

Magnetic Field Effects on Second-Grade Non-Newtonian Ferrofluids Flow Over Stretching Surfaces

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ABSTRACT

The current article aims to investigate how the flow of ferromagnetic non-Newtonian fluid is affected by elastic deformation when an external magnetic field is present on a stretching sheet. To achieve this target, we have converted the governing Partial Differential Equations (PDEs) into Ordinary Differential Equations (ODEs) by using suitable similarity transformations. The role of pertinent flow parameters like second-grade fluid parameters, the coefficient of elastic deformation, Prandtl number, and viscoelastic parameters on velocity, pressure, and temperature profiles are drawn pictorially and discussed in detail. Computations for skin friction number and local Nusselt number are also discussed graphically. Some highlights of the findings in terms of the influences of different parameters on velocity, temperature, and pressure profiles are given, and concluded that by raising the values of the second-grade parameter (K) and Ferrohydrodynamic interaction parameter (β) the velocity profiles raise. The velocity profile decreases by increasing the value of the Prandtl number Pr . Increasing the Prandtl number results in a significant decrease in temperature profiles. Also, the pressure increases with higher values of β and Pr . The findings from this research could be utilized in the development of advanced cooling systems for electronics, as well as optimization of processing magnetic materials in which better efficiency and quality of products can be reached by controlling the flow of ferromagnetic fluid under magnetic fields.

Keywords: Magnetic Dipole, Ferrofluid, Non-Newtonian Fluid, Curie Temperature, Magnetohydrodynamics (MHD)

(MSC2020) Codes: 76A05, 76W05

Table 1: Nomenclature

A	Distance	K	thermal conductivity
C	Constant	K_1	Constant
c_p	Specific heat	K	2 nd -grade parameter
e	constant	M	Magnetization
F	dimensionless stream function	Nu_x	Nusselt quantity
H	Magnetic field	P	pressure
P	dimensionless pressure	Pr	Prandtl number, $\frac{\mu c_p}{k}$
Re	local Reynolds number, $\frac{\rho c x^2}{\mu}$	T	temperature
X	coordinate along the sheet	v	velocity factor normal to the area
a	dimensionless distance	y	coordinate normal to the area
Y	Constant	u	velocity component along the area
η	dimensionless coordinate	ε	dimensionless Curie temperature
θ	dimensionless temperature	λ	viscid intemperance factor
μ_0	permeability	μ	vigorous viscidness
β	Ferrohydrodynamic interaction parameter	ξ	dimensionless coordinate

1 | INTRODUCTION

The investigation of fluid flow behavior in the presence of magnetic fields has been given great attention for its widespread applications across engineering and industry. One such unique group of fluids is ferrofluids which change their flow properties in the presence of an external magnetic field. These fluids are colloidal suspensions of ferromagnetic nanoparticles usually magnetite, which are suspended in a conventional carrier fluid (such as water or oil) and thus lose their magnetic properties when the fluid is at rest. Ferrofluids have the unique property of forming dipoles in the direction of magnetic field giving rise to changes in viscosity and flow behaviour properties.

The heat transfer characteristics of ferrofluid flow inside a micro fin tube under the influence of rotating magnetic fields were analyzed and found that rotating magnetic fields drastically improved the overall heat transfer efficiency in ferrofluid-based systems [1]. The external magnetic field significantly influences the displacement of ferrofluid particles, altering their movement and alignment within the fluid. This displacement enhances the thermal conductivity of the ferrofluid, promoting more efficient heat transfer [2]. The convective heat transfer of a ferrofluid in partially porous annulus geometry which involved interaction of magnetic fields and melting phenomena was investigated and discovered that forcing magnetic fields into the configurations increased both convective and melting heat transfer [3]. The effect of magneto diffusion on ferrofluids and nano-ferrofluids due to magnetic dipoles, particularly in the heat transfer phenomena and boundary layer flows has also been discussed [4], and observed that when the ferrofluid contained magnetic dipoles, it became a significantly better heat carrier than any

current thermal fluid. The slip effects of magnetized ferrofluid in the presence of radiative heating and magnetic forces, flowing through a shrinking/stretching surface were examined by [5]. It was shown that the heat transfer rate and the fluid velocity depended on non-constant slip factors. Some other important results related to ferrofluid can also be seen [6], [7], [8], [9].

A numerical investigation was conducted into the heat transfer characteristics of ferrofluid flow over a nonlinear stretching sheet situated in a porous medium. By modeling the complexities of ferrofluid behavior under varying conditions, valuable insights were provided into optimizing heat transfer [10]. The optimization of fluid flow can be effectively optimized by varying the magnetic field strength and the parameters associated with slip. This highlights the potential for applications in engineering and industrial processes where efficient heat transfer is crucial [11]. Researchers also investigated the influence of magnetic polarization and Stefan blowing on hybrid ferrofluid flow over a stretching sheet embedded in a porous medium [12]. The critical discovery that is made inside the essay signifies, that the addition of these attributes gives improved fluid manipulation and heat transfer. The unsteady flow of ferrofluid over an oscillatory stretching sheet with the influence of magnetic fields and heat source/sink has been discussed by [13].

