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A Group Decision-making Algorithm to Analyses Risk Evaluation of Hepatitis with Sine Trigonometric Laws under Bipolar Complex Fuzzy Sets Information.

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ABSTRACT

The risk evaluation of Hepatitis can be regarded as a decision-making problem since it requires the analysis of some factors to identify the probability of the development or the progression of the disease in the patient. This process involves the use of medical information including the patient's medical history, family history, and the results of the tests among other factors alongside clinical knowledge and the available treatments. These factors are rather intricate and coupled with the uncertainty of the disease and treatment efficacy, the evaluation becomes a multi-criteria decision-making problem.

In this article, we develop a decision-making technique for risk evaluation of Hepatitis and solve other problems, where the criteria or attribute contain bipolarity and 2nd dimension. For this methodology, we first devise a sine trigonometric bipolar complex fuzzy set along with sine trigonometric operational laws. Afterward, we devise sine trigonometric bipolar complex fuzzy weighted averaging and geometric operators to aggregate the information.

To reveal the practical use of the devised approach, we discuss a numerical example related to hepatitis. Moreover, in this article, for displaying the dominance and benefits of the characterized approach we do the comparison of the inaugurated approach with certain prevailing ones.

Keywords: Hepatitis, aggregation operators, bipolar complex fuzzy sets, decision-making algorithm, fuzzy sets

(MSC2020) Codes: 03B52, 03E72, 28E10

1 | INTRODUCTION

The word hepatitis represents the liver's inflammation situation, which is usually because of viral infections, but also other reasons that can cause hepatitis. These involve hepatitis which happens as a subordinate outcome of the drugs, alcohol, medications, etc., and autoimmune hepatitis which happens when the body creates antibodies against the tissues of the liver. There are five kinds of hepatitis A, hepatitis B, hepatitis C, hepatitis D, and hepatitis E which are caused by numerous sorts of viruses.

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In recent words, 345 million people are suffering from hepatitis B and C according to the World Health Organization. In hepatitis B and C, a patient can't see any sort of symptoms till hepatitis B and C disturb the function of the liver. On the other hand, patients with intense hepatitis will see the symptoms soon after catching the hepatitis virus. The usual symptoms of hepatitis are dark urine, fatigue, loss of appetite, yellow skin and eyes, flu-like symptoms, abdominal pain, weight loss, and pale stool. For treating hepatitis, it is significant to find what is the cause of hepatitis, for this doctor will do a series of tests such as liver function tests, blood tests, liver biopsy, ultrasound, etc., check the medical history, and physical examination. Risk assessment of Hepatitis is always probabilistic because Hepatitis is a chronic disease with a highly unpredictable course, individual patient response to treatment, and the nature of medical data. Fuzzy DM is vital in this regard since it enables dealing with uncertainty and imprecision, which are not addressed well by conventional DM techniques. Fuzzy theory helps to model the medical inputs that are inherently vague such as symptoms, or slightly abnormal test results, as fuzzy sets (FSs) [1], which in turn makes the risk assessment more flexible. Before 1965, the researchers and scholars merely considered the crisp theory (CT) for decision-making (DM) and modeling information, and in the CT the scholars had only two options i.e., 1 means yes and 0 means no but there were certain situations where the scholars need more options for modeling a real-life problem than 1 and 0. Therefore, 1965 a novel theory was developed and interpreted as FSs [1]. FSs provide more options for the decision analysts or experts to present their opinions in the shape of truth grade (TG) belonging to $[0, 1]$.

Because of the rich structure, FSs are generalized by various scholars to make the theory richer but before 1994, the researchers, scholars, and decision analysts considered the FSs, and their few generalizations for DM and modeling information but these theories had their limitations and drawbacks. The biggest problem was that these theories couldn't cope with negative opinions or aspects of the decision-analysts. Therefore, a theory was developed in 1994 and called bipolar FSs (BFSs) [2]. BFSs provide the range for both opinions i.e., positive opinion in the shape of positive TG (PTG) belongs to $[0, 1]$ and negative opinion in the shape of negative TG (NTG) belongs to $[-1, 0]$. Bipolar fuzzy (BF) matrices diagnosed were in [13]. The structures of the above theories model the awkward information containing positive and negative aspects but still, these theories can't model or cope with the information which contains the 2nd dimension. Therefore, complex FSs (CFSs) developed in [4]. CFSs provide a range to the decision analysts or experts to present their opinions in two dimensions which in the shape of TG belongs to a unit disc in a complex plane. Cartesian structure of CFSs proposed in [5]. Before 2022, the structures of the above-mentioned theories and other prevailing theories couldn't model or cope with awkward information containing positive and negative aspects in two dimensions. Therefore, a theory for bipolar CFSs (BCFSs) discussed in [6]. BCFSs provide the range for both opinions i.e., positive opinion in the shape of PTG belongs to $[0,1]+\iota\,[0,1]$ and negative opinion in the shape of NTG belongs to $[-1,0]+\iota\,[-1,0]$.

2 | LITERATURE REVIEW

Since the emergence of the FS theory, the field has evolved and has been applied extensively. A brief description of the early uses of the theory in different fields which shows the applicability of the theory in [7]. A detailed analysis of the principles of FSs and their uses discussed in [8]. FS theory was employed to show its applicability in the industrial environment [9]. A new operation was added to the FS arithmetic by presenting the concept of symmetric summation [10]. An axiomatic approach to operations on FSs was offered, contributing to the theoretical advancement of the field [11]. The concept of FSs and their operations was extended to provide more information on fuzzy logic manipulations [12]. The use of fuzzy relations in control systems was examined, focusing on the real-life application of the theory [13]. An interactive fuzzy linguistic term set and its use in MADM were recently presented, demonstrating that the development of FS theory continues [3]. An example of using fuzzy algorithms for the assessment of the bond strength of fiber-reinforced cementitious matrix to concrete was provided, illustrating how the theory can be applied in engineering practice [14]. The theory of FS was used in the context of pharmacological therapy for type 2 diabetes, showing its utility in the medical decision-making process [15].

Bipolar fuzzy (BF) matrices were introduced, expanding the theoretical framework of BFSs [16]. In the realm of DM, the VIKOR method was extended to BF environments [17], while further enhancements were made using connection numbers [18]. BF TOPSIS and ELECTRE-I methods were applied to diagnosis [19], and ELECTRE II was explored under a BF model [20]. Equivalent BF relations were investigated, contributing to the theoretical foundations [21]. Distance measures and similarity for BFSs were explored [22], [23]. BF concepts were applied to linear systems of equations [24]. In graph theory, notes on BF graphs were provided [25], while antipodal BF graphs were introduced [26]. Applications of BFSs in planar graphs were explored [27], and BF circuits were examined [28]. Various BF aggregation operators (AOs) for MADM were developed [29], [30], [31]. BF soft mappings were applied to bipolar disorders, demonstrating the concept's relevance in medical contexts [32]. Shifting focus to CFS, operational properties and equalities were investigated [33]. Distances of CFSs and the continuity of complex fuzzy (CF) operations were explored [34], while the orthogonality between CFSs and their application to signal detection were examined [35]. CF arithmetic and geometric AOs were developed [36],[37]. A fuzzy center-based solution for fuzzy complex linear systems of equations was proposed [38]. Bipolar complex fuzzy (BCF) Hamacher AOs for MADM were introduced [39]. A method for MADM based on Dombi AOs under BCF information was proposed [40]. AOs were identified and classified using BCF settings [41]. BCF soft sets and their applications in DM were explored [42].

2.1 | Motivation and Contribution

The risk assessment of Hepatitis has to be done in a very complicated way because it is not clear whether a person has Hepatitis or how the disease will progress. BCFS offers a powerful framework for this task by addressing two critical aspects: bipolarity and 2^{nd} dimension. The bipolarity allows for the representation of the positive and negative impact of the various attributes, which is a true reflection of the picture of diseases where attributes may have both positive and negative aspects. The complex component which adds the second dimension permits to gathering of additional information including the level of confidence, the time-related aspects of the disease, or the severity of the symptoms. This kind of approach is particularly appropriate in Hepatitis risk assessment where the interactions of the factors involved are complex and cannot be explained by simple binary or one-dimensional fuzzy logic. Therefore, the BCFS with the second dimension can be more useful in capturing the nature of Hepatitis and can help to make better decisions in diagnosis and treatment.

Moreover, operational laws have a significant role in the development of any sort of aggregation procedure. There are various functions such as logarithm and exponential have been employed in the developing aggregation procedure in the prevailing theories. Sin trigonometry function (STF) has a significant role in the process of the combination of data. The biggest benefits of the STF are symmetric origin and periodicity. Consequently, over the assessment of the object, it holds the expert preferences. Taking into account, the benefits of STF and BCFSs, there is a requirement to develop a few sin trigonometry (ST) operational laws in the setting of BCFSs and analyze their consequences. Therefore, in this article, we are going to establish STAOs based on STOLs for the BCFSs. The targets of this article are as follows.

1. To analyze the hepatitis along with its different types with the assistance of STAOs based on BCFSs.
2. To introduce the ST-BCFS and STOLs.
3. To investigate STAOs based on STOLs in the environment of BCFSs and named them ST-BCFWA, ST-BCFOWA, ST-BCFHA, ST-BCFWG, ST-BCFOWG, ST-BCFHG operators.
4. To analyze hepatitis and its types we investigate a descriptive example and assess the types of hepatitis with the help of the proposed DM procedure and operators.

2.2 | Structure of the Article

The following article is managed as: In Section 2, we did a brief discussion about hepatitis and BCFSs and their properties. In Section 3, we developed sine trigonometric BCFS and then interpreted some elementary operational laws for BCFNs. In Section 4, we established some new weighted averaging and weighted geometric operators based on STOLs of BCFNs such as ST-BCFWA, ST-BCFOWA, ST-BCFWG, and ST-BCFOWG operators. In Section 5, we introduced a DM technique and a numerical example to assess hepatitis and its types. The comparative analysis of the established work with certain prevailing theories for showing the authentication and potential of the diagnosed approaches is contained in Section 6, and the conclusion statement is presented in Section 7.

3 | PRELIMINARIES

Here, we devise some basic concepts of BCFS.

Definition 1: [6] A structure $\chi = \{(\lambda, \Phi_{P-X}(\lambda), \Phi_{N-X}(\lambda)) \mid \lambda \in \mathcal{U}\}$ is said to be BCF sets, where $\Phi_{P-X}(\lambda) = \Phi_{RP-X}(\lambda) + \iota\, \Phi_{IP-X}(\lambda)$ labels a positive satisfying grade (PSG) and $\Phi_{N-X}(\lambda) = \Phi_{RN-X}(\lambda) + \iota\, \Phi_{IN-X}(\lambda)$, labels the negative satisfying grade (NSG), $\Phi_{RP-X}(\lambda), \Phi_{IP-X}(\lambda) \in [0,1]$ and $\Phi_{RN-X}(\lambda), \Phi_{IN-X}(\lambda) \in [-1,0]$. The set $X = (\Phi_{P-X}(\lambda), \Phi_{N-X}(\lambda)) = (\Phi_{RP-X}(\lambda) + \iota\Phi_{IP-X}(\lambda), \Phi_{RN-X}(\lambda) + \iota\, \Phi_{IN-X}(\lambda))$, will be utilized as BCF numbers (BCFNs) in this analysis.

