right)$ denote some IVSFVs. Then using the IVSFDOWA operator, we get an IVSFV given as:

$$IVSFDOWA(\aleph_1, \aleph_2, \aleph_3, \dots \aleph_n) = \bigoplus_{\mathfrak{s}=1}^{n} \left(\Phi_{\mathfrak{s}} \aleph_{\sigma(\mathfrak{s})}\right)$$

$$= \left(\begin{array}{c} \left[\sqrt{1 - \dfrac{1}{1 + \left\{ \sum_{\mathfrak{s}=1}^{n} \Phi_{\mathfrak{s}} \left(\dfrac{\dot{\mathrm{s}}^{l\,2}_{\sigma(\mathfrak{s})}}{1 - \dot{\mathrm{s}}^{l\,2}_{\sigma(\mathfrak{s})}} \right)^{\mathcal{F}} \right\}^{\frac{1}{\mathcal{F}}}}}, \sqrt{1 - \dfrac{1}{1 + \left\{ \sum_{\mathfrak{s}=1}^{n} \Phi_{\mathfrak{s}} \left(\dfrac{\dot{\mathrm{s}}^{u\,2}_{\sigma(\mathfrak{s})}}{1 - \dot{\mathrm{s}}^{u\,2}_{\sigma(\mathfrak{s})}} \right)^{\mathcal{F}} \right\}^{\frac{1}{\mathcal{F}}}}} \right], \\ \left[\sqrt{\dfrac{1}{1 + \left\{ \sum_{\mathfrak{s}=1}^{n} \Phi_{\mathfrak{s}} \left(\dfrac{1 - \dot{\mathrm{i}}^{l\,2}_{\sigma(\mathfrak{s})}}{\dot{\mathrm{i}}^{l\,2}_{\sigma(\mathfrak{s})}} \right)^{\mathcal{F}} \right\}^{\frac{1}{\mathcal{F}}}}}, \sqrt{\dfrac{1}{1 + \left\{ \sum_{\mathfrak{s}=1}^{n} \Phi_{\mathfrak{s}} \left(\dfrac{1 - \dot{\mathrm{i}}^{u\,2}_{\sigma(\mathfrak{s})}}{\dot{\mathrm{i}}^{u\,2}_{\sigma(\mathfrak{s})}} \right)^{\mathcal{F}} \right\}^{\frac{1}{\mathcal{F}}}}} \right], \\ \left[\sqrt{\dfrac{1}{1 + \left\{ \sum_{\mathfrak{s}=1}^{n} \Psi_{\mathfrak{s}} \left(\dfrac{1 - \dot{\mathrm{d}}^{l\,2}_{\sigma(\mathfrak{s})}}{\dot{\mathrm{d}}^{l\,2}_{\sigma(\mathfrak{s})}} \right)^{\mathcal{F}} \right\}^{\frac{1}{\mathcal{F}}}}}, \sqrt{\dfrac{1}{1 + \left\{ \sum_{\mathfrak{s}=1}^{n} \Psi_{\mathfrak{s}} \left(\dfrac{1 - \dot{\mathrm{d}}^{u\,2}_{\sigma(\mathfrak{s})}}{\dot{\mathrm{d}}^{u\,2}_{\sigma(\mathfrak{s})}} \right)^{\mathcal{F}} \right\}^{\frac{1}{\mathcal{F}}}}} \right] \end{array} \right),$$

The accompanying example shows the applicability of the IVSFDOWA operator.

Example 3: Let $\aleph_1 = \begin{pmatrix}[0.65,0.83],\\ [0.4,0.54],\\ [0.26,0.41]\end{pmatrix}, \aleph_2 = \begin{pmatrix}[0.43,0.53],\\ [0.29,\ 0.43],\\ [0.39,0.47]\end{pmatrix}$ and $\aleph_3 = \begin{pmatrix}[0.77,0.89],\\ [0.34,0.49],\\ [0.59,0.68]\end{pmatrix}$ and $\aleph_4 = \begin{pmatrix}[0.71,0.81],\\ [0.56,0.65],\\ [0.64,0.65]\end{pmatrix}$ be four IVSFVs and also let $\text{₣} = 1$ and $\Phi = (0.15, 0.25, 0.41, 0.19)^T$ denote the aggregation-associated weight vector. First, we compute the scores using Definition 3 as follows:

$$\mathrm{S}(\aleph_1) = 0.254, \mathrm{S}(\aleph_2) = 0.224, \mathrm{S}(\aleph_3) = 0.121, \mathrm{S}(\aleph_4) = 0.192$$

Based on scores, we have $\aleph_{\sigma(1)} = \aleph_3, \aleph_{\sigma(2)} = \aleph_1, \aleph_{\sigma(3)} = \aleph_4, \aleph_{\sigma(4)} = \aleph_2$. So, using the IVSFDOWA operator, we have

$$IVSFDOWA(\aleph_3,\aleph_1,\aleph_4,\aleph_2) = \begin{pmatrix} \left[\sqrt[4]{1 - \frac{1}{1 + \left\{ 0.15\left(\frac{0.77^4}{1-0.77^4}\right)^1 + 0.25\left(\frac{0.65^4}{1-0.65^4}\right)^1 + 0.41\left(\frac{0.71^4}{1-0.71^4}\right)^1 + 0.19\left(\frac{0.43^4}{1-0.43^4}\right)^1 \right\}^{\frac{1}{1}}}}, \sqrt[4]{1 - \frac{1}{1 + \left\{ 0.15\left(\frac{0.89^4}{1-0.89^4}\right)^1 + 0.25\left(\frac{0.83^4}{1-0.83^4}\right)^1 + 0.41\left(\frac{0.81^4}{1-0.81^4}\right)^1 + 0.19\left(\frac{0.53^4}{1-0.53^4}\right)^1 \right\}^{\frac{1}{1}}}} \right], \\ \left[\sqrt[4]{\frac{1}{1 + \left\{ 0.15\left(\frac{1-0.34^4}{0.34^4}\right)^1 + 0.25\left(\frac{1-0.4^4}{0.4^4}\right)^1 + 0.41\left(\frac{1-0.56^4}{0.56^4}\right)^1 + 0.19\left(\frac{1-0.29^4}{0.29^4}\right)^1 \right\}^{\frac{1}{1}}}}, \sqrt[4]{\frac{1}{1 + \left\{ 0.15\left(\frac{1-0.49^4}{0.49^4}\right)^1 + 0.25\left(\frac{1-0.54^4}{0.54^4}\right)^1 + 0.41\left(\frac{1-0.65^4}{0.65^4}\right)^1 + 0.19\left(\frac{1-0.43^4}{0.43^4}\right)^1 \right\}^{\frac{1}{1}}}} \right], \\ \left[\sqrt[4]{\frac{1}{1 + \left\{ 0.15\left(\frac{1-0.59^4}{0.59^4}\right)^1 + 0.25\left(\frac{1-0.26^4}{0.26^4}\right)^1 + 0.41\left(\frac{1-0.64^4}{0.64^4}\right)^1 + 0.19\left(\frac{1-0.39^4}{0.39^4}\right)^1 \right\}^{\frac{1}{1}}}}, \sqrt[4]{\frac{1}{1 + \left\{ 0.15\left(\frac{1-0.68^4}{0.68^4}\right)^1 + 0.25\left(\frac{1-0.41^4}{0.41^4}\right)^1 + 0.41\left(\frac{1-0.65^4}{0.65^4}\right)^1 + 0.19\left(\frac{1-0.47^4}{0.47^4}\right)^1 \right\}^{\frac{1}{1}}}}, \right] \end{pmatrix}$$

$$IVSFDOWA(\aleph_3,\aleph_1,\aleph_4,\aleph_2) = \begin{pmatrix}[0.685,0.817],\\ [0.3724,0.5227],\\ [0.35,0.502]\end{pmatrix}$$

Theorem 7: If $\aleph_{\text{\&}} = \left([\dot{s}^l_{\text{\&}}, \dot{s}^u_{\text{\&}}], [\dot{I}^l_{\text{\&}}, \dot{I}^u_{\text{\&}}], [\dot{d}^l_{\text{\&}}, \dot{d}^u_{\text{\&}}]\right)$ denote like IVSFVs. i.e., $\aleph_{\text{\&}} = \aleph$ for all $\text{\&}$, then