The partial slip and suction effects of ferrofluid flow over an inclined stretching sheet were discussed by [14] and determined that the Nusselt number, which are found to have significant effects on heat transfer and fluid flow behaviour. Researchers have also examined radiation and permeability influences on ferrofluid flow past a stretching/shrinking sheet [15]. It is concluded that there were two dual solutions and radiation effects improve the heat transfer, making this a common solution in a more efficient system. The unsteady hybrid ferrofluid flow over a stretching sheet in the presence of Stefan blowing and magnetic polarization affects the fluid flow and heat transfer [12]. The influence of magnetohydrodynamics (MHD) within a porous medium has been an effective way to control the fluid motion whereas both the magnetic forces manage to improve its thermal efficiency. By optimizing the behavior of the fluid in response to the applied magnetic field, significant improvements in heat transfer can be achieved. This synergy between MHD and porous media is important in various engineering applications that require efficient thermal management. Numerous studies are found on MHD flow of couple stress fluid with the help of flat and curved plates [16].

Some recent studies have considered the magnetic field effects on ferrofluid over-stretching surfaces for non-Newtonian models like Casson, Carreau, and Power-law fluids [17]. These models are crucial in determining the flow behavior of complex fluids in practical applications. Because of flow stability and heat transfer, the impact of external factors such as magnetic fields, thermal radiation, and chemical reactions on non-Newtonian ferrofluids has been extensively examined [18],[19],[20]. It is investigated that, the magnetic fields were capable of increasing the heat transfer in non-Newtonian ferrofluid over-stretching sheets, so this can be useful for different industrial processes [21]. In addition, progress in the

numerical simulation approach has allowed a deeper study of ferrofluid response to magnetic fields [22]. Furthermore, the study of ferrofluid flow with thermal radiation investigated how it influenced heat transfer efficiency. The studies indicated that the coupling of thermal radiation and magnetic fields may be utilized for improving heat transfer in the presence of non-Newtonian ferrofluids in systems [23].

In this paper, the effects of a magnetic field on the flow of non-Newtonian ferrofluids over-stretching surfaces are explored. The motivation behind this work is the necessity for a comprehensive study, especially on how magnetic fields have an impact on flow dynamics, heat transfer, and stability of non-Newtonian ferrofluids. Thereby, the outcome of this study will lead to new designs in any industrial and biomedical field that has flow control dependency. The authors have stated the problem in terms of ordinary differential equations resulting from the complex set of partial differential equations (PDEs) by using similarity transformations. With this approach, the authors can investigate the behaviour of ferrofluids by different parameters. The method entails solving the ODEs numerically with the shooting method to show how different flow as well as thermal parameters affect velocity, temperature, and pressure profiles.

2 | FORMULATION OF PROBLEM

The two-dimensional steady incompressible flow past a horizontal and impervious flexible sheet is presented in Figure (1). In the direction of the x-axis by applying two opposite and equal forces, the sheet is stressed from the origin. The 2nd-grade fluid with Cauchy stress T has the form which is mentioned below equation:

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2. \quad (1)$$

Where,

$$A_1 = (\text{grad } \vec{v}) + (\text{grad } \vec{v})^T, \quad (2)$$

$$A_2 = \frac{dA_1}{dt} + A_1(\text{grad } \vec{v}) + (\text{grad } \vec{v})^T A_1. \quad (3)$$

In equation (1) the circular stress $-pI$ is due to the constriction of inelasticity μ denotes the viscidness, α_1 and α_2 denotes material moduli, d/dt represents the time derivative, v represents the velocity field, A_1 and A_2 are the first two Rivlin - Ericksen tensors.

Now we consider the flow of an inelastic, viscoelastic second-order fluid following the above equations (1) to (3) past a smooth sheet. With velocity $u(x) = cx$ the sheet is stretched. On perpendicular to the x-axis, we choose the y-axis and x-axis itself along the enlarging surface. On above the sheet $y > 0$, the viscous incompressible ferrofluids are kept where the magnetic dipole is placed below the sheet. Within below the Curie temperature the elongating sheet is retained at the temperature T_w , while the temperature of fluid $T = T_c$ is at a distance from the sheet.

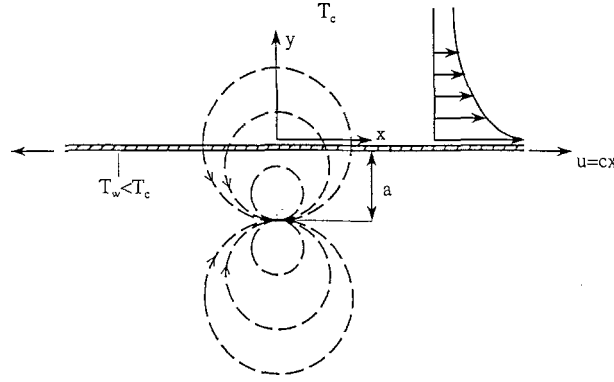


Figure 1: Geometric Illustration of Flow Problem

The equations of mass, momentum, and transport energy in a steady two dimension problem are follows as [24],

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (4)$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_o M \frac{\partial H}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \alpha_1 \left(\frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + u \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right), \quad (5)$$

$$c_p \rho \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) + \mu_o T \frac{\partial M}{\partial T} \left(u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \left[2 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right] - \sigma \alpha_1 \left(\frac{\partial u}{\partial y} \frac{\partial}{\partial y} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \right). \quad (6)$$

In these above Eqns. (4) - (6) u and v represent the velocity components, and the pressure field is represented by p . The appropriate boundary conditions along the sheet at $y = 0$ are [26]

$$u = cx, v = 0, T = T_w, \quad \text{at } y = 0 \quad (7)$$

$$u = 0, T = T_c, p + \frac{1}{2} \rho (u^2 + v^2) = \text{constant}, \quad \text{at } y \rightarrow \infty \quad (8)$$