Definition 2: [40] For a BCFN $X = (\Phi_{P-X}(\lambda), \Phi_{N-X}(\lambda)) = (\Phi_{RP-X}(\lambda) + \iota\Phi_{IP-X}(\lambda), \Phi_{RN-X}(\lambda) + \iota\, \Phi_{IN-X}(\lambda))$, the score value (SV) is signified and established as

$$\mathcal{S}_{SF}(X) = \frac{1}{4}\left(2 + \Phi_{RP-X}(\lambda) + \Phi_{IP-X}(\lambda) + \Phi_{RN-X}(\lambda) + \Phi_{IN-X}(\lambda)\right), \;\; \mathcal{S}_{SF} \in [0,1] \quad (1)$$

and the accuracy value is signified and established as

$$\mathcal{H}_{\mathcal{AF}}(X) = \frac{\Phi_{RP-X}(\lambda) + \Phi_{IP-X}(\lambda) - \Phi_{RN-X}(\lambda) - \Phi_{IN-X}(\lambda)}{4}, \mathcal{H}_{\mathcal{AF}} \in [0,1] \quad (2)$$

By employing Eq. (1) and Eq. (2), we achieved the following

1. if $\mathcal{S}_{SF}(X_1) < \mathcal{S}_{SF}(X_2)$, then $X_1 < X_2$;
2. if $\mathcal{S}_{SF}(X_1) > \mathcal{S}_{SF}(X_2)$, then $X_1 > X_2$;
3. if $\mathcal{S}_{SF}(X_1) = \mathcal{S}_{SF}(X_2)$, then
   (1) if $\mathcal{H}_{\mathcal{AF}}(X_1) < \mathcal{H}_{\mathcal{AF}}(X_2)$, then $X_1 < X_2$;
   (2) if $\mathcal{H}_{\mathcal{AF}}(X_1) > \mathcal{H}_{\mathcal{AF}}(X_2)$, then $X_1 > X_2$;
   (3) if $\mathcal{H}_{\mathcal{AF}}(X_1) = \mathcal{H}_{\mathcal{AF}}(X_2)$, then $X_1 = X_2$.

Definition 3: [40] For $X_1 = (\Phi_{P-X_1}, \Phi_{N-X_1}) = (\Phi_{RP-X_1} + \iota\Phi_{IP-X_1}, \Phi_{RN-X_1} + \iota\, \Phi_{IN-X_1})$ and $X_2 = (\Phi_{P-X_2}, \Phi_{N-X_2}) = (\Phi_{RP-X_2} + \iota\Phi_{IP-X_2}, \Phi_{RN-X_2} + \iota\, \Phi_{IN-X_2})$ with $\lambda > 0$, we have

1. $X_1 \oplus X_2 = \begin{pmatrix} \Phi_{RP-X_1} + \Phi_{RP-X_2} - \Phi_{RP-X_1}\Phi_{RP-X_2} + \iota\left(\Phi_{IP-X_1} + \Phi_{RP-X_2} - \Phi_{IP-X_1}\Phi_{IP-X_2}\right), \\ -(\Phi_{RN-X_1}\Phi_{RN-X_2}) + \iota\left(-(\Phi_{IN-X_1}\Phi_{IN-X_2})\right) \end{pmatrix}$
2. $X_1 \otimes X_2 = \begin{pmatrix} \Phi_{RP-X_1}\Phi_{RP-X_2} + \iota\, \Phi_{IP-X_1}\Phi_{IP-X_2}, \\ \Phi_{RN-X_1} + \Phi_{RN-X_2}\Phi_{RN-X_1} + \Phi_{RN-X_2} + \iota\left(\Phi_{IN-X_1} + \Phi_{IN-X_2}\Phi_{IN-X_1} + \Phi_{IN-X_2}\right) \end{pmatrix}$
3. $\lambda X_1 = \left(1 - (1 - \Phi_{RP-X_1})^{\lambda} + \iota\left(1 - (1 - \Phi_{IP-X_1})^{\lambda}\right), -|\Phi_{RN-X_1}|^{\lambda} + \iota\left(-|\Phi_{IN-X_1}|^{\lambda}\right)\right)$
4. $X_1{}^{\lambda} = \left(\left((\Phi_{RP-X_1})^{\lambda} + \iota\,(\Phi_{IP-X_1})^{\lambda}, -1 + (1 + \Phi_{RN-X_1})^{\lambda} + \iota\left(-1 + (1 + \Phi_{IN-X_1})^{\lambda}\right)\right)\right)$

Theorem 1: [40] For $X_1 = (\Phi_{P-X_1}, \Phi_{N-X_1}) = (\Phi_{RP-X_1} + \iota\Phi_{IP-X_1}, \Phi_{RN-X_1} + \iota\, \Phi_{IN-X_1})$ and $X_2 = (\Phi_{P-X_2}, \Phi_{N-X_2}) = (\Phi_{RP-X_2} + \iota\Phi_{IP-X_2}, \Phi_{RN-X_2} + \iota\, \Phi_{IN-X_2})$, with $\lambda, \lambda_1, \lambda_2 > 0$, we have

1. $X_1 \oplus X_2 = X_2 \oplus X_1$

2. $X_1 \otimes X_2 = X_2 \otimes X_1$
3. $\lambda(X_1 \oplus X_2) = \lambda X_1 \oplus \lambda X_2$
4. $(X_1 \otimes X_2)^{\lambda} = {X_1}^{\lambda} \otimes {X_2}^{\lambda}$
5. $\lambda_1 X_1 \oplus \lambda_2 X_1 = (\lambda_1 + \lambda_2) X_1$
6. ${X_1}^{\lambda_1} \otimes {X_1}^{\lambda_2} = {X_1}^{\lambda_1+\lambda_2}$
7. $\left({X_2}^{\lambda_1}\right)^{\lambda_2} = {X_2}^{\lambda_1 \lambda_2}$.

4 | SINE TRIGONOMETRIC OPERATIONAL LAWS FOR BCFSs

Here, firstly, we are going to establish sine trigonometric BCFS and then interpret some elementary operational laws for BCFNs.

Definition 4: For BCFS $\mathrm{X} = \{(ℷ, \Phi_{P-X}(ℷ), \Phi_{N-X}(ℷ)) \mid ℷ \in \mathcal{U}\}$ computed based on fixed set $\mathcal{U}$, the investigation theory:

$$\sin \mathrm{X} = \left\{\left(\begin{array}{c} ℷ, \sin\left(\frac{\pi}{2}\Phi_{RP-X}\right) + \iota\left(\frac{\pi}{2}\Phi_{IP-X}\right), \\ -1 + \sin\left(\frac{\pi}{2}(1+\Phi_{RN-X})\right) + \iota\left(-1 + \sin\left(\frac{\pi}{2}(1+\Phi_{IN-X})\right)\right) \end{array}\right) \mid ℷ \in \mathcal{U}\right\} \quad (3)$$

Called sine trigonometric operational laws (STOLs), where $\Phi_{RP-X}: \mathcal{U} \to [0,1]$, $\Phi_{IP-X}: \mathcal{U} \to [0,1]$, $\Phi_{RN-X}: \mathcal{U} \to [-1,0]$, $\Phi_{IN-X}: \mathcal{U} \to [-1,0]$ and the trigonometric function is defined by:

$\sin\left(\frac{\pi}{2}\Phi_{RP-X}\right): \mathcal{U} \to [0,1]$, $\sin\left(\frac{\pi}{2}\Phi_{IP-X}\right): \mathcal{U} \to [0,1]$ and $-1 + \sin\left(\frac{\pi}{2}(1+\Phi_{RN-X})\right): \mathcal{U} \to [0,1]$, $-1 + \sin\left(\frac{\pi}{2}(1+\Phi_{IN-X})\right): \mathcal{U} \to [-1,0]$, while,

$$\sin \mathrm{X} = \left\{\left(\begin{array}{c} ℷ, \sin\left(\frac{\pi}{2}\Phi_{RP-X}\right) + \iota\left(\frac{\pi}{2}\Phi_{IP-X}\right), \\ -1 + \sin\left(\frac{\pi}{2}(1+\Phi_{RN-X})\right) + \iota\left(-1 + \sin\left(\frac{\pi}{2}(1+\Phi_{IN-X})\right)\right) \end{array}\right) \mid ℷ \in \mathcal{U}\right\} \quad (4)$$

Represented the BCFS.

Definition 5: Suppose that $X = (\Phi_{P-X}, \Phi_{N-X}) = (\Phi_{RP-X} + \iota\Phi_{IP-X}, \Phi_{RN-X} + \iota\,\Phi_{IN-X})$ is a BCFS, then we call the function $\sin \mathrm{X}$ is a sine trigonometric operator if

$$\sin \mathrm{X} = \left(\begin{array}{c} \sin\left(\frac{\pi}{2}\Phi_{RP-X}\right) + \iota\left(\frac{\pi}{2}\Phi_{IP-X}\right), \\ -1 + \sin\left(\frac{\pi}{2}(1+\Phi_{RN-X})\right) + \iota\left(-1 + \sin\left(\frac{\pi}{2}(1+\Phi_{IN-X})\right)\right) \end{array}\right) \quad (5)$$

and we call sine trigonometric BCFN (ST-BCFN) to the value of $\sin \mathrm{X}$.

Theorem 2: The value of $\sin \mathrm{X}$ is again a BCFN.

Proof: To show that, we have to prove the following conditions

1. $0 \le \sin\left(\frac{\pi}{2}\Phi_{RP-X}\right) \le 1$,
2. $0 \le \sin\left(\frac{\pi}{2}\Phi_{IP-X}\right) \le 1$,
3. $-1 \le -1 + \sin\left(\frac{\pi}{2}(1+\Phi_{RN-X})\right) \le 0$,

4. $-1 \leq -1 + \sin\left(\frac{\pi}{2}(1+\Phi_{IN-X})\right) \leq 0.$

1. As $0 \leq \Phi_{RP-X} \leq 1$, then $0 \leq \frac{\pi}{2}\Phi_{RP-X} \leq \frac{\pi}{2}$. As we know in the first quadrant "sin" is an increasing function, Thus $0 \leq \sin\left(\frac{\pi}{2}\Phi_{RP-X}\right) \leq 1$.
2. As $0 \leq \Phi_{IP-X} \leq 1$, then $0 \leq \frac{\pi}{2}\Phi_{IP-X} \leq \frac{\pi}{2}$. As we know in the first quadrant "sin" is an increasing function, Thus $0 \leq \sin\left(\frac{\pi}{2}\Phi_{IP-X}\right) \leq 1$.
3. As $-1 \leq \Phi_{RN-X} \leq 0$, then $0 \leq (1+\Phi_{RN-X}) \leq 1$ which implies that $0 \leq \frac{\pi}{2}(1+\Phi_{RN-X}) \leq \frac{\pi}{2}$. As we know in the first quadrant "sin" is an increasing function, Thus $0 \leq \sin\left(\frac{\pi}{2}(1+\Phi_{RN-X})\right) \leq 1$, this implies that $-1 \leq -1 + \sin\left(\frac{\pi}{2}(1+\Phi_{RN-X})\right) \leq 0$.
4. As $-1 \leq \Phi_{IN-X} \leq 0$, then $0 \leq (1+\Phi_{IN-X}) \leq 1$ which implies that $0 \leq \frac{\pi}{2}(1+\Phi_{IN-X}) \leq \frac{\pi}{2}$. As we know in the first quadrant "sin" is an increasing function, Thus $0 \leq \sin\left(\frac{\pi}{2}(1+\Phi_{IN-X})\right) \leq 1$, this implies that $-1 \leq -1 + \sin\left(\frac{\pi}{2}(1+\Phi_{IN-X})\right) \leq 0$.

