ORIGINAL ARTICLE

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# Common Fixed-Point Theorems for Weakly Compatible Mapping in Neutrosophic Metric Space of Integral Type Using Common EA Property

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*Article History:*

Submitted: July 19, 2024
Revised: October 24, 2024
Accepted: November 07, 2024
Published online: Dec 30, 2024

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## ABSTRACT

This manuscript focuses on developing and generalizing fixed-point results within the framework of neutrosophic metric spaces of integral type. By employing the common property (EA) and four self-mappings the study investigates the conditions necessary for the existence and uniqueness of fixed and common fixed points in neutrosophic metric spaces. To guarantee the validity of the results, the mappings under examination must meet certain contractive conditions and implicit relations such as being Lebesgue integrable and summable and non-negative. This work builds on the foundational concepts of fuzzy metric spaces and intuitionistic fuzzy metric spaces extending their applicability to the neutrosophic context. Nontrivial examples with graphical representations are given to demonstrate the theoretical conclusions and demonstrating the correctness and practical significance of the proposed theorems. The manuscript also explores the relationships between various mappings and highlights the role of weak compatibility and continuity in achieving the common property (EA).

*Keywords*: Fixed point (FP); intuitionistic fuzzy metric spaces (IFMSs); neutrosophic metric spaces (NMSs); Intuitionistic fuzzy sets (IFSs.)

*(MSC2020) Codes:* 47H10, 54E35, 54E35, 03E72

## 1 | INTRODUCTION

Zadeh introduced the concept of fuzzy sets (FSs) [1], which fundamentally altered how imprecise and uncertain data is represented and had a significant impact on numerous scientific domains. Since its origin, fuzzy sets theory has seen significant development, giving rise to numerous extensions such as FSs and neutrosophic sets which offer improved approaches to handling challenging data. The concept of metric-like spaces, an extension of conventional metric spaces, was proposed [2]. B-metric-like spaces were introduced [3], and the work was advanced by developing controlled metric-type spaces (CMLS) and proving several significant contractive mapping results [4],[5].

Fuzzy metric spaces (FMSs) were first established [6], although a different definition was suggested [7]. The fuzzy version of the Banach contraction principle was provided [8], and the extension of the Banach fixed-point theorem (FPT) was given [9], both of which strengthened the basis of FMSs. Work on statistical metric spaces [11] extended fuzzy b-metric spaces [13], and fuzzy b-metric spaces have further expanded these ideas [10]. Controlled fuzzy metric spaces were classified and the concept of controlled metric spaces was generalized [14]. Fuzzy metric analogous spaces and fuzzy metric-like spaces were studied [15]. The technique using continuous t-norms and t-conorms was introduced, and research into intuitionistic fuzzy metric spaces (IFMSs) received considerable advancements [18]. This progress was followed by work on intuitionistic fuzzy b-metric spaces [21], intuitionistic fuzzy topological spaces [24], and related fixed-point results [19]–[23]. The concept of the common EA property was defined [25]. Some fixed-point theorems in complete metric spaces were generalized [26].

For integral-type inequalities, a fixed-point result under the method of the Banach contraction principle was obtained [27], while later works [28]–[36] expanded these results using generalized contractive conditions of integral type. A class of implicit functions was introduced to prove new common FP theorems [37]. We generalize and discuss fixed-point results with four self-mappings in NMS.

The primary objectives of this study:

1. This research aims to extend the understanding of fixed-point theory by introducing and proving new results within the framework of neutrosophic metric spaces. By utilizing advanced mathematical concepts such as integral-type contractive mappings and the common property (EA). This study explores the conditions necessary for the existence and uniqueness of fixed points and common fixed points in NMS.
2. To enhance existing literature on fuzzy metric spaces and intuitionistic fuzzy metric spaces in NMS.