$IVSFDOWA(\aleph_1, \aleph_2, \aleph_3, \ldots \aleph_n) = \aleph$

Theorem 8: Let $\aleph_{\text{\&}} = \left([\dot{s}^l_{\text{\&}}, \dot{s}^u_{\text{\&}}], [\dot{I}^l_{\text{\&}}, \dot{I}^u_{\text{\&}}], [\dot{d}^l_{\text{\&}}, \dot{d}^u_{\text{\&}}]\right)$ be a few IVSFVs. Let $\aleph^- = min(\aleph_1, \aleph_2, \aleph_3, \ldots \aleph_n)$ and $\aleph^+ = max(\aleph_1, \aleph_2, \aleph_3, \ldots \aleph_n)$. Then,

$$\aleph^- \leq IVSFDOWA(\aleph_1, \aleph_2, \aleph_3, \ldots \aleph_n) \leq \aleph^+.$$

Theorem 9: For two sets of IVSFVs $\aleph_{\text{\&}}$ and $\aleph'_{\text{\&}}$ $\aleph_{\text{\&}} \leq \aleph'_{\text{\&}}$ $\forall$ $\text{\&}$. We have

$$IVSFDOWA = (\aleph_1, \aleph_2, \aleph_3, \ldots \aleph_n) \leq IVSFDOWA(\aleph'_1, \aleph'_2, \aleph'_3, \ldots \aleph'_n)$$

In our next work, we aim to discuss the conception of the IVSFDHA operator and its several properties with examples.

Definition 10: Let $\aleph_{\text{\&}} = \left([\dot{s}^l_{\text{\&}}, \dot{s}^u_{\text{\&}}], [\dot{I}^l_{\text{\&}}, \dot{I}^u_{\text{\&}}], [\dot{d}^l_{\text{\&}}, \dot{d}^u_{\text{\&}}]\right)$ denote some IVSFVs. Then using the IVSFDHA operator, we get an IVSFV given as:

$$IVSFDHA(\aleph_1, \aleph_2, \ldots \aleph_n) = \bigoplus_{\text{\&}=1}^{n} \left(\Phi_{\text{\&}} \dot{\aleph}_{\sigma(\text{\&})}\right)$$

$$= \left(\begin{array}{c} \left[\sqrt{1 - \frac{1}{1 + \left\{ \sum_{\text{\&}=1}^{n} \Phi_{\text{\&}} \left(\frac{\dot{s}^{l\,2}_{\sigma(\text{\&})}}{1 - \dot{s}^{l\,2}_{\sigma(\text{\&})}} \right)^{₣} \right\}^{\frac{1}{₣}}}}, \sqrt{1 - \frac{1}{1 + \left\{ \sum_{\text{\&}=1}^{n} \Phi_{\text{\&}} \left(\frac{\dot{s}^{u\,2}_{\sigma(\text{\&})}}{1 - \dot{s}^{u\,2}_{\sigma(\text{\&})}} \right)^{₣} \right\}^{\frac{1}{₣}}}} \right], \\ \left[\sqrt{\frac{1}{1 + \left\{ \sum_{\text{\&}=1}^{n} \Phi_{\text{\&}} \left(\frac{1 - \dot{I}^{l\,2}_{\sigma(\text{\&})}}{\dot{I}^{l\,2}_{\sigma(\text{\&})}} \right)^{₣} \right\}^{\frac{1}{₣}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{\text{\&}=1}^{n} \Phi_{\text{\&}} \left(\frac{1 - \dot{I}^{u\,2}_{\sigma(\text{\&})}}{\dot{I}^{u\,2}_{\sigma(\text{\&})}} \right)^{₣} \right\}^{\frac{1}{₣}}}} \right], \\ \left[\sqrt{\frac{1}{1 + \left\{ \sum_{\text{\&}=1}^{n} \Phi_{\text{\&}} \left(\frac{1 - \dot{d}^{l\,2}_{\sigma(\text{\&})}}{\dot{d}^{l\,2}_{\sigma(\text{\&})}} \right)^{₣} \right\}^{\frac{1}{₣}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{\text{\&}=1}^{n} \Phi_{\text{\&}} \left(\frac{1 - \dot{d}^{u\,2}_{\sigma(\text{\&})}}{\dot{d}^{u\,2}_{\sigma(\text{\&})}} \right)^{₣} \right\}^{\frac{1}{₣}}}} \right] \end{array} \right)$$

Where $\dot{\aleph}_{\sigma(\text{\&})}$ the jth is the biggest weighted IVSF values $\dot{\aleph_{\text{\&}}}$ $\left(\dot{\aleph}_{\text{\&}} \dot{=} n\, \Psi_{\text{\&}} \aleph_{\text{\&}}\right)$, and $\Psi = (\Psi_1, \Psi_2, \ldots \Psi_n)^T$ is the weight vector of $\dot{\aleph_{\text{\&}}}$ with $\sum_{\text{\&}=1}^{n} \Psi_{\text{\&}} = 1$.

At the point when $\Psi = ({}^1/_n, {}^1/_n, \ldots, {}^1/_n)$, IVSFDWA and IVSFDOWA administrator becomes unique instances of the IVSFDHA operator.

The accompanying model delineates the pertinence of the IVSFDOWA operator.

Example: Let $\aleph_1 = \begin{pmatrix} [0.65,0.83], \\ [0.4,0.54], \\ [0.26,0.41] \end{pmatrix}, \aleph_2 = \begin{pmatrix} [0.43,0.53], \\ [0.29,\ 0.43], \\ [0.39,0.47] \end{pmatrix}$ and $\aleph_3 = \begin{pmatrix} [0.77,0.89], \\ [0.34,0.49], \\ [0.59,0.68] \end{pmatrix}$ and $\aleph_4 = \begin{pmatrix} [0.71,0.81], \\ [0.56,0.65], \\ [0.64,0.65] \end{pmatrix}$ be four IVSFVs and let $\text{Ғ} = 1$ and $\Phi = (0.15, 0.25, 0.41, 0.19)^T$ denote the aggregation associated weight vector and $\Psi_{\mathfrak{s}} = (0.1, 0.19, 0.41, 0.3)^T$. Then first we compute $\dot{\aleph}_{\mathfrak{s}} = n\,\Psi_{\mathfrak{s}}\aleph_{\mathfrak{s}}$ as follows:

$$\dot{\aleph}_1 = n\,\Psi_1\aleph_1 = \big((4)(0.1)([0.65,0.83],[0.4,0.54],[0.26,0.41])\big) = ([0.020,0.066],[0.015,0.047],[0.003,0.017])$$

$$\dot{\aleph}_2 = n\,\Psi_2\aleph_2 = \big((4)(0.19)([0.43,0.53],[0.29,0.43],[0.39,0.47])\big)$$
$$= ([0.007,0.015],[0.002,0.011],[0.008,0.016])$$

$$\dot{\aleph}_3 = n\,\Psi_3\aleph_3 = \big((4)(0.41)([0.77,0.89],[0.34,0.49],[0.59,0.68])\big)$$
$$= ([0.118,0.184],[0.002,0.009],[0.019,0.036])$$

$$\dot{\aleph}_4 = n\,\Psi_4\aleph_4 = \big((4)(0.3)([0.71,0.81],[0.56,0.65],[0.64,0.65])\big) = ([0.073,0.119],[0.021,0.038],[0.036,0.038])$$

$$\text{S}(\dot{\aleph}_1) = 0.0000065, \text{S}(\dot{\aleph}_2) = 0.000000019, \text{S}(\dot{\aleph}_3) = 0.00044, \text{S}(\dot{\aleph}_4) = 0.000076$$