Thus, conditions 1-4 hold. Consequently, $\sin \mathrm{X}$ is a BCFN.

Definition 6: Suppose $\sin \mathrm{X}_1 = \begin{pmatrix} \sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right) + \iota\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right), \\ -1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_1})\right) + \iota\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_1})\right)\right) \end{pmatrix}$ and $\sin \mathrm{X}_2 = \begin{pmatrix} \sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_2}\right) + \iota\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_2}\right), \\ -1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_2})\right) + \iota\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_2})\right)\right) \end{pmatrix}$ are ST-BCFNs, then we introduce their operational laws as

$$1.\ \sin \mathrm{X}_1 \oplus \sin \mathrm{X}_2 = \begin{pmatrix} 1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right)\right)\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_2}\right)\right) \\ +\iota\left(1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right)\right)\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_2}\right)\right)\right), \\ -\left(\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_1})\right)\right)\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_2})\right)\right)\right) \\ +\iota\left(-\left(\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_1})\right)\right)\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_2})\right)\right)\right)\right) \end{pmatrix}$$

$$2.\ \sin \mathrm{X}_1 \otimes \sin \mathrm{X}_2 = \begin{pmatrix} \sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right)\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_2}\right) \\ +\iota\ \sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right)\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_2}\right), \\ -1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_1})\right)\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_2})\right) \\ +\iota\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_1})\right)\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_2})\right)\right) \end{pmatrix}$$

$$3.\ \lambda \sin \mathrm{X}_1 = \begin{pmatrix} 1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right)\right)^{\lambda} + \iota\left(1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right)\right)^{\lambda}\right) \\ -\left|\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_1})\right)\right)\right|^{\lambda} + \iota\left(-\left|\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_1})\right)\right)\right|^{\lambda}\right) \end{pmatrix}$$

$$4.\ (\sin \mathrm{X}_1)^{\lambda} = \begin{pmatrix} \left(\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right)\right)^{\lambda} + \iota\left(\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right)\right)^{\lambda} \\ -1 + \left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_1}\right)\right)\right)^{\lambda} + \iota\left(-1 + \left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_1}\right)\right)\right)^{\lambda}\right) \end{pmatrix}$$

Following, the elementary properties of ST-BCFN depend on the STOLs are established.

Theorem 3: Suppose $X_{j} = \left(\Phi_{P-X_{j}}, \Phi_{N-X_{j}}\right) = \left(\Phi_{RP-X_{j}} + \iota\Phi_{IP-X_{j}}, \Phi_{RN-X_{j}} + \iota\,\Phi_{IN-X_{j}}\right)$ $(j = 1,2,3)$ are three BCFNs, then

1. $\sin \mathrm{X}_1 \oplus \sin \mathrm{X}_2 = \sin \mathrm{X}_2 \oplus \sin \mathrm{X}_1$;
2. $\sin \mathrm{X}_1 \otimes \sin \mathrm{X}_2 = \sin \mathrm{X}_2 \otimes \sin \mathrm{X}_1$;
3. $(\sin \mathrm{X}_1 \oplus \sin \mathrm{X}_2) \oplus \sin \mathrm{X}_3 = \sin \mathrm{X}_1 \oplus (\sin \mathrm{X}_2 \oplus \sin \mathrm{X}_3)$;
4. $(\sin \mathrm{X}_1 \otimes \sin \mathrm{X}_2) \otimes \sin \mathrm{X}_3 = \sin \mathrm{X}_1 \otimes (\sin \mathrm{X}_2 \otimes \sin \mathrm{X}_3)$

Proof: Omitted the proof as it is straightforward from Definition 6.

Theorem 4: Suppose $X_1 = \left(\Phi_{P-X_1}, \Phi_{N-X_1}\right) = \left(\Phi_{RP-X_1} + \iota\Phi_{IP-X_1}, \Phi_{RN-X_1} + \iota\,\Phi_{IN-X_1}\right)$ and $X_2 = \left(\Phi_{P-X_2}, \Phi_{N-X_2}\right) = \left(\Phi_{RP-X_2} + \iota\Phi_{IP-X_2}, \Phi_{RN-X_2} + \iota\,\Phi_{IN-X_2}\right)$ are two BCFNs with $\lambda, \lambda_1, \lambda_2 > 0$, are any real number, then we have

1. $\lambda(\sin \mathrm{X}_1 \oplus \sin \mathrm{X}_2) = \lambda \sin \mathrm{X}_1 \oplus \lambda \sin \mathrm{X}_2$
2. $(\sin \mathrm{X}_1 \otimes \sin \mathrm{X}_2)^{\lambda} = (\sin \mathrm{X}_1)^{\lambda} \otimes (\sin \mathrm{X}_2)^{\lambda}$
3. $\lambda_1 \sin \mathrm{X}_1 \oplus \lambda_2 \sin \mathrm{X}_1 = (\lambda_1 + \lambda_2) \sin \mathrm{X}_1$
4. $(\sin \mathrm{X}_1)^{\lambda_1} \otimes (\sin \mathrm{X}_1)^{\lambda_2} = (\sin \mathrm{X}_1)^{\lambda_1+\lambda_2}$
5. $\left((\sin \mathrm{X}_1)^{\lambda_1}\right)^{\lambda_2} = (\sin \mathrm{X}_1)^{\lambda_1\lambda_2}$.

Proof: Here, we are going to prove merely 1 and 3. The rest can prove similar.

1. By using Def 6 we have

$$\sin \mathrm{X}_1 \oplus \sin \mathrm{X}_2 = \begin{pmatrix} 1 - \left(1 - \sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right)\right)\left(1 - \sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_2}\right)\right) \\ +\iota\left(1 - \left(1 - \sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right)\right)\left(1 - \sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_2}\right)\right)\right), \\ -\left(\left(-1 + \sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_1}\right)\right)\right)\left(-1 + \sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_2}\right)\right)\right)\right) \\ +\iota\left(-\left(\left(-1 + \sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_1}\right)\right)\right)\left(-1 + \sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_2}\right)\right)\right)\right)\right) \end{pmatrix}$$

Now for $\lambda > 0$, we have

$$\lambda(\sin \mathrm{X}_1 \oplus \sin \mathrm{X}_2) = \begin{pmatrix} 1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right)\right)^{\lambda}\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_2}\right)\right)^{\lambda} \\ +\iota\left(1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right)\right)^{\lambda}\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_2}\right)\right)^{\lambda}\right), \\ -\left(\left|-\left(\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_1})\right)\right)\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_2})\right)\right)\right)\right|^{\lambda}\right) \\ +\iota\left(-\left(\left|-\left(\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_1})\right)\right)\left(-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_2})\right)\right)\right)\right|^{\lambda}\right)\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right)\right)^{\lambda}+\iota\left(1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right)\right)^{\lambda}\right), \\ -\left(\left|-1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_1})\right)\right|^{\lambda}\right)+\iota\left(-\left(\left|-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_1})\right)\right|^{\lambda}\right)\right) \end{pmatrix}$$

$$\oplus \begin{pmatrix} 1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_2}\right)\right)^{\lambda}+\iota\left(1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_2}\right)\right)^{\lambda}\right), \\ -\left(\left|-1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_2})\right)\right|^{\lambda}\right)+\iota\left(-\left(\left|-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_2})\right)\right|^{\lambda}\right)\right) \end{pmatrix}$$

$$= \lambda \sin \mathrm{X}_1 \oplus \lambda \sin \mathrm{X}_2$$

2. we have

$$\lambda_1 \sin \mathrm{X}_1 \oplus \lambda_2 \sin \mathrm{X}_1$$

$$= \begin{pmatrix} 1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right)\right)^{\lambda_1}+\iota\left(1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right)\right)^{\lambda_1}\right), \\ -\left(\left|-1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_1})\right)\right|^{\lambda_1}\right)+\iota\left(-\left(\left|-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_1})\right)\right|^{\lambda_1}\right)\right) \end{pmatrix}$$

$$\oplus \begin{pmatrix} 1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right)\right)^{\lambda_2}+\iota\left(1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right)\right)^{\lambda_2}\right), \\ -\left(\left|-1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_1})\right)\right|^{\lambda_2}\right)+\iota\left(-\left(\left|-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_1})\right)\right|^{\lambda_2}\right)\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right)\right)^{\lambda_1+\lambda_2}+\iota\left(1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right)\right)^{\lambda_1+\lambda_2}\right), \\ -\left(\left|-1+\sin\left(\frac{\pi}{2}(1+\Phi_{RN-\mathrm{X}_1})\right)\right|^{\lambda_1+\lambda_2}\right)+\iota\left(-\left(\left|-1+\sin\left(\frac{\pi}{2}(1+\Phi_{IN-\mathrm{X}_1})\right)\right|^{\lambda_1+\lambda_2}\right)\right) \end{pmatrix}$$

$$= (\lambda_1+\lambda_2)\sin \mathrm{X}_1$$

Theorem 5: For any two BCFNs $X_1 = (\Phi_{P-X_1}, \Phi_{N-X_1}) = (\Phi_{RP-X_1} + \iota\Phi_{IP-X_1}, \Phi_{RN-X_1} + \iota\,\Phi_{IN-X_1})$ and $X_2 = (\Phi_{P-X_2}, \Phi_{N-X_2}) = (\Phi_{RP-X_2} + \iota\Phi_{IP-X_2}, \Phi_{RN-X_2} + \iota\,\Phi_{IN-X_2})$ if $\Phi_{RP-X_1} \geq \Phi_{RP-X_2}$, $\Phi_{IP-X_1} \geq \Phi_{IP-X_2}$, $\Phi_{RN-X_1} \leq \Phi_{RN-X_2}$ and $\Phi_{IN-X_1} \leq \Phi_{IN-X_2}$ then $\sin \mathrm{X}_1 \geq \sin \mathrm{X}_2$

Proof: As $\Phi_{RP-X_1} \geq \Phi_{RP-X_2}$ and in $\left[0, \frac{\pi}{2}\right]$ "sin" is an increasing function, so we get $\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right) \geq \sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_2}\right)$. Likewise, $\Phi_{IP-X_1} \geq \Phi_{IP-X_2}$ and in $\left[0, \frac{\pi}{2}\right]$ "sin" is an increasing function, so we get

$\sin\left(\frac{\pi}{2}\Phi_{IP-X_1}\right) \geq \sin\left(\frac{\pi}{2}\Phi_{IP-X_2}\right)$. Now as $\Phi_{RN-X_1} \leq \Phi_{RN-X_2}$ so we have $\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-X_1}\right)\right) \geq \sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-X_2}\right)\right)$ which shows that $-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-X_1}\right)\right) \leq -1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-X_2}\right)\right)$. Likewise, as $\Phi_{IN-X_1} \leq \Phi_{IN-X_2}$ so we have $\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-X_1}\right)\right) \geq \sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-X_2}\right)\right)$ which shows that $-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-X_1}\right)\right) \leq -1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-X_2}\right)\right)$

5 | SINE TRIGONOMETRIC BCF AOs

In this section, some new weighted averaging and weighted geometric operators based on STOLs of BCFNs such as ST-BCFWA, ST-BCFOWA, ST-BCFWG, and ST-BCFOWG operators are defined. Let $\mathbb{w} = \left(\mathbb{w}_1, \mathbb{w}_2, \dots, \mathbb{w}_{\underline{n}}\right)^T$ be the weight vector with the condition that $\sum_{j=1}^{\underline{n}} \mathbb{w}_j = 1$ and $0 \leq \mathbb{w}_j \leq 1$ for all j in this manuscript.