The effect of a magnetic field on the flow of ferrofluids is given by [26],

$$\phi = \frac{\alpha}{2\pi} \frac{x}{x^2 + (y+a)^2}. \quad (9)$$

The components of the magnetic field $\bar{H}$ are as follows [26],

$$H_x = -\frac{\partial \phi}{\partial x} = \frac{Y}{2\pi} \frac{x^2 - (y+a)^2}{[x^2 + (y+a)^2]^2}, \quad (10)$$

$$H_y = -\frac{\partial \phi}{\partial y} = \frac{Y}{2\pi} \frac{2x(y+a)}{[x^2 + (y+a)^2]^2}. \quad (11)$$

The gradient of the magnitude of H which is proportional to the magnetic body force is as follows [26],

$$H = \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 \right]^{\frac{1}{2}} \quad (12)$$

After expanding the terms up to an order of x^2 . Let us consider that the applied magnetic field H is properly tough to preserve the ferrofluids. And the linear equation which denotes the deviation of magnetization is as follows [26],

$$\frac{\partial H}{\partial x} = -\frac{Y}{2\pi} \frac{2x}{(y+a)^4}, \quad (13)$$

$$\frac{\partial H}{\partial y} = \frac{Y}{2\pi} \left[-\frac{2}{(y+a)^3} + \frac{4x^2}{(y+a)^5} \right]. \quad (14)$$

$$M = K_1(T_c - T). \quad (15)$$

Now let us define the non-dimensional variables [26],

$$\psi(\xi, \eta) = \left(\frac{\mu}{\rho}\right) \xi \cdot f(\eta), \quad (16)$$

$$P(\xi, \eta) = \frac{p}{c\mu} = -P_1(\eta) - \xi^2 P_2(\eta), \quad (17)$$

$$\theta(\xi, \eta) = \frac{T - T_c}{T_c - T_w} = \theta_1(\eta) + \xi^2 \theta_2(\eta). \quad (18)$$

Equations (4), (5), and (6) have solutions of the form [26],

$$\eta = \left(\frac{c\rho}{\mu}\right)^{\frac{1}{2}} y, \xi = \left(\frac{c\rho}{\mu}\right)^{\frac{1}{2}} x, u = \frac{\partial \psi}{\partial y} = cx \cdot f'(\eta), v = -\frac{\partial \psi}{\partial x} = -\left(\frac{c\mu}{\rho}\right)^{\frac{1}{2}} \cdot f(\eta). \quad (19)$$

Where the prime signifies differentiation with respect to η . Confirmly u and v satisfied the continuity Eqn. (4). Given Eqns. (10) – (19) the above Eqns. (5) - (6) gives,

$$f''' + ff'' - (f')^2 + 2P_2 - \frac{2\beta\theta_1}{(\eta + \alpha)^4} + K(ff'' - f'^2 - ff''') = 0, \quad (20)$$

$$\theta_1'' + Pr f \theta_1' + \frac{2\lambda\beta(\theta_1 - \varepsilon)}{(\eta + \alpha)^3} f + 2\theta_2 - 4\lambda(f')^2 = 0, \quad (21)$$

$$\theta_2'' - Pr(2f'\theta_2 - f\theta_2') + \frac{2\lambda\beta\theta_2}{(\eta + \alpha)^3} f - \lambda\beta(\theta_1 - \varepsilon) \left[\frac{2f'}{(\eta + \alpha)^4} + \frac{4f}{(\eta + \alpha)^5} \right] - \lambda(f'')^2 - \lambda\delta K(f'f''^2 - ff''f''') = 0. \quad (22)$$

Satisfying the conditions

$$f=0, \quad \theta_1=1, \theta_2=0, f'=1. \quad \text{at } \eta=0 \quad (23)$$

$$\theta_1 \rightarrow 1, \theta_2 \rightarrow 0, P_1 \rightarrow -P_\infty, P_2 \rightarrow 0, f' \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (24)$$

Where $Pr = \frac{\mu c_p}{k}$ is the Prandtl number,

$K = \frac{\alpha_1 c}{\mu}$ is second-grade parameter

$\lambda = \frac{c\mu^2}{k\rho(T_c - T_w)}$ is the viscous dissipation parameter,

$\varepsilon = \frac{T_c}{(T_c - T_w)}$ is the dimensionless Curie temperature,

$\beta = \left(\frac{Y}{2\pi}\right) \frac{\mu_0 K (T_c - T_w) \rho}{\mu^2}$ is the Ferro hydrodynamic interaction parameter,

$\alpha = \left(\frac{c\rho}{\mu}\right)^{\frac{1}{2}}$ a is the dimensionless distance,

$\delta K = \frac{\sigma \alpha_1 c}{\mu}$ is the product of elastic deformation and second-grade parameters.

3 | NUMERICAL EFFECTS AND DISCUSSION

The set of non-linear ODEs from Eqns. (20) – (22) have been solved numerically by using the standard shooting method with the help of boundary conditions (23) – (24). The calculations were done by the program bvph 2.0 by using MATLAB. Boundary value problem solver BVPh 2.0, a MATLAB program that solves boundary value problems (BVPs) for ordinary differential equations (ODE). In this work shooting method was performed with BVPh 2.0. The validation is usually done by verifying the solution convergence and comparing the numerical results to previously validated numerical solutions. The results were validated in the study by comparing against suitable references, and the method showed consistent trends to previous work under different parameters, yielding physically sound profiles for velocity fields/pressures/temperature distributions.