The manuscript is structured as follows:

a) In section 2, we study the fundamental definitions and preliminary results like continuous t-norm (CTN), continuous t-conorm, intuitionistic fuzzy metric spaces, neutrosophic metric spaces, common property (EA), and Lamma which support our main result.

b) In section 3, we discuss Implicit relations and their significance in the context of fixed-point theory. Furthermore, illustrative examples are provided to demonstrate the practical application of these relations and offer insights into their behavior and utility within NMS.

c) The essential results, including corollaries, lemmas, and fixed-point theorems, are presented in section 4. These results are accompanied by detailed proofs and supported by nontrivial examples some of which are illustrated graphically to enhance clarity.

## 2 | PRELIAMINARIES

In this section, some basic definitions are given that are helpful to understand the main results.

**Definition 2.1**: [31] A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is said to be CTN if the following conditions are satisfied:

1. $*$ is commutative and associative,
2. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$,
3. $a * 1 = a$ for all $a \in [0,1]$,
4. $*$ is continuous.

**Definition 2.2**: [31] A binary operation $\diamond: [0,1] \times [0,1] \to [0,1]$ is called the CTCN if the following conditions are satisfied:

1. $a \diamond 0 = 0$ for all $a \in [0,1]$,
2. $\diamond$ is continuous,
3. $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0,1]$,
4. $\diamond$ is commutative and associative.

**Definition 2.3**: [35] A 5-tuple $(C, \chi, \omega, *, \diamond)$ is called IFMS if $C$ is an arbitrary nonempty set, $\diamond$ CTCN and $*$ is a CTN, $\chi, \omega$ are FSs on $C^2 \times [0, \infty]$ and verify the following conditions. For each $\tau, \sigma, \sigma, \in C$ and $t, s > 0$

1. $\chi(\tau, \sigma, t) + \omega(\tau, \sigma, t) \leq 1$,
2. $\chi(\tau, \sigma, t) = 0$, for all $\tau, \sigma$ in $C$,
3. $\chi(\tau, \sigma, t) = 1$ for all $\tau, \sigma$ in $C$ and $t > 0$ if and only if $\tau = \sigma$,
4. $\chi(\tau, \sigma, t) = \chi(\sigma, \tau, t)$, for all $\tau, \sigma$ in $C$ and $t > 0$,
5. $\chi(\tau, \sigma, t) * \chi(\sigma, \sigma, s) \leq \chi(\tau, \sigma, t + s)$,
6. $\chi(\tau, \sigma, \cdot): [0, \infty] \to [0,1]$ is left continuous,
7. $\lim_{t \to \infty} \chi(\tau, \sigma, t) = 1$,
8. $\omega(\tau, \sigma, 0) = 1$, for all $\tau, \sigma$ in $C$,
9. $\omega(\tau, \sigma, t) = 0$, for all $\tau, \sigma$ in $C$ and $t > 0$ if and only if $\tau = \sigma$,
10. $\omega(\tau, \sigma, t) = \omega(\sigma, \tau, t)$, for all $\tau, \sigma$ in$C$and $t > 0$,
11. $\omega(\tau, \sigma, t) \diamond \omega(\sigma, \sigma, s) \geq \omega(\tau, \sigma, t + s)$,
12. $\omega(\tau, \sigma, \cdot): [0, \infty] \to [0,1]$ is right continuous,
13. $\lim_{t \to \infty} \omega(\tau, \sigma, t) = 0$, for all $\tau, \sigma$ in $C$ and $t > 0$.

Then $(\chi, \omega)$ is said to be an intuitionistic fuzzy metric on $C$.

**Example 2.1**: [32] Suppose that $(C, d)$ is an IFMS and CTCN, CTN are $a \diamond b = \min\{1, a + b\}$, $a * b = a \cdot b$, respectively for all $a, b \in [0,1]$. Let $\chi$ and $\omega$ are FSs on $C^2 \times (0, \infty)$ such that

$$\chi(\tau, \sigma, t) = \frac{t}{t + d(\tau, \sigma)},$$

$$\omega(\tau,\sigma,t)=\frac{d(\tau,\sigma)}{t+d(\tau,\sigma)},$$

for all $\tau,\sigma \in X$ and all $t>0$. Property EA in IFMS on $C$.