Based on scores, we have $\dot{\aleph}_{\sigma(1)} = \dot{\aleph}_3$, $\dot{\aleph}_{\sigma(2)} = \dot{\aleph}_4$, $\dot{\aleph}_{\sigma(3)} = \dot{\aleph}_1$, $\dot{\aleph}_{\sigma(4)} = \dot{\aleph}_2$. So, using the IVSFDHA operator, we have:

$$IVSFDHA(\dot{\aleph}_3,\dot{\aleph}_4,\dot{\aleph}_1,\dot{\aleph}_2)=\left(\begin{matrix}\left[\begin{matrix}\sqrt[4]{1-\dfrac{1}{1+\left\{\begin{matrix}0.15\left(\frac{0.118^4}{1-0.118^4}\right)^2+0.25\left(\frac{0.073^4}{1-0.073^4}\right)^2+\\0.41\left(\frac{0.02^4}{1-0.02^4}\right)^2+0.19\left(\frac{0.007^4}{1-0.007^4}\right)^2\end{matrix}\right\}^{\frac{1}{2}}}},\\ \sqrt[4]{1-\dfrac{1}{1+\left\{\begin{matrix}0.15\left(\frac{0.184^4}{1-0.184^4}\right)^2+0.25\left(\frac{0.119^4}{1-0.119^4}\right)^2+\\0.41\left(\frac{0.066^4}{1-0.066^4}\right)^2+0.19\left(\frac{0.015^4}{1-0.015^4}\right)^2\end{matrix}\right\}^{\frac{1}{2}}}}\end{matrix}\right],\\ \left[\begin{matrix}\sqrt[4]{\dfrac{1}{1+\left\{\begin{matrix}0.15\left(\frac{1-0.002^4}{0.002^4}\right)^2+0.25\left(\frac{1-0.021^4}{0.021^4}\right)^2+\\0.41\left(\frac{1-0.015^4}{0.015^4}\right)^2+0.19\left(\frac{1-0.002^4}{0.002^4}\right)^2\end{matrix}\right\}^{\frac{1}{2}}}},\\ \sqrt[4]{\dfrac{1}{1+\left\{\begin{matrix}0.15\left(\frac{1-0.009^4}{0.009^4}\right)^2+0.25\left(\frac{1-0.038^4}{0.038^4}\right)^2+\\0.41\left(\frac{1-0.047^4}{0.047^4}\right)^2+0.19\left(\frac{1-0.011^4}{0.011^4}\right)^2\end{matrix}\right\}^{\frac{1}{2}}}}\end{matrix}\right],\\ \left[\begin{matrix}\sqrt[4]{\dfrac{1}{1+\left\{\begin{matrix}0.15\left(\frac{1-0.019^4}{0.019^4}\right)^2+0.25\left(\frac{1-0.036^4}{0.036^4}\right)^2+\\0.41\left(\frac{1-0.003^4}{0.003^4}\right)^2+0.19\left(\frac{1-0.008^4}{0.008^4}\right)^2\end{matrix}\right\}^{\frac{1}{2}}}},\\ \sqrt[4]{\dfrac{1}{1+\left\{\begin{matrix}0.15\left(\frac{1-0.036^4}{0.036^4}\right)^2+0.25\left(\frac{1-0.038^4}{0.038^4}\right)^2+\\0.41\left(\frac{1-0.017^4}{0.017^4}\right)^2+0.19\left(\frac{1-0.016^4}{0.016^4}\right)^2\end{matrix}\right\}^{\frac{1}{2}}}}\end{matrix}\right]\end{matrix}\right)$$

$$IVSFDHA(\dot{\aleph}_3,\dot{\aleph}_4,\dot{\aleph}_1,\dot{\aleph}_2)=\left(\begin{matrix}[0.078,0.123],\\ [0.0026,0.0129],\\ [0.004,0.019]\end{matrix}\right)$$

5 | INTERVAL-VALUED SPHERICAL FUZZY DOMBI GEOMETRIC AGGREGATION OPERATORS

In this section, we develop a series of robust aggregation methodologies based on IVSF information.

Definition 11: Let $\aleph_{\mathfrak{s}}=\left([\dot{s}^l_{\mathfrak{s}},\dot{s}^u_{\mathfrak{s}}],[\mathring{I}^l_{\mathfrak{s}},\mathring{I}^u_{\mathfrak{s}}],[d^l_{\mathfrak{s}},d^u_{\mathfrak{s}}]\right)$ be IVSFVs. Then IVSFDWG operator is given by:

$$IVSFDWG(\aleph_1,\aleph_2\ldots.\aleph_n)=\bigoplus_{\mathfrak{s}=1}^{n}(\aleph_{\mathfrak{s}})^{\Phi_{\mathfrak{s}}}$$

Theorem 10: Let $\aleph_{\mathfrak{s}}=\left([\dot{s}^l_{\mathfrak{s}},\dot{s}^u_{\mathfrak{s}}],[\mathring{I}^l_{\mathfrak{s}},\mathring{I}^u_{\mathfrak{s}}],[d^l_{\mathfrak{s}},d^u_{\mathfrak{s}}]\right)$ denote some IVSFVs. Then using the IVSFDWG operator, we get an IVSFV given as:

$$IVSFDWG(\aleph_1,\aleph_2,\aleph_3\ldots\aleph_n)=\left(\begin{array}{c}\left[\sqrt{\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{1-\dot{s}^{l\,2}_{\mathfrak{s}}}{\dot{s}^{l\,2}_{\mathfrak{s}}}\right)^{\mathcal{F}}\right\}^{\frac{1}{\mathcal{F}}}}},\sqrt{\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{1-\dot{s}^{u\,2}_{\mathfrak{s}}}{\dot{s}^{u\,2}_{\mathfrak{s}}}\right)^{\mathcal{F}}\right\}^{\frac{1}{\mathcal{F}}}}}\right],\\ \left[\sqrt{1-\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{\dot{I}^{l\,2}_{\mathfrak{s}}}{1-\dot{I}^{l\,2}_{\mathfrak{s}}}\right)^{\mathcal{F}}\right\}^{\frac{1}{\mathcal{F}}}}},\sqrt{1-\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{\dot{I}^{u\,2}_{\mathfrak{s}}}{1-\dot{I}^{u\,2}_{\mathfrak{s}}}\right)^{\mathcal{F}}\right\}^{\frac{1}{\mathcal{F}}}}}\right],\\ \left[\sqrt{1-\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{\underset{,}{d}^{l\,2}_{\mathfrak{s}}}{1-\underset{,}{d}^{l\,2}_{\mathfrak{s}}}\right)^{\mathcal{F}}\right\}^{\frac{1}{\mathcal{F}}}}},\sqrt{1-\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{\underset{,}{d}^{u\,2}_{\mathfrak{s}}}{1-\underset{,}{d}^{u\,2}_{\mathfrak{s}}}\right)^{\mathcal{F}}\right\}^{\frac{1}{\mathcal{F}}}}}\right]\end{array}\right)$$

Proof: Straightforward.

Theorem 11: If $\aleph_{\mathfrak{s}}=\left(\left[\dot{s}^{l}_{\mathfrak{s}},\dot{s}^{u}_{\mathfrak{s}}\right],\left[\dot{I}^{l}_{\mathfrak{s}},\dot{I}^{u}_{\mathfrak{s}}\right],\left[\underset{,}{d}^{l}_{\mathfrak{s}},\underset{,}{d}^{u}_{\mathfrak{s}}\right]\right)$ denote like IVSFVs. i.e., $\aleph_{\mathfrak{s}}=\aleph$ for all $\mathfrak{s}$, then $IVSFDWG(\aleph_1,\aleph_2,\aleph_3,\ldots\aleph_n)=\aleph$

Theorem 12: Let $\aleph_{\mathfrak{s}}=\left(\left[\dot{s}^{l}_{\mathfrak{s}},\dot{s}^{u}_{\mathfrak{s}}\right],\left[\dot{I}^{l}_{\mathfrak{s}},\dot{I}^{u}_{\mathfrak{s}}\right],\left[\underset{,}{d}^{l}_{\mathfrak{s}},\underset{,}{d}^{u}_{\mathfrak{s}}\right]\right)$ denote few IVSFVs and $\aleph^{-}=min(\aleph_1,\aleph_2,\aleph_3,\ldots\aleph_n)$ and $\aleph^{+}=max(\aleph_1,\aleph_2,\aleph_3,\ldots\aleph_n)$.Then

$$\aleph^{-}\leq IVSFDWG(\aleph_1,\aleph_2,\aleph_3,\ldots\aleph_n)\leq\aleph^{+}$$

Theorem 13: For two sets of IVSFVs $\aleph_{\mathfrak{s}}$ and $\aleph'_{\mathfrak{s}}$ $\aleph_{\mathfrak{s}}\leq\aleph'_{\mathfrak{s}}$ $\forall$ $\mathfrak{s}$. We have

$$IVSFDWG(\aleph_1,\aleph_2,\aleph_3,\ldots\aleph_n)\leq\ IVSFDWG(\aleph'_1,\aleph'_2,\aleph'_3,\ldots\aleph'_n)$$

We introduce the IVSFDOWG operator in the definition due to the importance of the ordered location of the uncertain information given below.