The effect of different parameters is discussed with the help of figures. For the fixed parameters in the system, we assigned numerical values to the flow structure for velocity, pressure, and temperature profiles. The impact of an external magnetic field is established over the ferromagnetic interaction parameter β . The existence of magnetic field performance the fluctuation force on the velocity profile. Figure (2) – (15) show the effect of different inserted flow parameters β , λ , ε , α , Pr , K , δ , f' and θ . The value of $\lambda=0.01$, $\varepsilon=0.1$, and $\alpha=2$ is fixed throughout the whole discussion. The effect of K , δ , β , and Pr on velocity sketch f' are presented in figures (2) to (5). In figure (2) and (3) the velocity f' rises when enhancing the value of K and β respectively, while in figures (4) and (5) the fluid velocity declines for the increment in the value of δ and Pr respectively because, for a larger value of Prandtl number, the fluid has comparatively lower thermal conductivity. We observed that the velocity is increased when a higher value of the second-grade fluid parameter (K) and ferromagnetic interaction parameter (β). Where, the increasing viscous potential of flow, means that the more fluidity of liquid and strength field intensity causes a high rate of flow. Velocity rises with the parameter K and because of ferromagnetic interaction as β respectively as second-grade fluid up to portion point, from where it reduces for bigger and β values. On the other hand, the velocity decreases with increasing Prandtl number (Pr) and elastic

deformation/second-grade parameter product (δ), reflecting a reduction in flow speed due to lower thermal conductivity and stronger elastic resistance.

Figures (6) - (9) show the effect of K , δ , β , and Pr on the temperature profile respectively. From the figures it is perceived that the temperature profile is progressively decreasing for all embedded parameter values K , δ , β , and Pr . The reason is that the temperature field drops because the Curie temperature loses the original magnetic behavior. Increasing the Prandtl number results in a significant decrease in temperature profiles, with a reduction in heat transfer for higher Pr values due to lower thermal diffusivity.

Figures (10) – (13) show the effect of K , δ , β , and Pr on the pressure profile correspondingly. From Figure (10) it's clear that the pressure falls for the value of K and rises for other parameters δ , β and Pr . When the parameter values β , δ , and Pr are larger, the pressure increases due to the interactive magnetic body force, large elastic deformation, and higher thermal resistance respectively. Conversely, as K increases the pressure decreases, implying that the nature of fluid becomes more elastic and therefore causes lesser pressure delivered within the system.

Now we discuss the skin friction and Nusselt quantity. From Figure (14) it can be concluded that the skin friction quantity with the variation of 2nd grade fluid factor K for the different values of Prandtl number, enhancing with the rising value of K . In Figure (15) it is shown that the larger the value of β , the larger the wall temperature this is because the temperature gradient is negative for all values of the β . As the fluid with a larger Prandtl number has higher heat capacity so therefore heat transfer rate increases. This is why, heat flows from the wall.

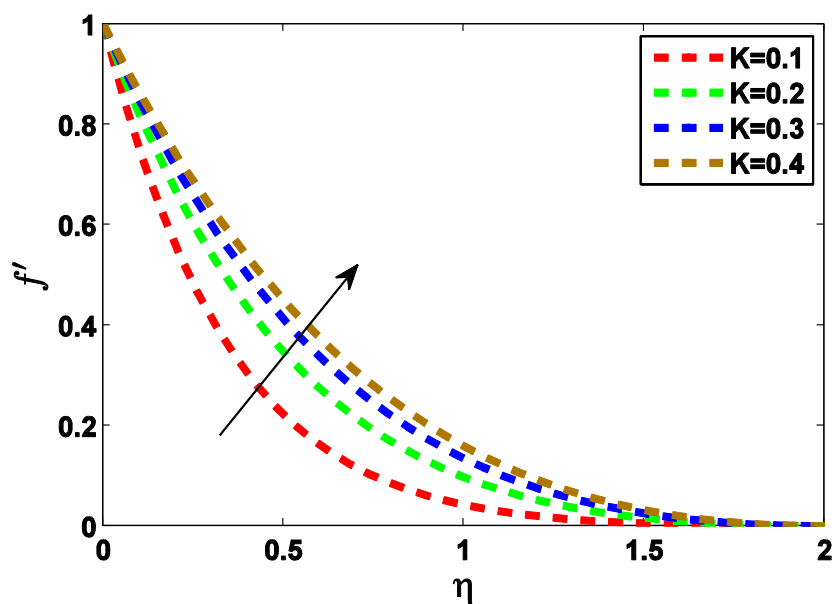


Figure 2: Effect of K on the Velocity Profile for Fixed Values of $\beta = 0$, $Pr = 0.7$, and $\delta = 4$