**Lemma 2.1**: [15] Suppose that $(C,\chi,\omega,*,\diamond)$ IFMS, where, $k\in(0,1)$ that is, $\tau,\sigma\in C$

$$\chi(\tau,\sigma,kt)\geq\chi(\tau,\sigma,t),$$
$$\omega(\tau,\sigma,kt)\leq\omega(\tau,\sigma,t),$$

for all $t>0$, then $\tau=\sigma$.

**Definition 2.4**: [34] Two pairs $(\varDelta,S)$ and $(B,\varUpsilon)$ of self-mappings of an IFMS $(C,\chi,\omega,*,\diamond)$ is called the common EA property, if the two sequence $\{\tau_n\}$ and $\{\sigma_n\}$ in $C$ such that $\lim_{n\to\infty}\varDelta\tau_n=\lim_{n\to\infty}S\tau_n=\lim_{n\to\infty}B\sigma_n=\lim_{n\to\infty}\varUpsilon\sigma_n=\varsigma$ for some $\varsigma\in C$.

**Definition 2.5**: [34] A pair $(f,g)$ of self-mappings of an IFMS $(C,\chi,\omega,*,\diamond)$ is called the mappings that are just marginally compatible if every point they commute of coincidence, i.e., $f\tau=g\tau$ for some $\tau\in C$ implies $fg\tau=gf\tau$.

**Definition 2.6**: [35] A 6-tuple $(C,\chi,\omega,\varTheta,*,\diamond)$ is called to be NMSs if $C$ is an arbitrary non empty set, $*$ is a CTN, $\diamond$ CTCN, $\chi,\omega,\varTheta$ are NSs on $C^2\times[0,\infty]$ satisfied the following conditions, for each $\tau,\sigma,\varsigma,\in C$ and $t,s>0$.

1. $\chi(\tau,\sigma,t)+\omega(\tau,\sigma,t)+\varTheta(\tau,\sigma,t)\leq 3$,
2. $\chi(\tau,\sigma,t)=0$, for all $\tau,\sigma$ in $C$,
3. $\chi(\tau,\sigma,t)=1$ for all $\tau,\sigma$ in $C$ and $t>0$ if and only if $\tau=\sigma$,
4. $\chi(\tau,\sigma,t)=\chi(\sigma,\tau,t)$, for all $\tau,\sigma$ in $C$ and $t>0$,
5. $\chi(\tau,\sigma,t)*\chi(\sigma,\varsigma,s)\leq\chi(\tau,\varsigma,t+s)$,
6. $\chi(\tau,\sigma,\cdot):[0,\infty]\to[0,1]$ is left continuous,
7. $\lim_{t\to\infty}\chi(\tau,\sigma,t)=1$,
8. $\omega(\tau,\sigma,0)=1$, for all $\tau,\sigma$ in $C$,
9. $\omega(\tau,\sigma,t)=0$, for all $\tau,\sigma$ in $C$ and $t>0$ if and only if $\tau=\sigma$,
10. $\omega(\tau,\sigma,t)=\omega(\sigma,\tau,t)$, for all $\tau,\sigma$ in $C$ and $t>0$,
11. $\omega(\tau,\sigma,t)\diamond\omega(\sigma,\varsigma,s)\geq\omega(\tau,\varsigma,t+s)$,
12. $\omega(\tau,\sigma,\cdot):[0,\infty]\to[0,1]$ is right continuous,
13. $\lim_{t\to\infty}\omega(\tau,\sigma,t)=0$, for all $\tau,\sigma$ in $C$ and $t>0$,
14. $\varTheta(\tau,\sigma,0)=1$, for all $\tau,\sigma$ in $C$,
15. $\varTheta(\tau,\sigma,t)=0$, for all $\tau,\sigma$ in $C$ and $t>0$ if and only if $\tau=\sigma$,
16. $\varTheta(\tau,\sigma,t)=\omega(\sigma,\tau,t)$, for all $\tau,\sigma$ in $C$ and $t>0$,
17. $\varTheta(\tau,\sigma,t)\diamond\omega(\sigma,\varsigma,s)\geq\omega(\tau,\varsigma,t+s)$,
18. $\varTheta(\tau,\sigma,\cdot):[0,\infty]\to[0,1]$ is right continuous,
19. $\lim_{t\to\infty}\omega(\tau,\sigma,t)=0$, for all $\tau,\sigma$ in $C$ and $t>0$.