Definition 12: Let $\aleph_{\mathfrak{s}}=\left(\left[\dot{s}^{l}_{\mathfrak{s}},\dot{s}^{u}_{\mathfrak{s}}\right],\left[\dot{I}^{l}_{\mathfrak{s}},\dot{I}^{u}_{\mathfrak{s}}\right],\left[\underset{,}{d}^{l}_{\mathfrak{s}},\underset{,}{d}^{u}_{\mathfrak{s}}\right]\right)$ be few IVSFVs. Then IVSFDOWG operator is given as:

$$IVSFDOWG(\aleph_1,\aleph_2,\aleph_3,\ldots\aleph_n)=\bigotimes_{\mathfrak{s}=1}^{n}\left(\aleph_{\sigma(\mathfrak{s})}\right)^{\Phi_{\mathfrak{s}}}$$

Where $\sigma(\mathfrak{s})$ ($\mathfrak{s}=1,2,3,\ldots n$) are permutations such that $\aleph_{\sigma(\mathfrak{s}=1)}\geq\aleph_{\sigma(\mathfrak{s})}$.

Using the four Dombi operations and the above definition, we have the following theorem.

Theorem 14: Let $\aleph_{\mathfrak{s}}=\left(\left[\dot{s}^{l}_{\mathfrak{s}},\dot{s}^{u}_{\mathfrak{s}}\right],\left[\dot{I}^{l}_{\mathfrak{s}},\dot{I}^{u}_{\mathfrak{s}}\right],\left[\underset{,}{d}^{l}_{\mathfrak{s}},\underset{,}{d}^{u}_{\mathfrak{s}}\right]\right)$ denote some IVSFVs. Then using the IVSFDOWG operator, we get an IVSFV given as:

$$IVSFDOWG(\aleph_1,\aleph_2,\aleph_3,\dots\aleph_n)=\bigotimes_{\mathfrak{s}=1}^{n}\left(\aleph_{\sigma(\mathfrak{s})}\right)^{\Phi_{\mathfrak{s}}}$$

$$=\left(\begin{array}{c}\left[\sqrt{\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{1-\dot{\mathrm{s}}{l}^{2}_{\sigma(\mathfrak{s})}}{\dot{\mathrm{s}}{l}^{2}_{\sigma(\mathfrak{s})}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{1-\dot{\mathrm{s}}{u}^{2}_{\sigma(\mathfrak{s})}}{\dot{\mathrm{s}}{u}^{2}_{\sigma(\mathfrak{s})}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{1-\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{\dot{\mathrm{I}}{l}^{2}_{\sigma(\mathfrak{s})}}{1-\dot{\mathrm{I}}{l}^{2}_{\sigma(\mathfrak{s})}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{1-\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{\dot{\mathrm{I}}{u}^{2}_{\sigma(\mathfrak{s})}}{1-\dot{\mathrm{I}}{u}^{2}_{\sigma(\mathfrak{s})}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right],\\ \left[\sqrt{1-\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{\dot{\mathrm{d}}{l}^{2}_{\sigma(\mathfrak{s})}}{1-\dot{\mathrm{d}}{l}^{2}_{\sigma(\mathfrak{s})}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}},\sqrt{1-\dfrac{1}{1+\left\{\sum_{\mathfrak{s}=1}^{n}\Phi_{\mathfrak{s}}\left(\dfrac{\dot{\mathrm{d}}{u}^{2}_{\sigma(\mathfrak{s})}}{1-\dot{\mathrm{d}}{u}^{2}_{\sigma(\mathfrak{s})}}\right)^{\text{₣}}\right\}^{\frac{1}{\text{₣}}}}}\right]\end{array}\right),$$

Theorem 15: If $\aleph_{\mathfrak{s}}=\left([\dot{\mathrm{s}}^{l}_{\mathfrak{s}},\dot{\mathrm{s}}^{u}_{\mathfrak{s}}],[\dot{\mathrm{I}}^{l}_{\mathfrak{s}},\dot{\mathrm{I}}^{u}_{\mathfrak{s}}],[\dot{\mathrm{d}}^{l}_{\mathfrak{s}},\dot{\mathrm{d}}^{u}_{\mathfrak{s}}]\right)$ denote like IVSFVs. i.e., $\aleph_{\mathfrak{s}}=\aleph$ for all $\mathfrak{s}$, then $IVSFDOWG(\aleph_1,\aleph_2,\aleph_3,\dots\aleph_n)=\aleph$

Theorem 16: Let $\aleph_{\mathfrak{s}}=\left([\dot{\mathrm{s}}^{l}_{\mathfrak{s}},\dot{\mathrm{s}}^{u}_{\mathfrak{s}}],[\dot{\mathrm{I}}^{l}_{\mathfrak{s}},\dot{\mathrm{I}}^{u}_{\mathfrak{s}}],[\dot{\mathrm{d}}^{l}_{\mathfrak{s}},\dot{\mathrm{d}}^{u}_{\mathfrak{s}}]\right)$ be a few IVSFVs. Let $\aleph^{-}=min(\aleph_1,\aleph_2,\aleph_3,\dots\aleph_n)$ and $\aleph^{+}=max(\aleph_1,\aleph_2,\aleph_3,\dots\aleph_n)$. Then

$$\aleph\leq IVSFDOWA(\aleph_1,\aleph_2,\aleph_3,\dots\aleph_n)\leq\aleph^{+}$$

Theorem 17: For two sets of IVSFVs $\aleph_{\mathfrak{s}}$ and $\aleph'_{\mathfrak{s}}$ such that $\aleph_{\mathfrak{s}}\leq\aleph'_{\mathfrak{s}}\ \forall\ \mathfrak{s}$. We have

$$IVSFDOWA=(\aleph_1,\aleph_2,\aleph_3,\dots\aleph_n)\leq\ IVSFDOWA(\aleph'_1,\aleph'_2,\aleph'_3,\dots\aleph'_n)$$

In our next work, we aim to discuss the conception of the IVSFDHG operator and its several properties with examples.

Definition 13: Let $\aleph_{\mathfrak{s}}=\left([\dot{\mathrm{s}}^{l}_{\mathfrak{s}},\dot{\mathrm{s}}^{u}_{\mathfrak{s}}],[\dot{\mathrm{I}}^{l}_{\mathfrak{s}},\dot{\mathrm{I}}^{u}_{\mathfrak{s}}],[\dot{\mathrm{d}}^{l}_{\mathfrak{s}},\dot{\mathrm{d}}^{u}_{\mathfrak{s}}]\right)$ denote some IVSFVs. Then using the IVSFDHG operator, we get an IVSFV given as:

$$IVSFDHG(\aleph_1, \aleph_2, \ldots \aleph_n) = \bigotimes_{\S=1}^{n} \left(\dot{\aleph}_{\sigma(\S)}\right)^{\Phi_{\S}}$$