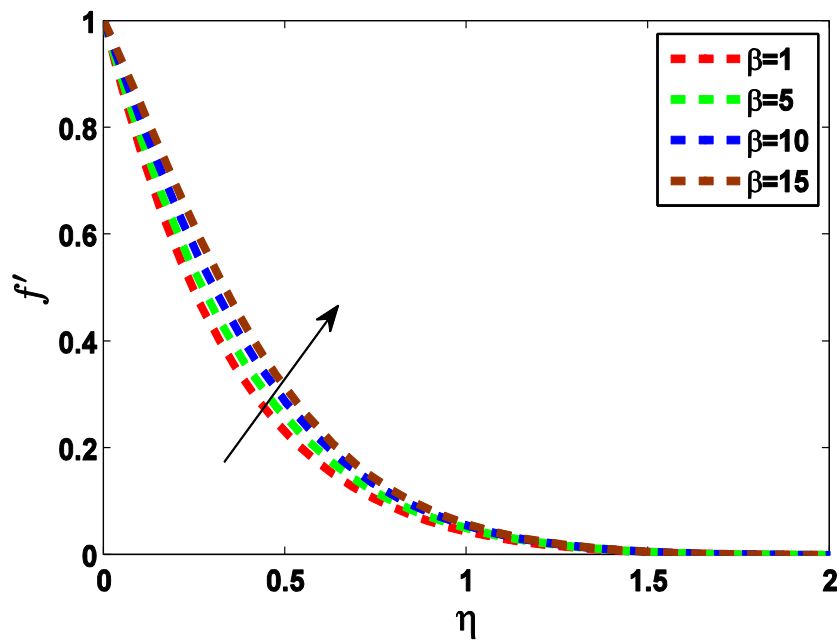


Figure 3: Effect of β on Velocity Profile for Fixed Value of $Pr=0.7$, $K=0.1$ and $\delta=4$

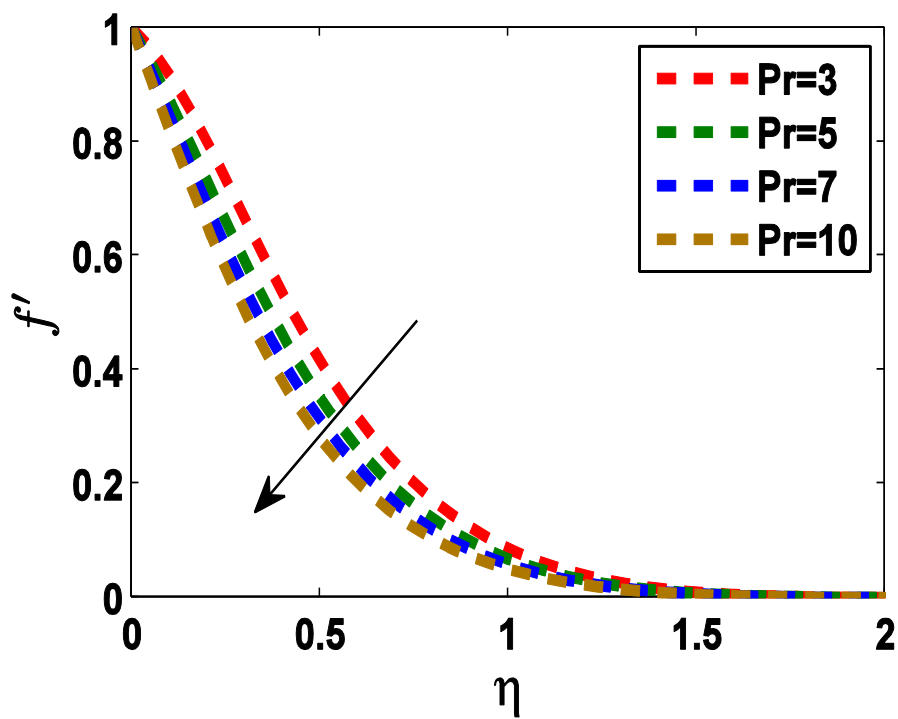


Figure 4: Effect of Pr on Velocity Profile for Fixed Value of $\beta=0$, and $\delta=4$

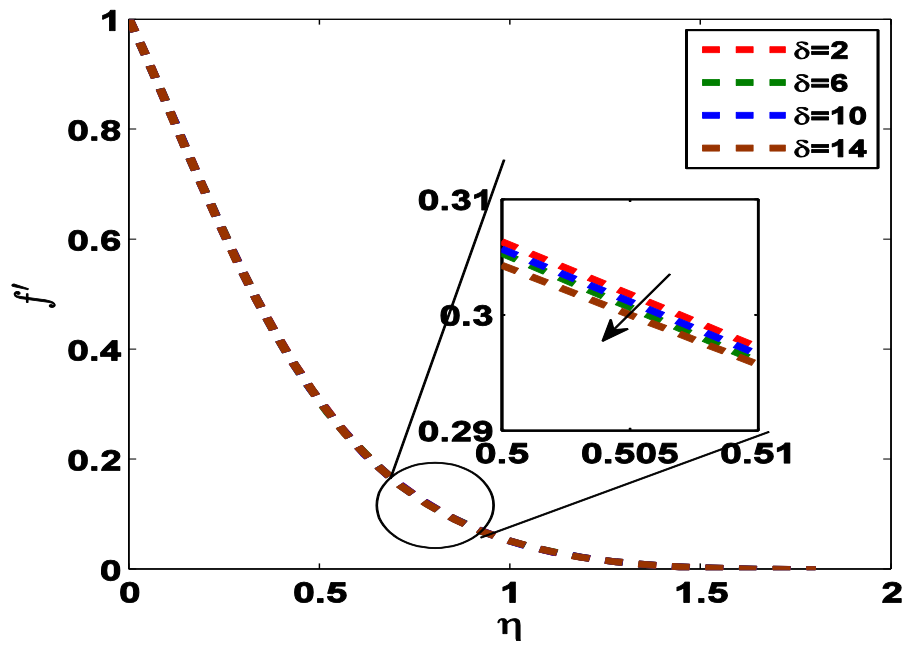


Figure 5: Influence of δ on Velocity Profile for Fixed Value of $K = 0.1$, $\beta = 0$, $Pr = 0.7$