5.1 | Sine Trigonometric Weighted Averaging AOs

In this subsection, we establish weighted and ordered weighted averaging AOs based on STOLs of BCFNs.

Definition 7: For any group of BCFNs, $X_j = \left(\Phi_{P-X_j}, \Phi_{N-X_j}\right) = \left(\Phi_{RP-X_j} + \iota\Phi_{IP-X_j}, \Phi_{RN-X_j} + \iota\,\Phi_{IN-X_j}\right)$ $(j = 1,2,3,\dots,\underline{n})$ the sine trigonometric weighted averaging AO for the group of BCFNs is identified and defined as

$$ST-BCFWA\left(X_1, X_2, X_3, \dots, X_{\underline{n}}\right) = \mathbb{w}_1 \sin X_1 \oplus \mathbb{w}_2 \sin X_2 \oplus \dots \oplus \mathbb{w}_{\underline{n}} \sin X_{\underline{n}} \quad (6)$$

And which is called ST-BCFWA operator.

Theorem 6: By employing the ST-BCFWA operator, we get again the BCFN and

$$ST-BCFWA\left(X_1, X_2, X_3, \dots, X_{\underline{n}}\right) = \begin{pmatrix} 1-\prod_{j=1}^{\underline{n}}\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-X_j}\right)\right)^{\mathbb{w}_j} \\ +\iota\left(1-\prod_{j=1}^{\underline{n}}\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-X_j}\right)\right)^{\mathbb{w}_j}\right), \\ -\prod_{j=1}^{\underline{n}}\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-X_j}\right)\right)\right|^{\mathbb{w}_j} \\ +\iota\left(-\prod_{j=1}^{\underline{n}}\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-X_j}\right)\right)\right|^{\mathbb{w}_j}\right) \end{pmatrix} \quad (7)$$

Proof: We will prove this theorem with the help of mathematical induction.

Step 1: Suppose $\underline{n} = 2$, we have

$$\mathbb{w}_1 \sin X_1 = \begin{pmatrix} 1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-X_1}\right)\right)^{\mathbb{w}_1} + \iota\left(1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-X_1}\right)\right)^{\mathbb{w}_1}\right) \\ ,-\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-X_1}\right)\right)\right|^{\mathbb{w}_1} + \iota\left(-\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-X_1}\right)\right)\right|^{\mathbb{w}_1}\right) \end{pmatrix}$$

$$\mathbb{w}_2 \sin \mathrm{X}_2 = \left(\begin{array}{c} 1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_2}\right)\right)^{\mathbb{w}_2} + \iota\left(1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_2}\right)\right)^{\mathbb{w}_2}\right) \\ ,-\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_2}\right)\right)\right|^{\mathbb{w}_2} + \iota\left(-\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_2}\right)\right)\right|^{\mathbb{w}_2}\right) \end{array} \right)$$

$$\mathbb{w}_1 \sin \mathrm{X}_1 \oplus \mathbb{w}_2 \sin \mathrm{X}_2$$

$$= \left(\begin{array}{c} 1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right)\right)^{\mathbb{w}_1}\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_2}\right)\right)^{\mathbb{w}_2} \\ +\iota\left(1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right)\right)^{\mathbb{w}_1}\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_2}\right)\right)^{\mathbb{w}_2}\right), \\ -\left(\left|-\left(\left(-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_1}\right)\right)\right)\left(-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_2}\right)\right)\right)\right)\right|^{\mathbb{w}_1}\right) \\ +\iota\left(-\left(\left|-\left(\left(-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_1}\right)\right)\right)\left(-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_2}\right)\right)\right)\right)\right|^{\mathbb{w}_2}\right)\right) \end{array} \right)$$

$$= \left(\begin{array}{c} 1-\prod_{j=1}^{2}\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_j}\right)\right)^{\mathbb{w}_j} + \iota\left(1-\prod_{j=1}^{2}\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_j}\right)\right)^{\mathbb{w}_j}\right), \\ -\prod_{j=1}^{2}\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_j}\right)\right)\right|^{\mathbb{w}_j} + \iota\left(-\prod_{j=1}^{2}\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_j}\right)\right)\right|^{\mathbb{w}_j}\right) \end{array} \right)$$

Step 2: Assume that Eq. (7) holds for $\underline{\mathrm{n}} = \mathrm{L}$ i.e.

$$ST-BCFWA(X_1, X_2, X_3, \dots, X_L) = \left(\begin{array}{c} 1-\prod_{j=1}^{L}\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_j}\right)\right)^{\mathbb{w}_j} \\ +\iota\left(1-\prod_{j=1}^{L}\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_j}\right)\right)^{\mathbb{w}_j}\right), \\ -\prod_{j=1}^{L}\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_j}\right)\right)\right|^{\mathbb{w}_j} \\ +\iota\left(-\prod_{j=1}^{L}\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_j}\right)\right)\right|^{\mathbb{w}_j}\right) \end{array} \right)$$

Step 3: In this step, we show that Eq. (7) is true for $\underline{\mathrm{n}} = \mathrm{L}+1$, we have

$$ST-BCFWA(X_1,X_2,X_3,\ldots,X_L,X_{L+1})$$

$$=\begin{pmatrix} 1-\prod_{j=1}^{L}\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-X_j}\right)\right)^{w_j} \\ +\iota\left(1-\prod_{j=1}^{L}\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-X_j}\right)\right)^{w_j}\right), \\ -\prod_{j=1}^{L}\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-X_j}\right)\right)\right|^{w_j} \\ +\iota\left(-\prod_{j=1}^{L}\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-X_j}\right)\right)\right|^{w_j}\right) \end{pmatrix}$$

$$\oplus\begin{pmatrix} 1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-X_{L+1}}\right)\right)^{w_{L+1}} \\ +\iota\left(1-\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-X_{L+1}}\right)\right)^{w_{L+1}}\right), \\ -\left(\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-X_{L+1}}\right)\right)\right|^{w_{L+1}}\right) \\ +\iota\left(-\left(\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-X_{L+1}}\right)\right)\right|^{w_{L+1}}\right)\right) \end{pmatrix}$$

$$=\begin{pmatrix} 1-\prod_{j=1}^{L+1}\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-X_j}\right)\right)^{w_j} \\ +\iota\left(1-\prod_{j=1}^{L+1}\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-X_j}\right)\right)^{w_j}\right), \\ -\prod_{j=1}^{L+1}\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-X_j}\right)\right)\right|^{w_j} \\ +\iota\left(-\prod_{j=1}^{L+1}\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-X_j}\right)\right)\right|^{w_j}\right) \end{pmatrix}$$

i.e. when $\underline{n}=L+1$, Eq. (7) is true. Thus, Eq. (7) is true for any $\underline{n}$.

We provide certain properties of the explored ST-BCFWA operator. The above-defined AO relies on the sine trigonometric function and satisfies the monotonicity, idempotency, and boundedness.

1. **Idempotency**: For any group of BCFNs, $X_j=\left(\Phi_{P-X_j},\Phi_{N-X_j}\right)=\left(\Phi_{RP-X_j}+\iota\Phi_{IP-X_j},\Phi_{RN-X_j}+\iota\,\Phi_{IN-X_j}\right)$ $(j=1,2,3,\ldots,\underline{n})$ if $X_j=X$ for all j, then

$$ST-BCFWA\left(X_1,X_2,..,X_{\underline{n}}\right)=\sin X$$

2. **Boundedness:** For any group of BCFNs, $X_j=\left(\Phi_{P-X_j},\Phi_{N-X_j}\right)=\left(\Phi_{RP-X_j}+\iota\Phi_{IP-X_j},\Phi_{RN-X_j}+\iota\,\Phi_{IN-X_j}\right)$, let $X^-=\left(\min_j\left\{\Phi_{RP-X_j}\right\}+\iota\,\min_j\left\{\Phi_{IP-X_j}\right\},\max_j\left\{\Phi_{RN-X_j}\right\}+\right.$

$\iota \max_{j}\left\{\Phi_{IP-X_j}\right\}\Big)$, and $X^{+}=\left(\max_{j}\left\{\Phi_{RP-X_j}\right\}+\iota \max_{j}\left\{\Phi_{IP-X_j}\right\}, \min_{j}\left\{\Phi_{RN-X_j}\right\}+\iota \min_{j}\left\{\Phi_{IP-X_j}\right\}\right)$ then

$$\sin X^{-} \leq ST-BCFWA\left(X_1, X_2, \ldots, X_{\mathrm{n}}\right) \leq \sin X^{+}$$

3. **Monotonicity:** For any two groups of BCFNs $X_j=\left(\Phi_{P-X_j}, \Phi_{N-X_j}\right)=\left(\Phi_{RP-X_j}+\iota\Phi_{IP-X_j}, \Phi_{RN-X_j}+\iota\,\Phi_{IN-X_j}\right)$ and $X_j'=\left(\Phi_{P-X_j'}, \Phi_{N-X_j'}\right)=\left(\Phi_{RP-X_j'}+\iota\Phi_{IP-X_j'}, \Phi_{RN-X_j'}+\iota\,\Phi_{IN-X_j'}\right)$, $(j=1,2,\ldots,\mathrm{n})$, if $\Phi_{RP-X_j} \leq \Phi_{RP-X_j'}$, $\Phi_{IP-X_j} \leq \Phi_{IP-X_j'}$, $\Phi_{RN-X_j} \leq \Phi_{RN-X_j'}$, $\Phi_{IN-X_j} \leq \Phi_{IN-X_j'}$ for all j then

$$ST-BCFWA\left(X_1, X_2, \ldots, X_{\mathrm{n}}\right) \leq ST-BCFWA\left(X_1', X_2', \ldots, X_{\mathrm{n}}'\right)$$

Definition 8: For any group of BCFNs, $X_j=\left(\Phi_{P-X_j}, \Phi_{N-X_j}\right)=\left(\Phi_{RP-X_j}+\iota\Phi_{IP-X_j}, \Phi_{RN-X_j}+\iota\,\Phi_{IN-X_j}\right)$ $(j=1,2,3,\ldots,\mathrm{n})$ the sine trigonometric ordered weighted averaging AO for the group of BCFNs is identified and defined as

$$ST-BCFOWA\left(X_1, X_2, X_3, \ldots, X_{\mathrm{n}}\right)=\mathbb{w}_1 \sin \mathrm{X}_{\mathfrak{T}(1)} \oplus \mathbb{w}_2 \sin \mathrm{X}_{\mathfrak{T}(2)} \oplus \ldots \oplus \mathbb{w}_{\mathrm{n}} \sin \mathrm{X}_{\mathfrak{T}(\mathrm{n})} \quad (8)$$

And which is called ST-BCFOWA operator. where, $\left(\mathfrak{T}(1), \mathfrak{T}(2), \mathfrak{T}(3), \ldots, \mathfrak{T}(\mathrm{n})\right)$ is a permutation of $(1,2,\ldots,\mathrm{n})$ such that $\mathrm{X}_{\mathfrak{T}(j-1)} \geq \mathrm{X}_{\mathfrak{T}(j)}\ \forall j$

Theorem 7: By employing the ST-BCFOWA operator, we get again the BCFN and

$$ST-BCFOWA\left(X_1, X_2, X_3, \ldots, X_{\mathrm{n}}\right)=\begin{pmatrix} 1-\prod_{j=1}^{\mathrm{n}}\left(1-\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_{\mathfrak{T}(j)}}\right)\right)^{\mathbb{w}_j} \\ +\iota\left(1-\prod_{j=1}^{\mathrm{n}}\left(1-\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_{\mathfrak{T}(j)}}\right)\right)^{\mathbb{w}_j}\right), \\ -\prod_{j=1}^{\mathrm{n}}\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_{\mathfrak{T}(j)}}\right)\right)\right|^{\mathbb{w}_j} \\ +\iota\left(-\prod_{j=1}^{\mathrm{n}}\left|-1+\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_{\mathfrak{T}(j)}}\right)\right)\right|^{\mathbb{w}_j}\right) \end{pmatrix} \quad (9)$$

Likewise, the ST-BCFWA operator, ST-BCFOWA satisfies the monotonicity, idempotency, and boundedness.