$$= \left(\begin{array}{l} \left[\sqrt{\dfrac{1}{1+\left\{\sum_{\S=1}^{n} \Phi_{\S}\left(\dfrac{1-\dot{\text{ṣ}}^{l\,2}_{\sigma(\S)}}{\dot{\text{ṣ}}^{l\,2}_{\sigma(\S)}}\right)^{\text{Ғ}}\right\}^{\frac{1}{\text{Ғ}}}}}, \sqrt{\dfrac{1}{1+\left\{\sum_{\S=1}^{n} \Phi_{\S}\left(\dfrac{1-\dot{\text{ṣ}}^{u\,2}_{\sigma(\S)}}{\dot{\text{ṣ}}^{u\,2}_{\sigma(\S)}}\right)^{\text{Ғ}}\right\}^{\frac{1}{\text{Ғ}}}}} \right], \\ \left[\sqrt{1-\dfrac{1}{1+\left\{\sum_{\S=1}^{n} \Phi_{\S}\left(\dfrac{\dot{\text{ỉ}}^{l\,2}_{\sigma(\S)}}{1-\dot{\text{ỉ}}^{l\,2}_{\sigma(\S)}}\right)^{\text{Ғ}}\right\}^{\frac{1}{\text{Ғ}}}}}, \sqrt{1-\dfrac{1}{1+\left\{\sum_{\S=1}^{n} \Phi_{\S}\left(\dfrac{\dot{\text{ỉ}}^{u\,2}_{\sigma(\S)}}{1-\dot{\text{ỉ}}^{u\,2}_{\sigma(\S)}}\right)^{\text{Ғ}}\right\}^{\frac{1}{\text{Ғ}}}}} \right], \\ \left[\sqrt{1-\dfrac{1}{1+\left\{\sum_{\S=1}^{n} \Phi_{\S}\left(\dfrac{\dot{\text{ḑ}}^{l\,2}_{\sigma(\S)}}{1-\dot{\text{ḑ}}^{l\,2}_{\sigma(\S)}}\right)^{\text{Ғ}}\right\}^{\frac{1}{\text{Ғ}}}}}, \sqrt{1-\dfrac{1}{1+\left\{\sum_{\S=1}^{n} \Phi_{\S}\left(\dfrac{\dot{\text{ḑ}}^{u\,2}_{\sigma(\S)}}{1-\dot{\text{ḑ}}^{u\,2}_{\sigma(\S)}}\right)^{\text{Ғ}}\right\}^{\frac{1}{\text{Ғ}}}}} \right] \end{array} \right)$$

Where $\dot{\aleph}_{\sigma(\S)}$ the jth is the biggest weighted IVSF values $\dot{\aleph}_{\S}$ $(\dot{\aleph}_{\S} = n\, \Psi_{\S} \aleph_{\S})$, and $\Psi = (\Psi_1, \Psi_2, \ldots \Psi_n)^T$ is the weight vector of $\dot{\aleph}_{\S}$ with $\sum_{\S=1}^{n} \Psi_{\S} = 1$.

6 | A STUDY OF THE MADM PROBLEM BASED ON IVSF INFORMATION

In this section, the author proposed a robust technique for the decision-making problem by considering some appropriate characteristics. To serve this purpose, we assume a class of finite alternatives $\eth = (\eth_1, \eth_2, \ldots, \eth_n)$ and a set of attributes represented by $\text{ʄ} = (\text{ʄ}_1, \text{ʄ}_2, \ldots, \text{ʄ}_m)$ with some specific degree of criteria $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$ such that $\lambda_j > 0$ and $\sum_{j=1}^{m} \lambda_j = 1$. The initial evaluation of the shortlisted choices is represented in the form of IVSFVs. The author listed acquired IVSF information in the form of decision matrix $D = \left(T_{ij}\right)_{n \times m}$ whose components are trios having the values of three components as closed subintervals of [0,1]. Decision-maker observed acquired information by using steps of the following algorithm of the MADM problem.

6.1 | ALGORITHM

The goal of this part is to apply the DAOs of IVSFVs in MADM issues. We likewise investigated the impact of variation in the values of F on the outcomes.

Step 1: The decision maker gathers information about an object in the form of IVSF information and list them in the decision matrix.

Step 2: Change the standard decision matrix into a normalized decision matrix if more than one type of attribute is involved in the given information; otherwise, this step is unnecessary.

Step 3: Utilized the following approaches of the IVSFDWA and IVSFDWG operators.

$$IVSFDWA(\aleph_1,\aleph_2,\aleph_3 \ldots \aleph_n) = \left(\begin{array}{c} \left[\sqrt{1-\frac{1}{1+\left\{\sum_{\text{ঌ}=1}^{n}\Phi_{\text{ঌ}}\left(\frac{\dot{s}l_{\text{ঌ}}^{2}}{1-\dot{s}l_{\text{ঌ}}^{2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}, \sqrt{1-\frac{1}{1+\left\{\sum_{\text{ঌ}=1}^{n}\Phi_{\text{ঌ}}\left(\frac{\dot{s}u_{\text{ঌ}}^{2}}{1-\dot{s}u_{\text{ঌ}}^{2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right], \\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\text{ঌ}=1}^{n}\Phi_{\text{ঌ}}\left(\frac{1-\text{Ỉ}l_{\text{ঌ}}^{2}}{\text{Ỉ}l_{\text{ঌ}}^{2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}, \sqrt{\frac{1}{1+\left\{\sum_{\text{ঌ}=1}^{n}\Phi_{\text{ঌ}}\left(\frac{1-\text{Ỉ}u_{\text{ঌ}}^{2}}{\text{Ỉ}u_{\text{ঌ}}^{2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right], \\ \left[\sqrt{\frac{1}{1+\left\{\sum_{\text{ঌ}=1}^{n}\Phi_{\text{ঌ}}\left(\frac{1-\text{ḑ}l_{\text{ঌ}}^{2}}{\text{ḑ}l_{\text{ঌ}}^{2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}, \sqrt{\frac{1}{1+\left\{\sum_{\text{ঌ}=1}^{n}\Phi_{\text{ঌ}}\left(\frac{1-\text{ḑ}u_{\text{ঌ}}^{2}}{\text{ḑ}u_{\text{ঌ}}^{2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right] \end{array}\right)$$

$$IVSFDWG(\aleph_1,\aleph_2,\aleph_3 \ldots \aleph_n) = \left(\begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{\sum_{\text{ঌ}=1}^{n}\Phi_{\text{ঌ}}\left(\frac{1-\dot{s}l_{\text{ঌ}}^{2}}{\dot{s}l_{\text{ঌ}}^{2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}, \sqrt{\frac{1}{1+\left\{\sum_{\text{ঌ}=1}^{n}\Phi_{\text{ঌ}}\left(\frac{1-\dot{s}u_{\text{ঌ}}^{2}}{\dot{s}u_{\text{ঌ}}^{2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right], \\ \left[\sqrt{1-\frac{1}{1+\left\{\sum_{\text{ঌ}=1}^{n}\Phi_{\text{ঌ}}\left(\frac{\text{Ỉ}l_{\text{ঌ}}^{2}}{1-\text{Ỉ}l_{\text{ঌ}}^{2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}, \sqrt{1-\frac{1}{1+\left\{\sum_{\text{ঌ}=1}^{n}\Phi_{\text{ঌ}}\left(\frac{\text{Ỉ}u_{\text{ঌ}}^{2}}{1-\text{Ỉ}u_{\text{ঌ}}^{2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right], \\ \left[\sqrt{1-\frac{1}{1+\left\{\sum_{\text{ঌ}=1}^{n}\Phi_{\text{ঌ}}\left(\frac{\text{ḑ}l_{\text{ঌ}}^{2}}{1-\text{ḑ}l_{\text{ঌ}}^{2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}, \sqrt{1-\frac{1}{1+\left\{\sum_{\text{ঌ}=1}^{n}\Phi_{\text{ঌ}}\left(\frac{\text{ḑ}u_{\text{ঌ}}^{2}}{1-\text{ḑ}u_{\text{ঌ}}^{2}}\right)^{₣}\right\}^{\frac{1}{₣}}}}\right] \end{array}\right)$$

Step 4: Compute the score values of all individuals.

Step 5: To explore a suitable optimal option from available optimal options.

The practicality of the proposed algorithm is described below example.

6.2 | NUMERICAL EXAMPLE

By using developed approaches, we resolved an application of complex challenges under consideration of IVSF information. Considered a model dependent on innovation commercialization wherein the choice of different programming bundles is analyzed depending on the accumulation of data under vulnerability. We follow a similar issue yet with interval-valued SF data to get ideal outcomes. Assume five programming bundles need to evaluate under four characteristics by utilizing some IVSF information. The ascribes dependent on which the four programming bundles signified by $\mathcal{A}_{1\leq k\leq 4}$ will be analyzed to incorporate Innovation G_1: Innovation of the technology G_2: The possible potential in the market, G_3: Financial development and Human resources. G_4: Future impact. Here we take the weight of the attribute as $\Phi_{\delta} = (0.15, 0.35, 0.1, 0.4)^T$is adjusted is a weight vector of the characteristics under perception. After beginning screening, data about the bundles from the chiefs is accumulated in the choice lattice given in Step 1 below. Given information about the programming bundle listed in Table 1.