Then $(\chi,\omega,\varTheta)$ is said to be a neutrosophic metric on $C$.

## 3 | IMPLICIT RELATION

Popa [37] introduced a class of implicit functions to prove new common FP theorems. Suppose that $\mathbb{R}_+^5$ and $\mathbb{R}$ represent continuous functions of real valued, $\phi, \psi: \mathbb{R}_+^5 \to \mathbb{R}$ mappings which are non-decreasing and satisfied the following conditions:

1. $\int_0^{\phi(\xi,1,\xi,1,\xi)} \rho(t)dt \geq 0$ implies $\xi \geq 1$, (1)
2. $\int_0^{\phi(\xi,1,1,\xi,\xi)} \rho(t)dt \geq 0$ implies $\xi \geq 1$, (2)
3. $\int_0^{\phi(\xi,\xi,1,1,\xi)} \rho(t)dt \geq 0$ implies $\xi \geq 1$, (3)
4. $\int_0^{\psi(\varsigma,0,\varsigma,0,\varsigma)} \rho(t)dt \leq 0$ implies $\varsigma \leq 0$, (4)
5. $\int_0^{\psi(\varsigma,0,0,\varsigma,\varsigma)} \rho(t)dt \leq 0$ implies $\varsigma \leq 0$, (5)
6. $\int_0^{\psi(\varsigma,\varsigma,0,0,\varsigma)} \rho(t)dt \leq 0$ implies $\varsigma \leq 0$, (6)
7. $\int_0^{\psi(\upsilon,0,\upsilon,0,\upsilon)} \rho(t)dt \leq 0$ implies $\upsilon \leq 0$, (7)
8. $\int_0^{\psi(\upsilon,0,0,\upsilon,\upsilon)} \rho(t)dt \leq 0$ implies $\upsilon \leq 0$, (8)
9. $\int_0^{\psi(\upsilon,\upsilon,0,0,\upsilon)} \rho(t)dt \leq 0$ implies $\upsilon \leq 0$. (9)

**Example 3.1**: Describe $\phi, \psi: \mathbb{R}_+^5 \to \mathbb{R}$ as

$$\phi(t_1, t_2, t_3, t_4, t_5) = 11t_1 - 12t_2 + 6t_3 - 8t_4 + 3t_5,$$
$$\psi(t_1, t_2, t_3, t_4, t_5, t_6) = 10t_1 - 9t_2 + 8t_3 - 11t_4 + 2t_5.$$

Clearly $\phi, \psi$ satisfies all conditions (1)-(9) and $\phi, \psi \in \mathbb{R}_+^5$.

## 4 | MAIN RESULTS

In this section, we discuss some FP results in the senses of NMS. We give four self-mapping and existence and uniqueness of the fixed point.

**Definition 4.1**: Two pairs $(\Delta, S)$ and $(B, \Upsilon)$ of self-mappings in an NMS $(C, \chi, \omega, \Theta, *, \diamond)$ is called the common EA property, if the two sequence $\{\tau_n\}$ and $\{\sigma_n\}$ in $C$ such that $\lim_{n\to\infty} \Delta\tau_n = \lim_{n\to\infty} S\tau_n = \lim_{n\to\infty} B\sigma_n = \lim_{n\to\infty} \Upsilon\sigma_n = \sigma$ for some $\sigma \in C$.