1. **Idempotency**: For any group of BCFNs, $X_j=\left(\Phi_{P-X_j}, \Phi_{N-X_j}\right)=\left(\Phi_{RP-X_j}+\iota\Phi_{IP-X_j}, \Phi_{RN-X_j}+\iota\,\Phi_{IN-X_j}\right)$ $(j=1,2,3,\ldots,\mathrm{n})$ if $X_j=X$ for all j, then

$$ST-BCFOWA\left(X_1, X_2, \ldots, X_{\mathrm{n}}\right)=\sin X$$

2. **Boundedness:** For any group of BCFNs, $X_{\jmath} = \left(\Phi_{P-X_{\jmath}}, \Phi_{N-X_{\jmath}}\right) = \left(\Phi_{RP-X_{\jmath}} + \iota\Phi_{IP-X_{\jmath}}, \Phi_{RN-X_{\jmath}} + \iota\,\Phi_{IN-X_{\jmath}}\right)$, let $X^{-} = \left(\min_{\jmath}\left\{\Phi_{RP-X_{\jmath}}\right\} + \iota\min_{\jmath}\left\{\Phi_{IP-X_{\jmath}}\right\}, \max_{\jmath}\left\{\Phi_{RN-X_{\jmath}}\right\} + \iota\max_{\jmath}\left\{\Phi_{IP-X_{\jmath}}\right\}\right)$, and $X^{+} = \left(\max_{\jmath}\left\{\Phi_{RP-X_{\jmath}}\right\} + \iota\max_{\jmath}\left\{\Phi_{IP-X_{\jmath}}\right\}, \min_{\jmath}\left\{\Phi_{RN-X_{\jmath}}\right\} + \iota\min_{\jmath}\left\{\Phi_{IP-X_{\jmath}}\right\}\right)$ then

$$\sin X^{-} \leq ST - BCFOWA\left(X_1, X_2, .., X_{\underline{n}}\right) \leq \sin X^{+}$$

3. **Monotonicity:** For any two groups of BCFNs $X_{\jmath} = \left(\Phi_{P-X_{\jmath}}, \Phi_{N-X_{\jmath}}\right) = \left(\Phi_{RP-X_{\jmath}} + \iota\Phi_{IP-X_{\jmath}}, \Phi_{RN-X_{\jmath}} + \iota\,\Phi_{IN-X_{\jmath}}\right)$ and $X'_{\jmath} = \left(\Phi_{P-X'_{\jmath}}, \Phi_{N-X'_{\jmath}}\right) = \left(\Phi_{RP-X'_{\jmath}} + \iota\Phi_{IP-X'_{\jmath}}, \Phi_{RN-X'_{\jmath}} + \iota\,\Phi_{IN-X'_{\jmath}}\right)$, $(\jmath = 1, 2, \ldots, \underline{n})$, if $\Phi_{RP-X_{\jmath}} \leq \Phi_{RP-X'_{\jmath}}$, $\Phi_{IP-X_{\jmath}} \leq \Phi_{IP-X'_{\jmath}}$, $\Phi_{RN-X_{\jmath}} \leq \Phi_{RN-X'_{\jmath}}$, $\Phi_{IN-X_{\jmath}} \leq \Phi_{IN-X'_{\jmath}}$ for all $\jmath$ then

$$ST - BCFOWA\left(X_1, X_2, .., X_{\underline{n}}\right) \leq ST - BCFOWA\left(X'_1, X'_2, .., X'_{\underline{n}}\right)$$

Definition 9: For any group of BCFNs, $X_{\jmath} = \left(\Phi_{P-X_{\jmath}}, \Phi_{N-X_{\jmath}}\right) = \left(\Phi_{RP-X_{\jmath}} + \iota\Phi_{IP-X_{\jmath}}, \Phi_{RN-X_{\jmath}} + \iota\,\Phi_{IN-X_{\jmath}}\right)$ $(\jmath = 1, 2, 3, \ldots, \underline{n})$ the sine trigonometric hybrid averaging AO for the group of BCFNs is identified and defined as

$$ST - BCFHA\left(X_1, X_2, X_3, \ldots, X_{\underline{n}}\right) = \mathfrak{P}_1 \sin \mathrm{X}^{*}_{\mathfrak{T}(1)} \oplus \mathfrak{P}_2 \sin \mathrm{X}^{*}_{\mathfrak{T}(2)} \oplus \ldots \oplus \mathfrak{P}_{\underline{n}} \sin \mathrm{X}^{*}_{\mathfrak{T}(\underline{n})} \quad (10)$$

And which is called ST-BCFHA operator. where, $\mathrm{X}^{*}_{\jmath} = \underline{n}\mathbb{w}_{\jmath}\mathrm{X}_{\jmath}$ $\left(\mathfrak{T}(1), \mathfrak{T}(2), \mathfrak{T}(3), \ldots, \mathfrak{T}(\underline{n})\right)$ is a permutation of $(1, 2, .., \underline{n})$ such that $\mathrm{X}_{\mathfrak{T}(\jmath-1)} \geq \mathrm{X}_{\mathfrak{T}(\jmath)}\ \forall\ \jmath$

Theorem 8: By employing the ST-BCFHA operator, we get again the BCFN and

$$ST - BCFHA\left(X_1, X_2, X_3, \ldots, X_{\underline{n}}\right) = \begin{pmatrix} 1 - \prod_{\jmath=1}^{\underline{n}}\left(1 - \sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}^{*}_{\mathfrak{T}(\jmath)}}\right)\right)^{\mathfrak{P}_{\jmath}} \\ +\iota\left(1 - \prod_{\jmath=1}^{\underline{n}}\left(1 - \sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}^{*}_{\mathfrak{T}(\jmath)}}\right)\right)^{\mathfrak{P}_{\jmath}}\right), \\ -\prod_{\jmath=1}^{\underline{n}}\left|-1 + \sin\left(\frac{\pi}{2}\left(1 + \Phi_{RN-\mathrm{X}^{*}_{\mathfrak{T}(\jmath)}}\right)\right)\right|^{\mathfrak{P}_{\jmath}} \\ +\iota\left(-\prod_{\jmath=1}^{\underline{n}}\left|-1 + \sin\left(\frac{\pi}{2}\left(1 + \Phi_{IN-\mathrm{X}^{*}_{\mathfrak{T}(\jmath)}}\right)\right)\right|^{\mathfrak{P}_{\jmath}}\right) \end{pmatrix} \quad (11)$$

5.2 | Sine Trigonometric Weighted Geometric AOs

In this subsection, we establish weighted and ordered weighted geometric AOs based on STOLs of BCFNs.

Definition 10: For any group of BCFNs, $X_{j} = \left(\Phi_{P-X_{j}}, \Phi_{N-X_{j}}\right) = \left(\Phi_{RP-X_{j}} + \iota\Phi_{IP-X_{j}}, \Phi_{RN-X_{j}} + \iota\,\Phi_{IN-X_{j}}\right)$ ($j = 1, 2, 3, \dots, \underline{n}$) the sine trigonometric weighted geometric AO for the group of BCFNs is identified and defined as

$$ST - BCFWG\left(X_1, X_2, X_3, \dots, X_{\underline{n}}\right) = (\sin X_1)^{\mathbb{w}_1} \oplus (\sin X_2)^{\mathbb{w}_2} \oplus \dots \oplus \left(\sin X_{\underline{n}}\right)^{\mathbb{w}_{\underline{n}}} \quad (12)$$

And which is called ST-BCFWG operator.

Theorem 9: By employing the ST-BCFWG operator, we get again the BCFN and

$$ST - BCFWG\left(X_1, X_2, X_3, \dots, X_{\underline{n}}\right) = \begin{pmatrix} \prod_{j=1}^{\underline{n}} \left(\sin\left(\frac{\pi}{2}\Phi_{RP-X_{j}}\right)\right)^{\mathbb{w}_{j}} \\ +\iota\left(\prod_{j=1}^{\underline{n}} \left(\sin\left(\frac{\pi}{2}\Phi_{IP-X_{j}}\right)\right)^{\mathbb{w}_{j}}\right), \\ -1 + \prod_{j=1}^{\underline{n}} \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{RN-X_{j}}\right)\right)\right)^{\mathbb{w}_{j}} \\ +\iota\left(-1 + \prod_{j=1}^{\underline{n}} \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{IN-X_{j}}\right)\right)\right)^{\mathbb{w}_{j}}\right) \end{pmatrix} \quad (13)$$

Proof: We will prove this theorem with the help of mathematical induction.