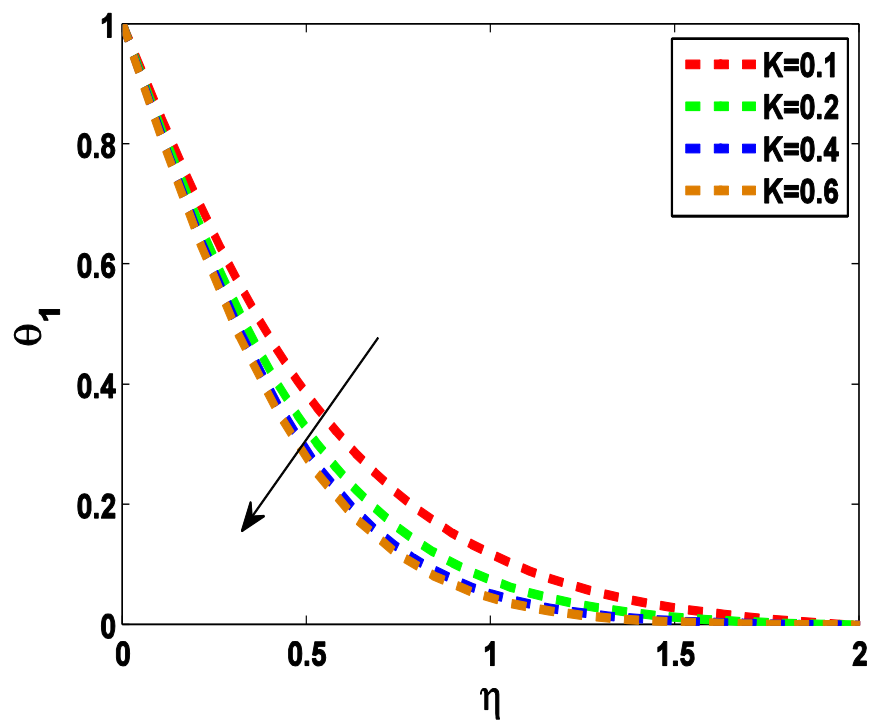


Figure 6: Effect of K on Temperature profile for a fixed value of $\beta = 0$, $Pr = 0.7$ and $\delta = 4$

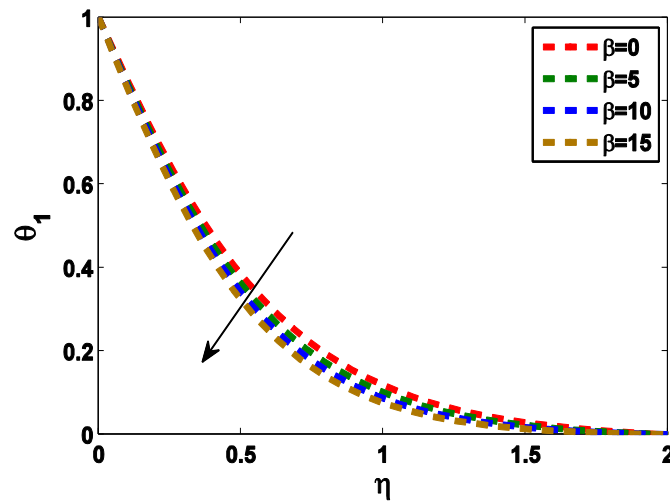


Figure 7: Effect of β on the Temperature profile for a fixed value of $Pr=0.7$, $K=0.1$ and $\delta=4$

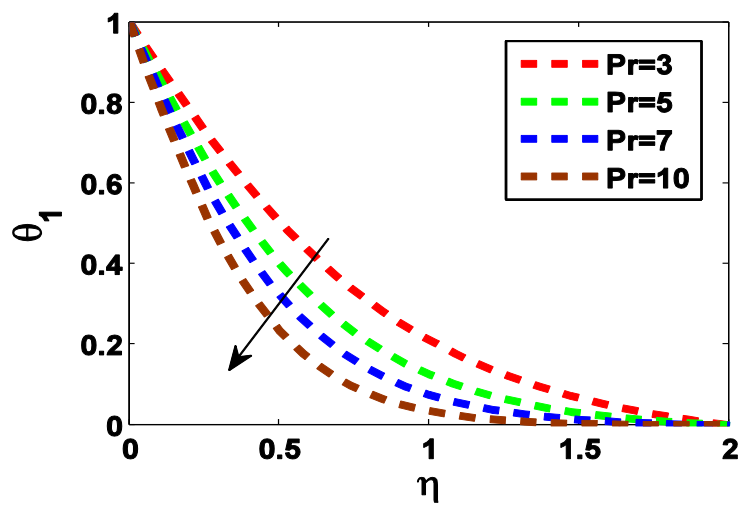


Figure 8: Effect of Pr on Temperature Profile for Fixed Value of $K=0.1$, $\beta=0$, and $\delta=4$