**Definition 4.2**: [34] A pair $(f, g)$ of self-mappings in NMS $(C, \chi, \omega, \Theta, *, \diamond)$ is called the mappings that are just marginally compatible if every point they commute of coincidence, i.e., $f\tau = g\tau$ for some $\tau \in C$ implies $fg\tau = gf\tau$.

**Lemma 4.1**: Suppose that $(C, \chi, \omega, \Theta, *, \diamond)$ NMS, where, $k \in (0,1)$ that is, $\tau, \sigma \in C$,

$$\chi(\tau, \sigma, kt) \geq \chi(\tau, \sigma, t),$$
$$\omega(\tau, \sigma, kt) \leq \omega(\tau, \sigma, t),$$
$$\Theta(\tau, \sigma, kt) \leq \Theta(\tau, \sigma, t),$$

for all $t > 0$, then $\tau = \sigma$.

**Theorem 4.1**: Suppose that $\Delta, S, B,$ and $\Upsilon$ be self-mappings of NMS $(C, \chi, \omega, \Theta, *, \diamond)$ fulfill the following conditions:

The pair $(\Delta, S)$ or $(B, \Upsilon)$ satisfied the EA property.

For any $\tau, \sigma \in C, \phi, \psi: \mathbb{R}_+^5 \to \mathbb{R}$ are mappings and for all $t > 0$, there exists $k \in (0,1)$ such that:

$$\int_0^{\phi(\chi(\Delta\tau,B\sigma,\alpha t),\chi(S\tau,\Upsilon\sigma,t),\chi(S\tau,\Delta\tau,t),\chi(\Upsilon\sigma,B\sigma,t),\chi(\Delta\tau,\Upsilon\sigma,t)*\chi(S\tau,B\sigma,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\Delta\tau,B\sigma,\alpha t),\omega(S\tau,\Upsilon\sigma,t),\omega(S\tau,\Delta\tau,t),\omega(\Upsilon\sigma,B\sigma,t),\omega(\Delta\tau,\Upsilon\sigma,t)\diamond\omega(S\tau,B\sigma,t))} \rho(t)dt \leq 0,$$

$$\int_0^{\psi(\Theta(\Delta\tau,B\sigma,\alpha t),\Theta(S\tau,\Upsilon\sigma,t),\Theta(S\tau,\Delta\tau,t),\Theta(\Upsilon\sigma,B\sigma,t),\Theta(\Delta\tau,\Upsilon\sigma,t)\diamond\Theta(S\tau,B\sigma,t))} \rho(t)dt \leq 0.$$

Now, $\rho: \mathbb{R}_+^5 \to \mathbb{R}_+^5$ is a Lebesgue integral mapping that is non-negative and summable such that $\int_0^\varepsilon \rho(t)dt > 0$ for each $\varepsilon > 0$.

$$\Delta(C) \subset \Upsilon(C)\big(\text{or } B(C) \subset S(C)\big).$$

**Proof**: Suppose that a pair $(\Delta, S)$ of mappings satisfied the EA property, then there exists a sequence $\{\tau_n\}$ in $C$ such that $\lim_{n\to\infty} \Delta\tau_n = \lim_{n\to\infty} S\tau_n = \sigma$ for some $\sigma \in$ C. Since $\Delta(C) \subset \Upsilon(C)$, therefore, for each $\tau_n$, there exist $\sigma_n$ in $C$ such that $\Delta\tau_n = \Upsilon\sigma_n$. Given that, $\lim_{n\to\infty} \Delta\tau_n = \lim_{n\to\infty} S\tau_n = \lim_{n\to\infty} \Upsilon\sigma_n = \sigma$. Currently, we assert that $\lim_{n\to\infty} B\sigma_n = \sigma$. Applying inequality (i), we obtain,