Step 1: Suppose $\underline{n} = 2$, we have

$$(\sin X_1)^{\mathbb{w}_1} = \begin{pmatrix} \left(\sin\left(\frac{\pi}{2}\Phi_{RP-X_1}\right)\right)^{\mathbb{w}_1} + \iota\left(\sin\left(\frac{\pi}{2}\Phi_{IP-X_1}\right)\right)^{\mathbb{w}_1}, \\ -1 + \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{RN-X_1}\right)\right)\right)^{\mathbb{w}_1} + \iota\left(-1 + \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{IN-X_1}\right)\right)\right)^{\mathbb{w}_1}\right) \end{pmatrix}$$

$$(\sin X_2)^{\mathbb{w}_2} = \begin{pmatrix} \left(\sin\left(\frac{\pi}{2}\Phi_{RP-X_2}\right)\right)^{\mathbb{w}_2} + \iota\left(\sin\left(\frac{\pi}{2}\Phi_{IP-X_2}\right)\right)^{\mathbb{w}_2}, \\ -1 + \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{RN-X_2}\right)\right)\right)^{\mathbb{w}_2} + \iota\left(-1 + \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{IN-X_2}\right)\right)\right)^{\mathbb{w}_2}\right) \end{pmatrix}$$

$$(\sin X_1)^{\mathbb{w}_1} \otimes (\sin X_2)^{\mathbb{w}_2}$$

$$= \begin{pmatrix} \left(\sin\left(\frac{\pi}{2}\Phi_{RP-X_1}\right)\right)^{\mathbb{w}_1} + \iota\left(\sin\left(\frac{\pi}{2}\Phi_{IP-X_1}\right)\right)^{\mathbb{w}_1}, \\ -1 + \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{RN-X_1}\right)\right)\right)^{\mathbb{w}_1} + \iota\left(-1 + \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{IN-X_1}\right)\right)\right)^{\mathbb{w}_1}\right) \end{pmatrix}$$

$$\otimes \begin{pmatrix} \left(\sin\left(\frac{\pi}{2}\Phi_{RP-X_2}\right)\right)^{\mathbb{w}_2} + \iota\left(\sin\left(\frac{\pi}{2}\Phi_{IP-X_2}\right)\right)^{\mathbb{w}_2}, \\ -1 + \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{RN-X_2}\right)\right)\right)^{\mathbb{w}_2} + \iota\left(-1 + \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{IN-X_2}\right)\right)\right)^{\mathbb{w}_2}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \left(\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_1}\right)\right)^{\mathbb{w}_1}\left(\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_2}\right)\right)^{\mathbb{w}_2} \\ +\iota\left(\left(\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_1}\right)\right)^{\mathbb{w}_1}\left(\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_2}\right)\right)^{\mathbb{w}_2}\right) \\ -1+\left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_1}\right)\right)\right)^{\mathbb{w}_1}\left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_2}\right)\right)\right)^{\mathbb{w}_2} \\ +\iota\left(-1+\left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_1}\right)\right)\right)^{\mathbb{w}_1}\left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_2}\right)\right)\right)^{\mathbb{w}_2}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \prod_{j=1}^{2}\left(\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_j}\right)\right)^{\mathbb{w}_j} + \iota\left(\prod_{j=1}^{2}\left(\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_j}\right)\right)^{\mathbb{w}_j}\right), \\ -1+\prod_{j=1}^{2}\left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_j}\right)\right)\right)^{\mathbb{w}_j} + \iota\left(-1+\prod_{j=1}^{2}\left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_j}\right)\right)\right)^{\mathbb{w}_j}\right) \end{pmatrix}$$

Step 2: Assume that Eq. (13) holds for $\underline{n} = L$ i.e.

$$ST-BCFWG(X_1, X_2, X_3, \dots, X_L) = \begin{pmatrix} \prod_{j=1}^{L}\left(\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_j}\right)\right)^{\mathbb{w}_j} \\ +\iota\left(\prod_{j=1}^{L}\left(\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_j}\right)\right)^{\mathbb{w}_j}\right), \\ -1+\prod_{j=1}^{L}\left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_j}\right)\right)\right)^{\mathbb{w}_j} \\ +\iota\left(-1+\prod_{j=1}^{L}\left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_j}\right)\right)\right)^{\mathbb{w}_j}\right) \end{pmatrix}$$

Step 3: In this step, we show that Eq (13) is true for $\underline{n} = L + 1$, we have

$$ST-BCFWG(X_1, X_2, X_3, \dots, X_L, X_{L+1})$$

$$= \begin{pmatrix} \prod_{j=1}^{L}\left(\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_j}\right)\right)^{\mathbb{w}_j} \\ +\iota\left(\prod_{j=1}^{L}\left(\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_j}\right)\right)^{\mathbb{w}_j}\right), \\ -1+\prod_{j=1}^{L}\left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_j}\right)\right)\right)^{\mathbb{w}_j} \\ +\iota\left(-1+\prod_{j=1}^{L}\left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_j}\right)\right)\right)^{\mathbb{w}_j}\right) \end{pmatrix}$$

$$\otimes \begin{pmatrix} \left(\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}_{L+1}}\right)\right)^{\mathbb{w}_{L+1}} \\ +\iota\left(\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}_{L+1}}\right)\right)^{\mathbb{w}_{L+1}}, \\ -1+\left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-\mathrm{X}_{L+1}}\right)\right)\right)^{\mathbb{w}_{L+1}} \\ +\iota\left(-1+\left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-\mathrm{X}_{L+1}}\right)\right)\right)^{\mathbb{w}_{L+1}}\right) \end{pmatrix}$$

$$= \left(\begin{array}{c} \prod_{\jmath=1}^{L+1} \left(\sin\left(\frac{\pi}{2}\Phi_{RP-X_{\jmath}}\right)\right)^{w_{\jmath}} \\ +\iota\left(\prod_{\jmath=1}^{L+1} \left(\sin\left(\frac{\pi}{2}\Phi_{IP-X_{\jmath}}\right)\right)^{w_{\jmath}}\right), \\ -1+\prod_{\jmath=1}^{L+1} \left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{RN-X_{\jmath}}\right)\right)\right)^{w_{\jmath}} \\ +\iota\left(-1+\prod_{\jmath=1}^{L+1} \left(\sin\left(\frac{\pi}{2}\left(1+\Phi_{IN-X_{\jmath}}\right)\right)\right)^{w_{\jmath}}\right) \end{array} \right)$$

i.e., when $\underline{n} = L+1$, Eq. (13) is true. Thus, Eq. (13) is true for any $\underline{n}$.

We provide certain properties of the explored ST-BCFWG operator. The above-defined AO relies on the sine trigonometric function and satisfies the monotonicity, idempotency, and boundedness.

1. **Idempotency**: For any group of BCFNs, $X_{\jmath} = \left(\Phi_{P-X_{\jmath}}, \Phi_{N-X_{\jmath}}\right) = \left(\Phi_{RP-X_{\jmath}} + \iota\Phi_{IP-X_{\jmath}}, \Phi_{RN-X_{\jmath}} + \iota\,\Phi_{IN-X_{\jmath}}\right)$ $(\jmath = 1,2,3,\dots,\underline{n})$ if $X_{\jmath} = X$ for all $\jmath$, then

$$ST-BCFWG\left(X_1, X_2, \dots, X_{\underline{n}}\right) = \sin X$$

2. **Boundedness:** For any group of BCFNs, $X_{\jmath} = \left(\Phi_{P-X_{\jmath}}, \Phi_{N-X_{\jmath}}\right) = \left(\Phi_{RP-X_{\jmath}} + \iota\Phi_{IP-X_{\jmath}}, \Phi_{RN-X_{\jmath}} + \iota\,\Phi_{IN-X_{\jmath}}\right)$, let $X^{-} = \left(\min_{\jmath}\left\{\Phi_{RP-X_{\jmath}}\right\} + \iota\min_{\jmath}\left\{\Phi_{IP-X_{\jmath}}\right\}, \max_{\jmath}\left\{\Phi_{RN-X_{\jmath}}\right\} + \iota\max_{\jmath}\left\{\Phi_{IP-X_{\jmath}}\right\}\right)$, and $X^{+} = \left(\max_{\jmath}\left\{\Phi_{RP-X_{\jmath}}\right\} + \iota\max_{\jmath}\left\{\Phi_{IP-X_{\jmath}}\right\}, \min_{\jmath}\left\{\Phi_{RN-X_{\jmath}}\right\} + \iota\min_{\jmath}\left\{\Phi_{IP-X_{\jmath}}\right\}\right)$ then

$$\sin X^{-} \leq ST-BCFWG\left(X_1, X_2, \dots, X_{\underline{n}}\right) \leq \sin X^{+}$$

3. **Monotonicity:** For any two groups of BCFNs $X_{\jmath} = \left(\Phi_{P-X_{\jmath}}, \Phi_{N-X_{\jmath}}\right) = \left(\Phi_{RP-X_{\jmath}} + \iota\Phi_{IP-X_{\jmath}}, \Phi_{RN-X_{\jmath}} + \iota\,\Phi_{IN-X_{\jmath}}\right)$ and $X'_{\jmath} = \left(\Phi_{P-X'_{\jmath}}, \Phi_{N-X'_{\jmath}}\right) = \left(\Phi_{RP-X'_{\jmath}} + \iota\Phi_{IP-X'_{\jmath}}, \Phi_{RN-X'_{\jmath}} + \iota\,\Phi_{IN-X'_{\jmath}}\right)$, $(\jmath = 1,2,\dots,\underline{n})$, if $\Phi_{RP-X_{\jmath}} \leq \Phi_{RP-X'_{\jmath}}$, $\Phi_{IP-X_{\jmath}} \leq \Phi_{IP-X'_{\jmath}}$, $\Phi_{RN-X_{\jmath}} \leq \Phi_{RN-X'_{\jmath}}$, $\Phi_{IN-X_{\jmath}} \leq \Phi_{IN-X'_{\jmath}}$ for all $\jmath$ then

$$ST-BCFWG\left(X_1, X_2, \dots, X_{\underline{n}}\right) \leq ST-BCFWG\left(X'_1, X'_2, \dots, X'_{\underline{n}}\right)$$

Definition 11: For any group of BCFNs, $X_{\jmath} = \left(\Phi_{P-X_{\jmath}}, \Phi_{N-X_{\jmath}}\right) = \left(\Phi_{RP-X_{\jmath}} + \iota\Phi_{IP-X_{\jmath}}, \Phi_{RN-X_{\jmath}} + \iota\,\Phi_{IN-X_{\jmath}}\right)$ $(\jmath = 1,2,3,\dots,\underline{n})$ the sine trigonometric ordered weighted geometric AO for the group of BCFNs is identified and defined as

$$ST-BCFOWG\left(X_1, X_2, X_3, \dots, X_{\underline{n}}\right) = \left(\sin X_{\mathfrak{T}(1)}\right)^{w_1} \oplus \left(\sin X_{\mathfrak{T}(2)}\right)^{w_2} \oplus \dots \oplus \left(\sin X_{\mathfrak{T}(\underline{n})}\right)^{w_{\underline{n}}} \quad (14)$$

And which is called ST-BCFOWA operator. Where, $(\mathfrak{T}(1), \mathfrak{T}(2), \mathfrak{T}(3), \dots, \mathfrak{T}(\underline{n}))$ is a permutation of $(1, 2, \dots, \underline{n})$ such that $X_{\mathfrak{T}(j-1)} \geq X_{\mathfrak{T}(j)} \ \forall\, j$

Theorem 10: By employing the ST-BCFOWG operator, we get again the BCFN and

$$ST - BCFOWG(X_1, X_2, X_3, \dots, X_{\underline{n}}) = \begin{pmatrix} \prod_{j=1}^{\underline{n}} \left(\sin\left(\frac{\pi}{2}\Phi_{RP-X_{\mathfrak{T}(J)}}\right)\right)^{\mathbb{w}_j} \\ +\iota\left(\prod_{j=1}^{\underline{n}} \left(\sin\left(\frac{\pi}{2}\Phi_{IP-X_{\mathfrak{T}(J)}}\right)\right)^{\mathbb{w}_j}\right), \\ -1 + \prod_{j=1}^{\underline{n}} \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{RN-X_{\mathfrak{T}(J)}}\right)\right)\right)^{\mathbb{w}_j} \\ +\iota\left(-1 + \prod_{j=1}^{\underline{n}} \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{IN-X_{\mathfrak{T}(J)}}\right)\right)\right)^{\mathbb{w}_j}\right) \end{pmatrix} \quad (15)$$

Likewise, the ST-BCFWG operator, ST-BCFOWG satisfies the monotonicity, idempotency, and boundedness.