Step 1: Initial evaluation by the experts in the form of IVSFVs about the programming bundles.

Table 1. Decision Matrix based on IVSF information.

	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$
$\mathcal{A}_1$	$\begin{pmatrix}[0.11,0.17],\\ [0.1,0.21],\\ [0.3,0.4]\end{pmatrix}$	$\begin{pmatrix}[0.4,0.49],\\ [0.2,0.33],\\ [0.41,0.049]\end{pmatrix}$	$\begin{pmatrix}[0.6,0.71],\\ [0.1,0.15],\\ [0.3,0.39]\end{pmatrix}$	$\begin{pmatrix}[0.45,0.49],\\ [0.55,0.63],\\ [0.43,0.55]\end{pmatrix}$
$\mathcal{A}_2$	$\begin{pmatrix}[0.25,0.38],\\ [0.2,0.45],\\ [0.21,0.26]\end{pmatrix}$	$\begin{pmatrix}[0.42,0.59],\\ [0.39,0.44],\\ [0.44,0.57]\end{pmatrix}$	$\begin{pmatrix}[0.49,0.51],\\ [0.16,0.61],\\ [0.41,0.51]\end{pmatrix}$	$\begin{pmatrix}[0.31,0.41],\\ [0.55,0.63],\\ [0.64,0.65]\end{pmatrix}$
$\mathcal{A}_3$	$\begin{pmatrix}[0.21,0.41],\\ [0.54,0.61],\\ [0.31,0.41]\end{pmatrix}$	$\begin{pmatrix}[0.67,0.77],\\ [0.23,0.55],\\ [0.17,0.22]\end{pmatrix}$	$\begin{pmatrix}[0.33,0.67],\\ [0.09,0.27],\\ [0.44,0.66]\end{pmatrix}$	$\begin{pmatrix}[0.59,0.63],\\ [0.42,0.51],\\ [0.32,0.57]\end{pmatrix}$
$\mathcal{A}_4$	$\begin{pmatrix}[0.42,0.56],\\ [0.21,0.39],\\ [0.29,0.31]\end{pmatrix}$	$\begin{pmatrix}[0.38,0.46],\\ [0.39,0.44],\\ [0.46,0.61]\end{pmatrix}$	$\begin{pmatrix}[0.19,0.27],\\ [0.41,0.54],\\ [0.54,0.59]\end{pmatrix}$	$\begin{pmatrix}[0.22,0.45],\\ [0.43,0.52],\\ [0.41,0.69]\end{pmatrix}$
$\mathcal{A}_5$	$\begin{pmatrix}[0.27,0.32],\\ [0.41,0.45],\\ [0.21,0.35]\end{pmatrix}$	$\begin{pmatrix}[0.41,0.56],\\ [0.37,0.45],\\ [0.43,0.47]\end{pmatrix}$	$\begin{pmatrix}[0.15,0.18],\\ [0.29,0.33],\\ [0.37,0.41]\end{pmatrix}$	$\begin{pmatrix}[0.15,0.27],\\ [0.23,0.36],\\ [0.38,0.42]\end{pmatrix}$

Step 3: This progression includes the accumulation of the data offered by experts in Table 1. Both IVSFDWA and IVSFDWG operators are used to total the data, and the discoveries are given in Table 2:

Table 2. Aggregated data by using IVSFDWA and IVSFDWG operators.

	$IVSFDWA\ operator$	$IVSFDWG\ operator$
$\mathcal{A}_1$	$\begin{pmatrix}[0.5018,0.6084],\\ [0.1109,0.1882],\\ [0.3227,0.0786]\end{pmatrix}$	$\begin{pmatrix}[0.1423,0.2184],\\ [0.4038,0.4801],\\ [0.3621,0.4435]\end{pmatrix}$
$\mathcal{A}_2$	$\begin{pmatrix}[0.4096,0.4883],\\ [0.1949,0.4965],\\ [0.2652,0.3267]\end{pmatrix}$	$\begin{pmatrix}[0.2997,0.4269],\\ [0.4149,0.5664],\\ [0.5056,0.5462]\end{pmatrix}$
$\mathcal{A}_3$	$\begin{pmatrix}[0.5356,0.6634],\\ [0.1208,0.3491],\\ [0.2511,0.3289]\end{pmatrix}$	$\begin{pmatrix}[0.2615,0.4944],\\ [0.4561,0.5446],\\ [0.3662,0.5733]\end{pmatrix}$
$\mathcal{A}_4$	$\begin{pmatrix}[0.3567,0.4889],\\ [0.2633,0.4434],\\ [0.3516,0.3858]\end{pmatrix}$	$\begin{pmatrix}[0.2329,0.3442],\\ [0.3789,0.4885],\\ [0.4609,0.5972]\end{pmatrix}$
$\mathcal{A}_5$	$\begin{pmatrix}[0.2910,0.4026],\\ [0.2925,0.3760],\\ [0.2613,0.3888]\end{pmatrix}$	$\begin{pmatrix}[0.1749,0.2288],\\ [0.3566,0.4107],\\ [0.3584,0.4089]\end{pmatrix}$

The conglomeration data obtained in Step 3 are exposed to score values using Definition 4 for positioning reasons, and all results are drawn in Table 3.

Table 3. Scores values of all individuals.

Scores	$IVSFDWA\ operator$	$IVSFDWG\ operator$
$\mathcal{A}_1$	(0.3408)	(0.0234)
$\mathcal{A}_2$	(0.2010)	(0.0745)
$\mathcal{A}_3$	(0.3775)	(0.0755)
$\mathcal{A}_4$	(0.1549)	(0.0509)
$\mathcal{A}_5$	(0.1099)	(0.0344)

Step 5: In light of the scores obtained in Table 4, the options are positioned, and the positioning outcomes are given in Table 5 as follows.

Table 4. Ranking results of score values.

	$Ranking\ Analysis$
$IVSFPWA\ operator$	$\mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5$
$IVSFPWG\ operator$	$\mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_1$

Figure 1 also portrays the results of score values listed in Table 3, and the following figure shows the graphical behavior of the score values. $\mathcal{A}_3$ is an appropriate individual from all available alternative.

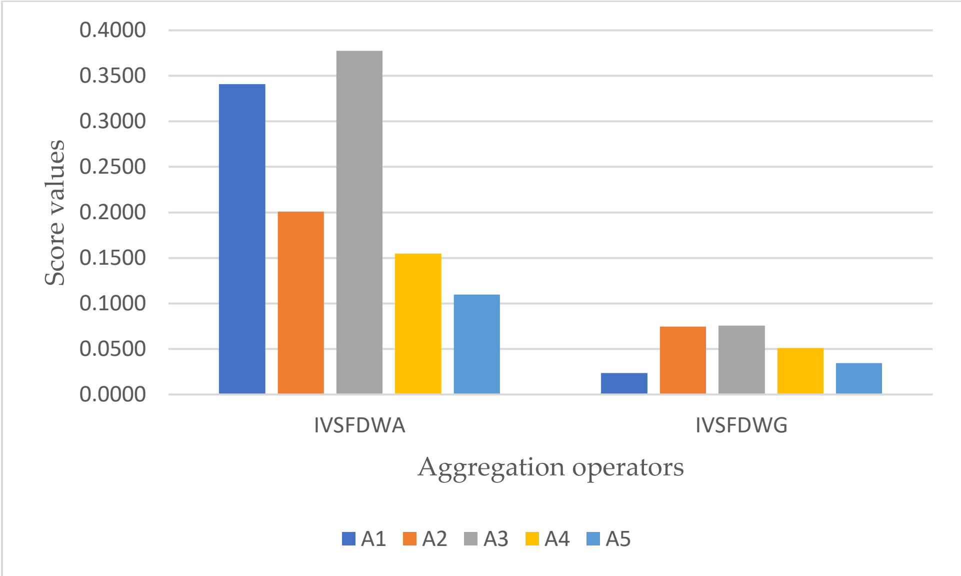

Figure 1. covered the ranking of score values obtained by the IVSFDWA and IVSFDWG operators.