$$\int_0^{\phi(\chi(\Delta\tau_n,B\sigma_n,\alpha t),\chi(S\tau_n,\Upsilon\sigma_n,t),\chi(S\tau_n,\Delta\tau_n,t),\chi(\Upsilon\sigma_n,B\sigma_n,t),\chi(\Delta\tau_n,\Upsilon\sigma_n,t)*\chi(S\tau_n,B\sigma_n,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\Delta\tau_n,B\sigma_n,\alpha t),\omega(S\tau_n,\Upsilon\sigma_n,t),\omega(S\tau_n,\Delta\tau_n,t),\omega(\Upsilon\sigma_n,B\sigma_n,t),\omega(\Delta\tau_n,\Upsilon\sigma_n,t)\diamond\omega(S\tau_n,B\sigma_n,t))} \rho(t)dt \leq 0,$$

$$\int_0^{\psi(\Theta(\Delta\tau_n,B\sigma_n,\alpha t),\Theta(S\tau_n,\Upsilon\sigma_n,t),\Theta(S\tau_n,\Delta\tau_n,t),\Theta(\Upsilon\sigma_n,B\sigma_n,t),\Theta(\Delta\tau_n,\Upsilon\sigma_n,t)\diamond\Theta(S\tau_n,B\sigma_n,t))} \rho(t)dt \leq 0.$$

Where, limit as $n \to \infty$

$$\int_0^{\phi\left(\chi\left(\sigma,\lim_{n\to\infty} B\sigma_n,\alpha t\right),\chi(\sigma,\sigma,t),\chi(\sigma,\sigma,t),\chi\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right),\chi(\sigma,\sigma,t)*\chi\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right)\right)} \rho(t)dt \geq 0,$$

$$\int_0^{\psi\left(\omega\left(\sigma,\lim_{n\to\infty} B\sigma_n,\alpha t\right),\omega(\sigma,\sigma,t),\omega(\sigma,\sigma,t),\omega\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right),\omega(\sigma,\sigma,t)\diamond\omega\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right)\right)} \rho(t) \leq 0,$$

and

$$\int_0^{\psi\left(\Theta\left(\sigma,\lim_{n\to\infty} B\sigma_n,\alpha t\right),\Theta(\sigma,\sigma,t),\Theta(\sigma,\sigma,t),\Theta\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right),\Theta(\sigma,\sigma,t)\diamond\Theta\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right)\right)} \rho(t) \leq 0.$$

Given that $\phi$ and $\psi$ are non-decreasing, we have

$$\int_0^{\phi\left(\chi\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right),1,1,\chi\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right),\chi\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right)\right)} \rho(t)dt \geq 0,$$

$$\int_0^{\psi\left(\omega\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right),0,0,\omega\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right),\omega\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right)\right)} \rho(t)dt \le 0,$$

and

$$\int_0^{\psi\left(\Theta\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right),0,0,\Theta\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right),\Theta\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right)\right)} \rho(t)dt \le 0.$$

Using (2), (5) and (8), we get,

$$\chi\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right) \ge 1,$$

$$\omega\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right) \le 0,$$

$$\Theta\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right) \le 0.$$

Hence

$$\chi\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right) = 1,$$

$$\omega\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right) = 0,$$

$$\Theta\left(\sigma,\lim_{n\to\infty} B\sigma_n,t\right) = 0.$$

Therefore $\lim_{n\to\infty} B\sigma_n = \sigma$. Then the pair of mappings $(\Delta, S)$ and $(B, \Upsilon)$ share common EA property.

**Example 4.1**: Suppose that $(C, \chi, \omega, \Theta, *, \diamond)$ be an NMS $C = [0, \infty)$ with CTN $a * b = \min\{a, b\}$ and CTCN $a \diamond b = \max\{a, b\}$. $\chi, \omega$, and $\Theta$ are NSs on $C^2 \times (0, \infty)$ defined by,

$$\chi(\tau,\sigma,t) = \frac{t}{t + d(\tau,\sigma)},$$

$$\omega(\tau,\sigma,t) = \frac{d(\tau,\sigma)}{t + d(\tau,\sigma)},$$

$$\Theta(\tau,\sigma,t) = \frac{d(\tau,\sigma)}{t}.$$

where $d(\tau,\sigma) = |\tau - \sigma|$, $\forall \tau, \sigma \in C, t > 0$, and $\rho(t) = e^{-t}$. The behavior of $\chi(\tau,\sigma,t), \omega(\tau,\sigma,t)$, and $\Theta(\tau,\sigma,t)$ are shown in Figure 1.