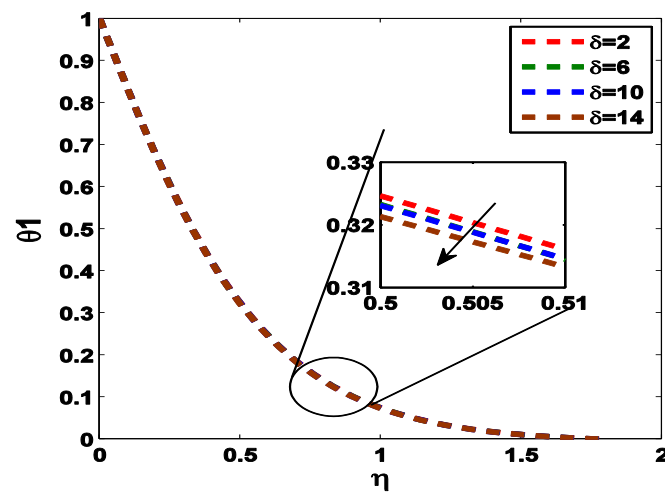


Figure 7: Effect of δ on Temperature Profile for Fixed Value of $K=0.1$, $\beta=0$, $Pr=0.7$

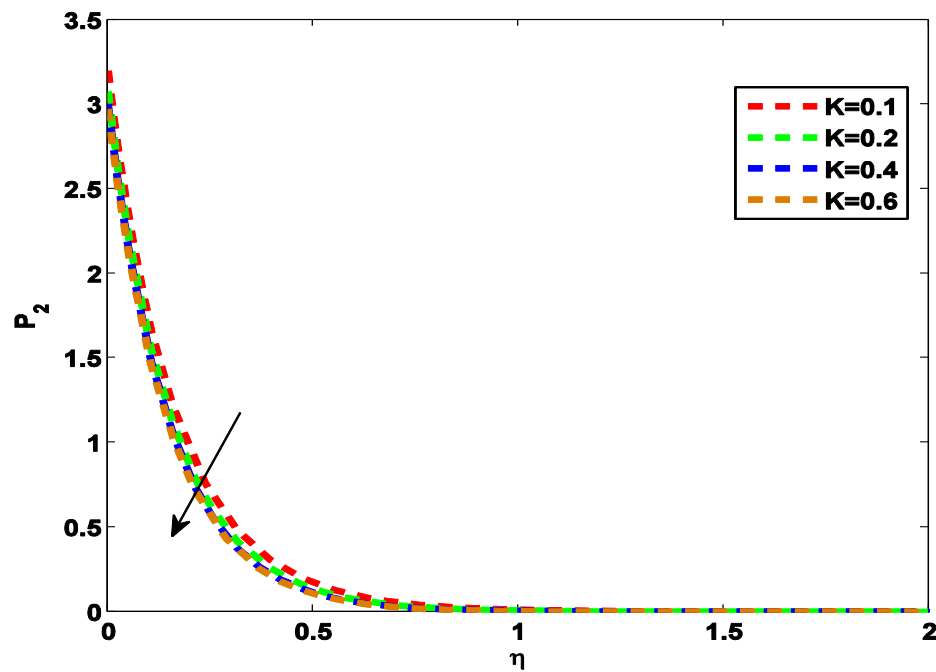


Figure 8: Effect of K on Pressure profile for a fixed value of $\beta = 0$, $Pr = 0.7$ and $\delta = 4$

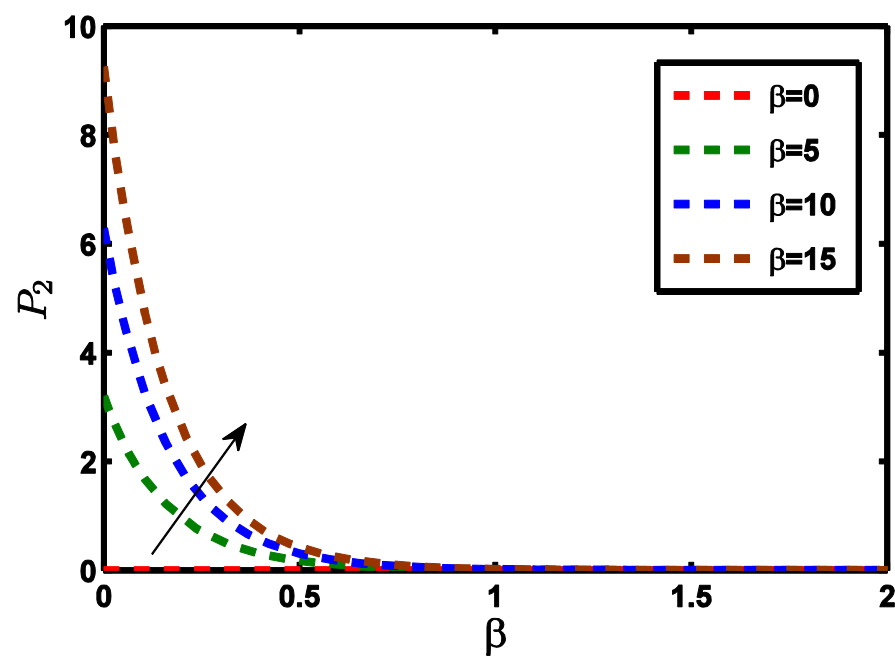


Figure 9: Effect of β on the Pressure profile for a fixed value of $K = 0.1$, $Pr = 0.7$ and $\delta = 4$