6.3 | INFLUENCE STUDY

In this section, the author analysis impact of several parametric values of the Dombi aggregation tools on the outcomes of the IVSFDWA and IVSFDWG operators. By setting different parametric values of the Dombi aggregation tools in step 4, compute consequences from currently developed aggregation methodologies. All commuted results are listed in Table 5. From Table 5, we examined the ranking of score values that remain unchanged by setting different parametric values of the Dombi aggregation tools. This study shows that our proposed methodologies are superior to existing aggregation approaches.

Figure 2 and Figure 3 cover the results of score values obtained by the IVSFDWA and IVSFDWG operators and show the graphical behavior of estimated values.

Table 5. covered results of score values obtained by the IVSFDWA and IVSFDWG operators for different parametric values of the Dombi aggregation tools.

	Ş($\mathcal{A}_1$)	Ş($\mathcal{A}_2$)	Ş($\mathcal{A}_3$)	Ş($\mathcal{A}_4$)	Ş($\mathcal{A}_5$)	Ranking and ordering
₣ = 1	(0.2705)	(0.1708)	(0.2884)	(0.1275)	(0.0809)	$\mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5$
₣ = 3	(0.3784)	(0.2224)	(0.4309)	(0.1725)	(0.1340)	$\mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5$
₣ = 15	(0.4636)	(0.2858)	(0.5630)	(0.2171)	(0.2072)	$\mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5$
₣ = 25	(0.4733)	(0.2948)	(0.5801)	(0.2231)	(0.2169)	$\mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5$
₣ = 35	(0.4774)	(0.2988)	(0.5875)	(0.2257)	(0.2212)	$\mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5$
₣ = 40	(0.4787)	(0.3000)	(0.5898)	(0.2265)	(0.2226)	$\mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5$
₣ = 65	(0.4822)	(0.3034)	(0.5960)	(0.2288)	(0.2262)	$\mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5$
₣ = 85	(0.4835)	(0.3048)	(0.5983)	(0.2296)	(0.2276)	$\mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5$
₣ = 100	(0.4842)	(0.3054)	(0.5995)	(0.2300)	(0.2283)	$\mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5$

	Ş($\mathcal{A}_1$)	Ş($\mathcal{A}_2$)	Ş($\mathcal{A}_3$)	Ş($\mathcal{A}_4$)	Ş($\mathcal{A}_5$)	Ranking and ordering
₣ = 1	(0.0408)	(0.0976)	(0.1037)	(0.0657)	(0.0416)	$\mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_1$

$₣ = 3$	(0.0181)	(0.0614)	(0.0625)	(0.0434)	(0.0306)	$\mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_1$
$₣ = 15$	(0.0107)	(0.0340)	(0.0391)	(0.0287)	(0.0229)	$\mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_1$
$₣ = 25$	(0.0101)	(0.0311)	(0.0368)	(0.0271)	(0.0221)	$\mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_1$
$₣ = 35$	(0.0098)	(0.0298)	(0.0358)	(0.0263)	(0.0217)	$\mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_1$
$₣ = 40$	(0.0097)	(0.0294)	(0.0355)	(0.0261)	(0.0216)	$\mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_1$
$₣ = 65$	(0.0095)	(0.0284)	(0.0347)	(0.0255)	(0.0213)	$\mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_1$
$₣ = 85$	(0.0094)	(0.0280)	(0.0344)	(0.0253)	(0.0212)	$\mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_1$
$₣ = 100$	(0.0093)	(0.0278)	(0.0343)	(0.0252)	(0.0211)	$\mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_1$

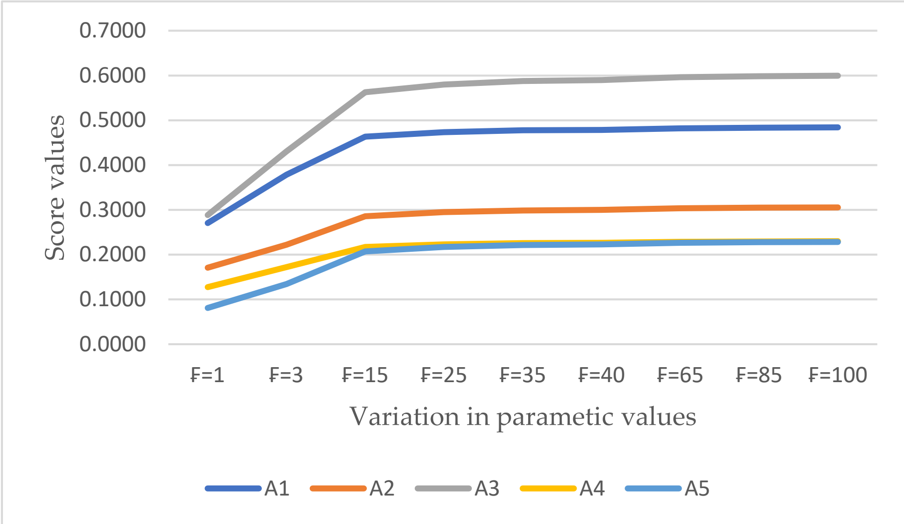

Figure 2. covers the ranking of score values obtained by the IVSFDWA operator.

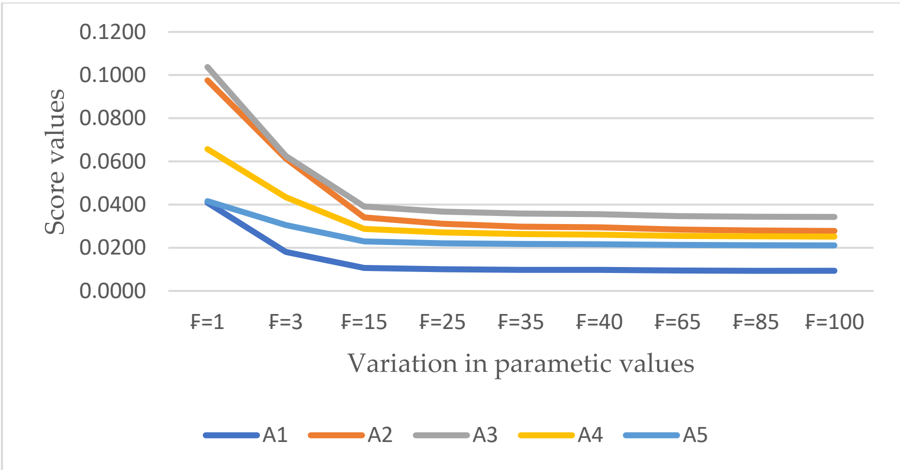

Figure 3. covers the ranking of score values obtained by the IVSFDWG operator.

7 | CONCLUSION

In this article, we expose the theory of IVSF information, which is the modified version of picture fuzzy sets and spherical fuzzy sets. The IVSFS has maintained vague and imprecision information on human opinion. The objectives of this article are illustrated as follows:

1. Some appropriate operations of Dombi aggregation tools under the system of IVSF information.
2. We developed a class of new approaches based on IVSF information, including IVSFDWA and IVSFDWG operators.
3. We discussed some notable features of proposed aggregation operators to express the effectiveness of proposed approaches.
4. To resolve complicated real-life situations, an algorithm for the MADM problem is established under our proposed methodologies.
5. Using our invented approaches, a numerical example gave to choose an appropriate programming bundle.
6. To reveal the applicability and effectiveness of proposed methodologies, examined the impact of different parametric values of the Dombi aggregation tools on the presented approaches' results.

In the future, we try to resolve complicated applications like renewable energy, waste management enterprises, selection of raw materials, game theory, and computational and environmental science. We will extend our invented research work in different fuzzy circumstances like t-spherical fuzzy sets [32], complex spherical fuzzy sets [33], similarity measures [34], and Linear diophantine fuzzy environments [35].

Acknowledgment:We are thankful to the editors and anonymous reviewers for their constructive suggestions. All authors contributed equally to this manuscript.

Declaration of Conflicting Interest: The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding: The author(s) received no financial support for the research, authorship, and/or publication of this article.