1. **Idempotency**: For any group of BCFNs, $X_j = \left(\Phi_{P-X_j}, \Phi_{N-X_j}\right) = \left(\Phi_{RP-X_j} + \iota\Phi_{IP-X_j}, \Phi_{RN-X_j} + \iota\,\Phi_{IN-X_j}\right)$ $(j = 1, 2, 3, \dots, \underline{n})$ if $X_j = X$ for all j, then

$$ST - BCFOWG(X_1, X_2, \dots, X_{\underline{n}}) = \sin X$$

2. **Boundedness:** For any group of BCFNs, $X_j = \left(\Phi_{P-X_j}, \Phi_{N-X_j}\right) = \left(\Phi_{RP-X_j} + \iota\Phi_{IP-X_j}, \Phi_{RN-X_j} + \iota\,\Phi_{IN-X_j}\right)$, let $X^- = \left(\min_j\left\{\Phi_{RP-X_j}\right\} + \iota\min_j\left\{\Phi_{IP-X_j}\right\}, \max_j\left\{\Phi_{RN-X_j}\right\} + \iota\max_j\left\{\Phi_{IP-X_j}\right\}\right)$, and $X^+ = \left(\max_j\left\{\Phi_{RP-X_j}\right\} + \iota\max_j\left\{\Phi_{IP-X_j}\right\}, \min_j\left\{\Phi_{RN-X_j}\right\} + \iota\min_j\left\{\Phi_{IP-X_j}\right\}\right)$ then

$$\sin X^- \leq ST - BCFOWG(X_1, X_2, \dots, X_{\underline{n}}) \leq \sin X^+$$

3. **Monotonicity:** For any two groups of BCFNs $X_j = \left(\Phi_{P-X_j}, \Phi_{N-X_j}\right) = \left(\Phi_{RP-X_j} + \iota\Phi_{IP-X_j}, \Phi_{RN-X_j} + \iota\,\Phi_{IN-X_j}\right)$ and $X'_j = \left(\Phi_{P-X'_j}, \Phi_{N-X'_j}\right) = \left(\Phi_{RP-X'_j} + \iota\Phi_{IP-X'_j}, \Phi_{RN-X'_j} + \iota\,\Phi_{IN-X'_j}\right)$, $(j = 1, 2, \dots, \underline{n})$, if $\Phi_{RP-X_j} \leq \Phi_{RP-X'_j}$, $\Phi_{IP-X_j} \leq \Phi_{IP-X'_j}$, $\Phi_{RN-X_j} \leq \Phi_{RN-X'_j}$, $\Phi_{IN-X_j} \leq \Phi_{IN-X'_j}$ for all j then

$$ST - BCFOWG(X_1, X_2, \dots, X_{\underline{n}}) \leq ST - BCFOWG(X'_1, X'_2, \dots, X'_{\underline{n}})$$

Definition 12: For any group of BCFNs, $X_j = \left(\Phi_{P-X_j}, \Phi_{N-X_j}\right) = \left(\Phi_{RP-X_j} + \iota\Phi_{IP-X_j}, \Phi_{RN-X_j} + \iota\,\Phi_{IN-X_j}\right)$ $(j = 1, 2, 3, \dots, \underline{n})$ the sine trigonometric hybrid geometric AO for the group of BCFNs is identified and defined as

$$ST - BCFHG(X_1, X_2, X_3, \dots, X_{\underline{n}}) = \left(\sin \mathrm{X}^*_{\mathfrak{T}(1)}\right)^{\mathfrak{P}_1} \oplus \left(\sin \mathrm{X}^*_{\mathfrak{T}(2)}\right)^{\mathfrak{P}_2} \oplus \dots \oplus \left(\sin \mathrm{X}^*_{\mathfrak{T}(\underline{n})}\right)^{\mathfrak{P}_{\underline{n}}} \quad (16)$$

And which is called ST-BCFHA operator. Where, $\mathrm{X}^*_{j} = \underline{n}\mathbb{w}_{j}\mathrm{X}_{j}$ $(\mathfrak{T}(1), \mathfrak{T}(2), \mathfrak{T}(3), \dots, \mathfrak{T}(\underline{n}))$ is a permutation of $(1, 2, \dots, \underline{n})$ such that $\mathrm{X}_{\mathfrak{T}(j-1)} \geq \mathrm{X}_{\mathfrak{T}(j)}\ \forall\, j$

Theorem 11: By employing the ST-BCFHG operator, we get again the BCFN and

$$ST - BCFHG(X_1, X_2, X_3, \dots, X_{\underline{n}}) = \begin{pmatrix} \prod_{j=1}^{\underline{n}} \left(\sin\left(\frac{\pi}{2}\Phi_{RP-\mathrm{X}^*_{\mathfrak{T}(J)}}\right)\right)^{\mathfrak{P}_{j}} \\ +\iota\left(\prod_{j=1}^{\underline{n}} \left(\sin\left(\frac{\pi}{2}\Phi_{IP-\mathrm{X}^*_{\mathfrak{T}(J)}}\right)\right)^{\mathfrak{P}_{j}}\right), \\ -1 + \prod_{j=1}^{\underline{n}} \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{RN-\mathrm{X}^*_{\mathfrak{T}(J)}}\right)\right)\right)^{\mathfrak{P}_{j}} \\ +\iota\left(-1 + \prod_{j=1}^{\underline{n}} \left(\sin\left(\frac{\pi}{2}\left(1 + \Phi_{IN-\mathrm{X}^*_{\mathfrak{T}(J)}}\right)\right)\right)^{\mathfrak{P}_{j}}\right) \end{pmatrix} \quad (17)$$

6 | APPLICATIONS

The word called hepatitis interprets liver inflammation. There are so many reasons for live inflammation such as viruses, alcohol chemicals, etc. Contingent upon its progress, hepatitis can be intense, which erupts unexpectedly and afterward disappears, or constant which is a drawn-out condition normally creating more unpretentious side effects and moderate liver harm. There are 5 basic classes of hepatitis which are hepatitis A, hepatitis B, hepatitis C, hepatitis D, and hepatitis E. Their detailed discussion is given below.

Hepatitis A: It is caused by the hepatitis A virus and it is an intense, brief-time disease. But in hepatitis A if a person faces discomfort, then bed rest would be needed. For sustaining the nutrition and hydration a doctor can suggest a dietary program in case of vomiting or diarrhea.

Hepatitis B: The cause of hepatitis B is the hepatitis B virus. For treating hepatitis B there is no such treatment but if one is facing chronic hepatitis B, then he may need antiviral medications. It would be a costly treatment if one continues it for months.

Hepatitis C: The cause of hepatitis C is the hepatitis C virus. The critical and lasting shapes of hepatitis C are treatable with antiviral medications. Usually, people with lasting hepatitis C employ a fusion of therapies of antiviral drugs. For the finest treatment, they require to do more medical tests. If someone faces liver disease because of hepatitis C then he may need a liver transplant.

Hepatitis D: It is a unique type of hepatitis that merely happens in combination with the infection of hepatitis B. The pegylated interferon alpha is the finest treatment for D according to WHO. However, there are numerous side effects of this treatment. Thus, this treatment is not ideal for people with liver damage, psychiatric conditions, and autoimmune diseases.

Hepatitis E: It is fundamentally tracked sown in regions with unfortunate sterilization and regularly comes about because of ingesting feces that pollute the water supply. The infection of hepatitis E usually settles on its own and presently, there is no particular medical treatment for hepatitis E.

Our main goal in this study is to examine hepatitis and various kinds of hepatitis by employing the interpreted sin operators. To achieve our goal, we need a DM technique which we as below.

6.1 | MADM technique under BCFS

For making a DM technique, consider the class of ņ-alternatives i.e., $X = \{X_1, X_2, \dots, X_{ņ}\}$ along with m-attributes $\mathbb{E} = \{\mathbb{E}_1, \mathbb{E}_2, \dots, \mathbb{E}_{ņ}\}$. For giving preference or significance to the alternatives, consider the weight vector $\mathbb{w}_j = (\mathbb{w}_1, \mathbb{w}_2, \dots, \mathbb{w}_{ņ})^T$ such that $\sum_{j=1}^{ņ} \mathbb{w}_j = 1$ and $\mathbb{w}_j \in [0,1]\ \forall\, j$. Now the expert will provide data on the structure of the BCF setting and make a DM matrix that would contain BCFNs. i.e. $X = (\Phi_{P-X}(\lambda), \Phi_{N-X}(\lambda)) = (\Phi_{RP-X}(\lambda) + \iota\Phi_{IP-X}(\lambda), \Phi_{RN-X}(\lambda) + \iota\, \Phi_{IN-X}(\lambda))$, where $\Phi_{P-X}(\lambda) = \Phi_{RP-X}(\lambda) + \iota\, \Phi_{IP-X}(\lambda)$ and $\Phi_{N-X}(\lambda) = \Phi_{RN-X}(\lambda) + \iota\, \Phi_{IN-X}(\lambda)$ with $\Phi_{RP-X}(\lambda), \Phi_{IP-X}(\lambda) \in [0,1]$ and $\Phi_{RN-X}(\lambda), \Phi_{IN-X}(\lambda) \in [-1,0]$. After that, this procedure would be followed by the following four stages

Stage 1: In daily life, the experts consider two sorts of information which are benefit and cost types. If the information is the benefits kind, then no one needs to normalize the DM matrix, otherwise, the DM matrix must be normalized by the following formula.

$$\mathfrak{N} = \begin{cases} (\Phi_{P-X}(\lambda), \Phi_{N-X}(\lambda)) & for\ beņefit \\ (\Phi_{P-X}(\lambda), \Phi_{N-X}(\lambda))^c & for\ cost \end{cases}$$

Stage 2: In this stage, the data given in the DM matrix would be aggregated by diagnosed ST-BCFWA, ST-BCFOWA, ST-BCFWG, and ST-BCFOWG operators.

Stage 3: The SVs of the aggregated values will be determined in this stage. If the SVs of any two aggregated values are the same then the accuracy value would be determined by both aggregated values.

Stage 4: The ranking order would be obtained in this stage by observing the SVs achieved in stage 3. The finest alternative would be finalized by ranking order.

To display the applicability of the diagnosis technique and examine hepatitis and its numerous types we are going to consider the following numerical example.

6.2 | Numerical Example

Consider four kinds of hepatitis i.e. $X_1 = hepatitis\ A$, $X_2 = hepatitis\ B$, $X_3 = hepatitis\ C$, and $X_4 = hepatitis\ D$ with four symptoms or attributes i.e. $\mathbb{E}_1 = yellow\ skiņ\ aņd\ eyes$, $\mathbb{E}_2 = fatigue$, $\mathbb{E}_3 = abdomiņal\ paiņ$, and $\mathbb{E}_4 = loss\ of\ appetite$ and for giving preference or significance to the alternatives, consider the weight vector $\mathbb{w} = (0.23, 0.32, 0.15, 0.3)$. Here, we are going to find the most viral type of hepatitis. Thus, we employ the diagnosed DM technique and operators to achieve our required result. The DM matrix containing information which is in the setting of BCF sets related to numerous types of hepatitis is displayed in Table 1.

Table 1. The assessment values of considered types of hepatitis.