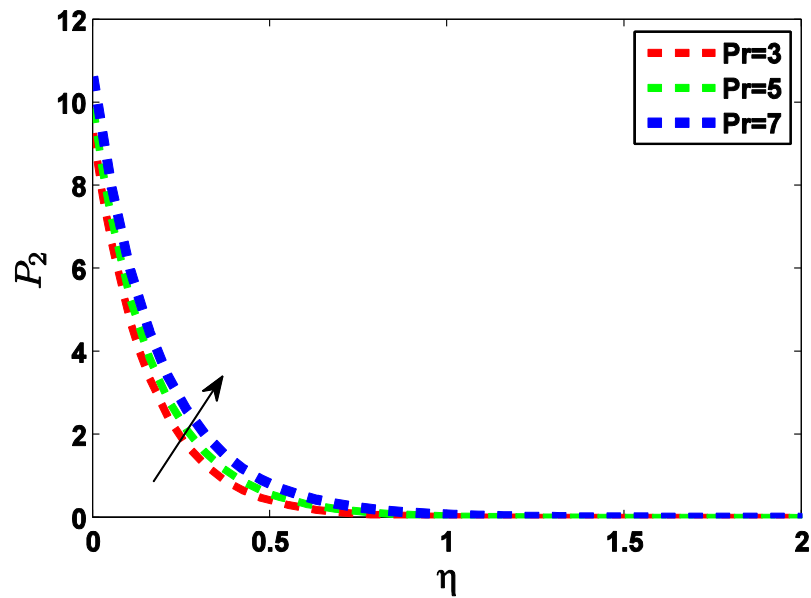


Figure 10: Impact of Pr on pressure profile for a fixed value of $K=0.1$, $\beta=0$, and $\delta=4$

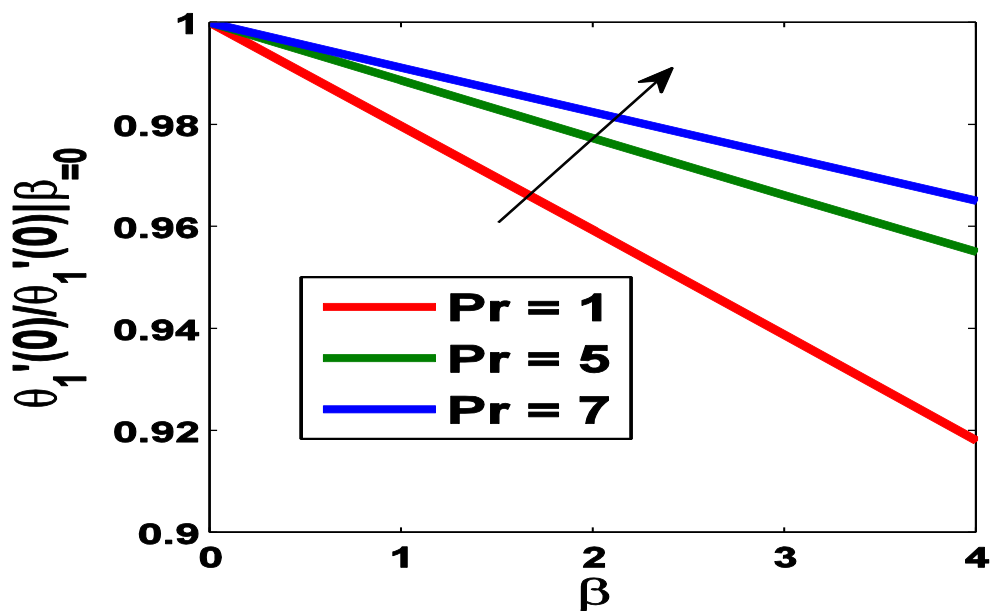


Figure 13: Variation of Nusselt Number With β and Pr

4 | CONCLUDING REMARKS

In the current work, we have reduced the PDEs into ODEs by using the resemblance transformations. The thermal energy equations and fluid momentum equations are expressed as a seven-parameter problem, and the impact of the magneto-thermo mechanical combinations is resolved mathematically. The presented research discussed the new prospects for employing this technique in areas ranging from magnetic drug targeting to imaging, which is applied for better accuracy in treatments and diagnostics. The effect of different factors such as K , β , δ , and Pr on temperature velocity and pressure are concluded. On raising the values of K and β the velocity profiles also rise and the velocity profile decreases when

we increase the value of Pr and δ . Increasing the Prandtl number results in a significant decrease in temperature profiles, with a reduction in heat transfer for higher Pr values due to lower thermal diffusivity. Increasing the parameters (second-grade factor K , ferromagnetic interaction parameter β , Prandtl number, and the combined elastic deformation/second-grade parameter) leads to a decline in the temperature profiles. The pressure increases with higher values of β , δ , and Pr , as these parameters enhance the magnetic interaction, elastic deformation, and thermal resistance, causing greater fluid pressure. The pressure decreases as K increases, indicating that the elastic nature of the fluid reduces the overall pressure exerted in the system. On raising the parameter Pr , the skin friction factor also increases. By increasing the value of Pr , the local Nusselt number also rises.

4.1. LIMITATION AND FUTURE RESEARCH

The study fixes values for parameters such as λ , ϵ and α the true consequences of these parameters on fluid behavior are not explored. Of course, that might not cover the full gamut of behaviors in various practical scenarios. Future work can explore the effects of varying λ , ϵ , and α to better understand how these variables impact the behavior of fluid across various scenarios. This would improve the current model and help in predicting the dynamics of ferrofluids under a magnetic field, especially for more complex industrial cases that contain three-dimensional or turbulent flows.

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