REFERENCES

[1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965, doi: 10.1016/S0019-9958(65)90241-X.
[2] K. T. Atanasov, "Intuitionistic fuzzy sets Fuzzy sets and systems," 1986.
[3] R. R. Yager, "Pythagorean fuzzy subsets," in *2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*, Jun. 2013, pp. 57–61. doi: 10.1109/IFSA-NAFIPS.2013.6608375.
[4] R. R. Yager, "Generalized Orthopair Fuzzy Sets," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 5, pp. 1222–1230, Oct. 2017, doi: 10.1109/TFUZZ.2016.2604005.
[5] B. C. Cuong, "Picture fuzzy sets-first results. part 1, seminar neuro-fuzzy systems with applications," *Institute of Mathematics, Hanoi*, 2013.

[6] T. Mahmood, K. Ullah, Q. Khan, and N. Jan, "An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets," *Neural Comput & Applic*, vol. 31, no. 11, pp. 7041–7053, Nov. 2019, doi: 10.1007/s00521-018-3521-2.

[7] C. Tian, J. Peng, S. Zhang, W. Zhang, and J. Wang, "Weighted picture fuzzy aggregation operators and their applications to multi-criteria decision-making problems," *Computers & Industrial Engineering*, vol. 137, p. 106037, 2019.

[8] C. Wang, X. Zhou, H. Tu, and S. Tao, "Some geometric aggregation operators based on picture fuzzy sets and their application in multiple attribute decision making," *Ital. J. Pure Appl. Math*, vol. 37, pp. 477–492, 2017.

[9] M. Qiyas, S. Abdullah, S. Ashraf, and M. Aslam, "Utilizing linguistic picture fuzzy aggregation operators for multiple-attribute decision-making problems," *International Journal of Fuzzy Systems*, vol. 22, pp. 310–320, 2020.

[10] J. Dombi, "A general class of fuzzy operators, the DeMorgan class of fuzzy operators and fuzziness measures induced by fuzzy operators," *Fuzzy sets and systems*, vol. 8, no. 2, pp. 149–163, 1982.

[11] M. Akram, W. A. Dudek, and J. M. Dar, "Pythagorean Dombi fuzzy aggregation operators with application in multicriteria decision-making," *International Journal of Intelligent Systems*, vol. 34, no. 11, pp. 3000–3019, 2019.

[12] P. Liu, J. Liu, and S.-M. Chen, "Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making," *Journal of the Operational Research Society*, pp. 1–26, 2017.

[13] A. Hussain, A. Alsanad, K. Ullah, Z. Ali, M. K. Jamil, and M. A. Mosleh, "Investigating the Short-Circuit Problem Using the Planarity Index of Complex q-Rung Orthopair Fuzzy Planar Graphs," *Complexity*, vol. 2021, 2021.

[14] C. Jana, G. Muhiuddin, and M. Pal, "Some Dombi aggregation of Q-rung orthopair fuzzy numbers in multiple-attribute decision making," *International Journal of Intelligent Systems*, vol. 34, no. 12, pp. 3220–3240, 2019.

[15] J.-W. Wang, "Multi-criteria decision-making method based on a weighted 2-tuple fuzzy linguistic representation model," *International Journal of Information Technology & Decision Making*, vol. 20, no. 02, pp. 619–634, 2021.

[16] Y. Liu and W. Jiang, "A new distance measure of interval-valued intuitionistic fuzzy sets and its application in decision making," *Soft Comput*, vol. 24, no. 9, pp. 6987–7003, May 2020, doi: 10.1007/s00500-019-04332-5.

[17] P. Liu, P. Wang, and J. Liu, "Normal neutrosophic frank aggregation operators and their application in multi-attribute group decision making," *International Journal of Machine Learning and Cybernetics*, vol. 10, pp. 833–852, 2019.

[18] T. Senapati and R. R. Yager, "Some New Operations Over Fermatean Fuzzy Numbers and Application of Fermatean Fuzzy WPM in Multiple Criteria Decision Making," *Informatica*, vol. 30, no. 2, pp. 391–412, Jan. 2019.

[19] T. Senapati and G. Chen, "Some novel interval-valued Pythagorean fuzzy aggregation operator based on Hamacher triangular norms and their application in MADM issues," *Computational and Applied Mathematics*, vol. 40, no. 4, pp. 1–27, 2021.

[20] C. Jana, T. Senapati, M. Pal, and R. R. Yager, "Picture fuzzy Dombi aggregation operators: application to MADM process," *Applied Soft Computing*, vol. 74, pp. 99–109, 2019.

[21] M. S. Sindhu, T. Rashid, and A. Kashif, "An approach to select the investment based on bipolar picture fuzzy sets," *International Journal of Fuzzy Systems*, vol. 23, no. 7, pp. 2335–2347, 2021.

[22] L. Shi and J. Ye, "Dombi aggregation operators of neutrosophic cubic sets for multiple attribute decision-making," *Algorithms*, vol. 11, no. 3, p. 29, 2018.

[23] Y. He, Z. He, and H. Chen, "Intuitionistic fuzzy interaction Bonferroni means and its application to multiple attribute decision making," *IEEE transactions on cybernetics*, vol. 45, no. 1, pp. 116–128, 2014.

[24] L.-P. Zhou, J.-Y. Dong, and S.-P. Wan, "Two new approaches for multi-attribute group decision-making with interval-valued neutrosophic Frank aggregation operators and incomplete weights," *IEEE Access*, vol. 7, pp. 102727–102750, 2019.

[25] C. Liu, G. Tang, P. Liu, and C. Liu, "Hesitant fuzzy linguistic archimedean aggregation operators in decision making with the dempster–shafer belief structure," *International Journal of Fuzzy Systems*, vol. 21, pp. 1330–1348, 2019.

[26] D. Nagarajan, J. Kavikumar, M. Lathamaheswari, and N. Kumaresan, "Fuzzy optimization techniques by hidden Markov model with interval type-2 fuzzy parameters," *International Journal of Fuzzy Systems*, vol. 22, pp. 62–76, 2020.

[27] M. Riaz, H. M. A. Farid, M. Aslam, D. Pamucar, and D. Bozanić, "Novel Approach for Third-Party Reverse Logistic Provider Selection Process under Linear Diophantine Fuzzy Prioritized Aggregation Operators," *Symmetry*, vol. 13, no. 7, Art. no. 7, Jul. 2021, doi: 10.3390/sym13071152.

[28] Z. Yang and J. Chang, "Interval-valued Pythagorean normal fuzzy information aggregation operators for multi-attribute decision making," *IEEE Access*, vol. 8, pp. 51295–51314, 2020.

[29] P. Liu and H. Gao, "Multicriteria Decision Making Based on Generalized Maclaurin Symmetric Means with Multi-Hesitant Fuzzy Linguistic Information," *Symmetry*, vol. 10, no. 4, Art. no. 4, Apr. 2018, doi: 10.3390/sym10040081.

[30] B. Cuong, "Picture fuzzy sets-first results. Part 1, in: seminar," *Neuro-Fuzzy Systems with Applications*, 2013.
[31] P. Liu, J. Liu, and S.-M. Chen, "Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making," *J Oper Res Soc*, Mar. 2017, doi: 10.1057/s41274-017-0190-y.
[32] M. Munir, H. Kalsoom, K. Ullah, T. Mahmood, and Y.-M. Chu, "T-spherical fuzzy Einstein hybrid aggregation operators and their applications in multi-attribute decision making problems," *Symmetry*, vol. 12, no. 3, p. 365, 2020.
[33] Z. Ali, T. Mahmood, and M.-S. Yang, "TOPSIS Method Based on Complex Spherical Fuzzy Sets with Bonferroni Mean Operators," *Mathematics*, vol. 8, no. 10, Art. no. 10, Oct. 2020, doi: 10.3390/math8101739.
[34] W. Yi, L. Sanyang, N. Wei, L. Kai, and L. Yong, "Threat assessment method based on intuitionistic fuzzy similarity measurement reasoning with orientation," *China Communications*, vol. 11, no. 6, pp. 119–128, Jun. 2014, doi: 10.1109/CC.2014.6879010.
[35] M. Riaz and M. R. Hashmi, "Linear Diophantine fuzzy set and its applications towards multi-attribute decision-making problems," *Journal of Intelligent & Fuzzy Systems*, vol. 37, no. 4, pp. 5417–5439, 2019.