	$\mathbb{E}_1$	$\mathbb{E}_2$	$\mathbb{E}_3$	$\mathbb{E}_4$
X_1	$\begin{pmatrix}0.18+\iota 0.28,\\-0.4-\iota 0.19\end{pmatrix}$	$\begin{pmatrix}0.78+\iota 0.79,\\-0.36-\iota 0.35\end{pmatrix}$	$\begin{pmatrix}0.49+\iota 0.61,\\-0.26-\iota 0.31\end{pmatrix}$	$\begin{pmatrix}0.25+\iota 0.57,\\-0.17-\iota 0.76\end{pmatrix}$
X_2	$\begin{pmatrix}0.28+\iota 0.92,\\-0.06-\iota 0.11\end{pmatrix}$	$\begin{pmatrix}0.67+\iota 0.66,\\-0.44-\iota 0.45\end{pmatrix}$	$\begin{pmatrix}0.25+\iota 0.16,\\-0.15-\iota 0.07\end{pmatrix}$	$\begin{pmatrix}0.27+\iota 0.55,\\-0.29-\iota 0.67\end{pmatrix}$
X_3	$\begin{pmatrix}0.39+\iota 0.20,\\-0.7-\iota 0.19\end{pmatrix}$	$\begin{pmatrix}0.17+\iota 0.27,\\-0.45-\iota 0.34\end{pmatrix}$	$\begin{pmatrix}0.45+\iota 0.3,\\-0.3-\iota 0.15\end{pmatrix}$	$\begin{pmatrix}0.29+\iota 0.57,\\-0.37-\iota 0.53\end{pmatrix}$
X_4	$\begin{pmatrix}0.8+\iota 0.7,\\-0.3-\iota 0.4\end{pmatrix}$	$\begin{pmatrix}0.19+\iota 0.7,\\-0.6-\iota 0.5\end{pmatrix}$	$\begin{pmatrix}0.9+\iota 0.6,\\-0.6-\iota 0.3\end{pmatrix}$	$\begin{pmatrix}0.31+\iota 0.6,\\-0.6-\iota 0.62\end{pmatrix}$

Stage 1: Normalization is not required as all the data is the benefit type.

Stage 2: In this stage, we aggregated the data given in the DM matrix displayed in Table 2 by employing diagnosed ST-BCFWA, ST-BCFOWA, ST-BCFWG, and ST-BCFOWG operators as demonstrated in Table 2.

Table 2. The aggregated values were achieved by diagnosed ST-BCFWA, ST-BCFOWA, ST-BCFWG, and ST-BCFOWG operators.

Methods	X_1	X_2	X_3	X_4
ST-BCFWA	$\begin{pmatrix}0.728+\iota 0.83,\\-0.095-\iota 0.167\end{pmatrix}$	$\begin{pmatrix}0.635+\iota 0.891,\\-0.053-\iota 0.091\end{pmatrix}$	$\begin{pmatrix}0.465+\iota 0.551,\\-0.23-\iota 0.109\end{pmatrix}$	$\begin{pmatrix}0.808+\iota 0.859,\\-0.304-\iota 0.259\end{pmatrix}$
ST-BCFOWA	$\begin{pmatrix}0.699+\iota 0.8,\\-0.108-\iota 0.119\end{pmatrix}$	$\begin{pmatrix}0.635+\iota 0.891,\\-0.053-\iota 0.091\end{pmatrix}$	$\begin{pmatrix}0.476+\iota 0.568,\\-0.201-\iota 0.107\end{pmatrix}$	$\begin{pmatrix}0.904+\iota 0.845,\\-0.304-\iota 0.219\end{pmatrix}$
ST-BCFWG	$\begin{pmatrix}0.519+\iota 0.727,\\-0.119-\iota 0.316\end{pmatrix}$	$\begin{pmatrix}0.521+\iota 0.711,\\-0.114-\iota 0.261\end{pmatrix}$	$\begin{pmatrix}0.421+\iota 0.474,\\-0.289-\iota 0.166\end{pmatrix}$	$\begin{pmatrix}0.531+\iota 0.853,\\-0.353-\iota 0.295\end{pmatrix}$
ST-BCFOWG	$\begin{pmatrix}0.518+\iota 0.69,\\-0.127-\iota 0.213\end{pmatrix}$	$\begin{pmatrix}0.521+\iota 0.711,\\-0.114-\iota 0.261\end{pmatrix}$	$\begin{pmatrix}0.429+\iota 0.495,\\-0.248-\iota 0.169\end{pmatrix}$	$\begin{pmatrix}0.652+\iota 0.839,\\-0.353-\iota 0.267\end{pmatrix}$

Stage 3: The SVs of the aggregated values have been determined in this stage and the required outcomes are demonstrated in Table 3.

Table 3. The SVs of the aggregated values.

Methods	$\mathcal{S}_{SF}(X_1)$	$\mathcal{S}_{SF}(X_2)$	$\mathcal{S}_{SF}(X_3)$	$\mathcal{S}_{SF}(X_4)$	Ranking
ST-BCFWA	0.824	0.456	0.669	0.776	$X_1 > X_4 > X_3 > X_2$
ST-BCFOWA	0.818	0.845	0.684	0.807	$X_2 > X_1 > X_4 > X_3$
ST-BCFWG	0.702	0.714	0.61	0.684	$X_2 > X_1 > X_4 > X_3$
ST-BCFOWG	0.717	0.714	0.627	0.718	$X_4 > X_1 > X_2 > X_3$

Stage 4: The ranking order is obtained in this stage by observing the SVs achieved in stage 3.

Table 4. The ranking order of SVs.

Methods	Ranking
ST-BCFWA	$X_1 > X_4 > X_3 > X_2$
ST-BCFOWA	$X_2 > X_1 > X_4 > X_3$
ST-BCFWG	$X_2 > X_1 > X_4 > X_3$
ST-BCFOWG	$X_4 > X_1 > X_2 > X_3$

According to above Table 4, ST-BCFWA is giving us that. $X_1 = hepatitis\ A$ is the most viral hepatitis and ST-BCFOWA, ST-BCFWG are giving us that $X_2 = hepatitis\ B$ is the most viral type of hepatitis while ST-BCFOWG is giving us that $X_4 = hepatitis\ D$ is the most viral type of hepatitis.

7 | COMPARISON

To show the worth and capacity of the proposed operators and approach, we are going to compare the diagnosed operators and technique with a few prevailing notions diagnosed by [29], [30], [36], [37], [43], [44], [39], [40]. We will discuss them one by one in detail as follows.

1. The DM technique was diagnosed based on Hamacher AOs (HAOs) for BF sets [29]. The technique and operators described in [29]are not applicable to cope with the information involving the second dimension. Thus, the data displayed in Table 2 can’t be handled by [29], but our technique and operators can effectively manage BF data.
2. The DM technique based on Dombi AOs (DAOs) for BF sets was diagnosed in [30]. The technique and operators described in [30] are also not applicable to cope with the information involving the second dimension. Thus, the data displayed in Table 1 can’t be handled by [30], but our technique and operators can effectively manage BF data.
3. A MADM approach using Aczel-Alsina power AOs within the framework of BFS was devised in [43]. However, this framework and MADM approach cannot handle the information containing the second dimension, so the BCF information cannot be addressed by this theory.
4. [44] interpreted a DM approach within BFS by employing Yager AOs. But this framework and MADM approach also can’t handle the information containing the 2nd dimension, thus the BCF information can’t be handled by this theory.
5. Arithmetic AOs (AAOs) were diagnosed in the context of CF sets in [36]. The structure of CF sets is not rich enough to handle negative human opinions; thus, the operators discussed in [36] cannot manage data with negative opinions. However, the proposed technique and operators can effectively address CF data by ignoring the negative opinions.
6. Geometric AOs (GAOs) were diagnosed in the context of CF sets in [37]. The structure of CF sets is not rich enough to handle negative human opinions; thus, the operators discussed in [37]cannot manage data with negative opinions. However, the proposed technique and operators can effectively address CF data by ignoring the negative opinions.
7. HAOs for BCF sets and the DM technique were interpreted in [39], while DAOs for BCF sets and the DM technique were explored in [40]. The methods described in [39] and [40] solved the data displayed in Table 2, and the required outcomes are shown in Table 5. Table 1. The SVs were obtained by a diagnosed approach and a few prevailing approaches.

Method	$\mathcal{S}_{\mathcal{S}F}(X_1)$	$\mathcal{S}_{\mathcal{S}F}(X_2)$	$\mathcal{S}_{\mathcal{S}F}(X_3)$	$\mathcal{S}_{\mathcal{S}F}(X_4)$	Ranking
Wei et al. [29]	⤫↣⤫	⤫↣⤫	⤫↣⤫	⤫↣⤫	⤫↣⤫
Jana et al. [30]	⤫↣⤫	⤫↣⤫	⤫↣⤫	⤫↣⤫	⤫↣⤫
Garg et al. [43]	⤫↣⤫	⤫↣⤫	⤫↣⤫	⤫↣⤫	⤫↣⤫
Rehman and Mahmood [44]	⤫↣⤫	⤫↣⤫	⤫↣⤫	⤫↣⤫	⤫↣⤫
Bi et al. [36]	⤫↣⤫	⤫↣⤫	⤫↣⤫	⤫↣⤫	⤫↣⤫
Bi et al. [37]	⤫↣⤫	⤫↣⤫	⤫↣⤫	⤫↣⤫	⤫↣⤫
Mahmood et al. [39]	0.619	0.661	0.483	0.567	$X_2 > X_1 > X_4 > X_3$
Mahmood et al. [39]	0.381	0.339	0.517	0.433	$X_3 > X_4 > X_1 > X_2$
Mahmood and Ur Rehman [40]	0.663	0.731	0.502	0.608	$X_2 > X_1 > X_4 > X_3$
Mahmood and Ur Rehman [40]	0.281	0.248	0.425	0.278	$X_3 > X_4 > X_1 > X_2$
ST-BCFWA	0.824	0.456	0.669	0.776	$X_1 > X_4 > X_3 > X_2$
ST-BCFOWA	0.818	0.845	0.684	0.807	$X_2 > X_1 > X_4 > X_3$
ST-BCFWG	0.702	0.714	0.61	0.684	$X_2 > X_1 > X_4 > X_3$
ST-BCFOWG	0.717	0.714	0.627	0.718	$X_4 > X_1 > X_2 > X_3$

Also, the ranking and the outcomes achieved by the diagnose operators and technique are displayed in Table 5. Most techniques give that $X_2 = hepatitis\ B$ is the viral type of hepatitis. Thus, the propounded technique and operators are applicable and stable for the DM.

8 | CONCLUSION

This study has contributed to the evaluation of the risk of hepatitis in a new approach by using BCFSs and sine trigonometric AOs. We first introduced ST-BCFSs and then developed the STOLs which form the foundation of the proposed work. According to this framework, we proposed a set of STAOs including ST-BCFWA, ST-BCFOWA, ST-BCFHA, ST-BCFWG, ST-BCFOWG, and ST-BCFHG operators to provide a brief in the case of BCFSs. Our study has demonstrated that this approach is useful for measuring the complexity of hepatitis risk factors where bipolarity and the second dimension of BCFSs can reflect the positive and negative impact of attributes, time, and intensity. In this paper, we demonstrated the use of the proposed DM technique by solving a real-life problem, which involved identifying the most viral type of hepatitis out of the four types: A, B, C, and D using a numerical example. Finally, the comparison with the existing methods was made and it was pointed out that our method is more suitable for modeling the intricacies and stochastic nature of the hepatitis risk assessment. In this study, we have incorporated the symmetric and periodic properties of sine trigonometric functions with multidimensional properties of BCFSs to develop a new model that can be more effective and accurate in the medical DM processes.

In the future, we aim to consider some mathematical frameworks such as BCF soft set [45], complex hesitant FS [46], and hesitant BCFS [47] to discuss the risk evaluation of hepatitis.

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