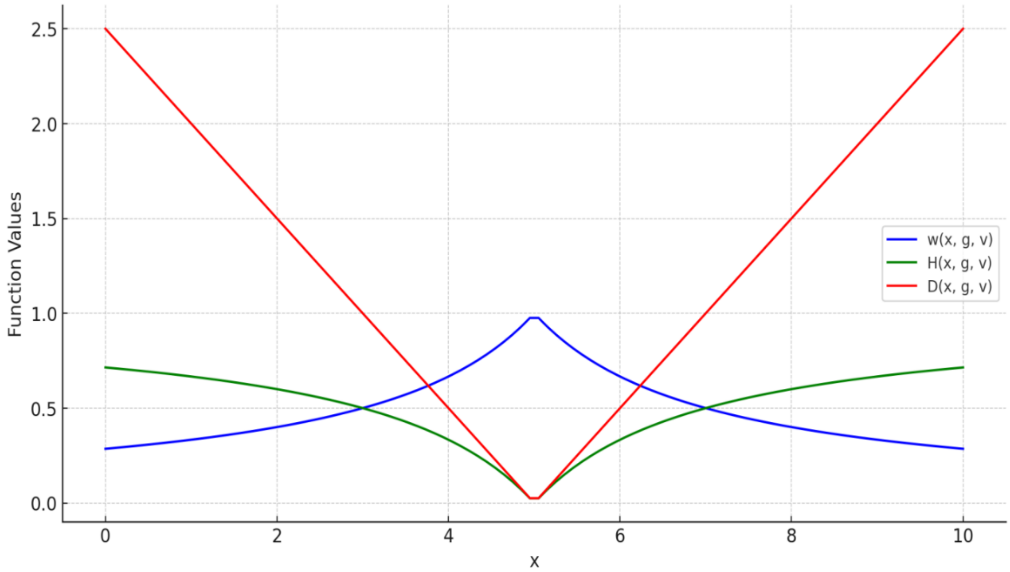


**Figure 1**. Shows the graphical behavior of $\chi(\tau,\sigma,t)$, $\omega(\tau,\sigma,t)$, and $\Theta(\tau,\sigma,t)$.

Define $\Delta, B, S, \Upsilon: C \to C$ by,

$$\Delta(\tau) = \frac{\tau}{2}, B(\tau) = \frac{\tau}{3}, S(\tau) = \frac{\tau}{6}, \Upsilon(\tau) = \frac{\tau}{4},$$

Suppose that sequence$\{\tau_n\}$ and $\{\sigma_n\}$, define by,

$$\tau_n = \frac{1}{2n}, \quad n \geq 1.$$

Clearly, $(\Delta, B)$, and $(S, \Upsilon)$ are satisfied the EA property. Then, we have $\lim_{n\to\infty} \Delta\tau_n = \lim_{n\to\infty} B\tau_n = \lim_{n\to\infty} S\tau_n = \lim_{n\to\infty} \Upsilon\tau_n = 0$. All the conditions of Theorem 4.1 are satisfied $\Delta, B, S$, and $\Upsilon$, have a unique fixed point which is 0.

**Theorem 4.2**: Suppose that $\Delta, B, S$ and $\Upsilon$ are four self-mappings in NMSs $(C, \chi, \omega, \Theta, *, \diamond)$ and satisfied the following conditions:

For any $\tau, \sigma \in C, \phi, \psi \in \chi^5$ and for all $t > 0$, there exists $k \in (0,1)$ such that

$$\int_0^{\phi(\chi(\Delta\tau,B\sigma,\alpha t),\chi(S\tau,\Upsilon\sigma,t),\chi(S\tau,\Delta\tau,t),\chi(\Upsilon\sigma,B\sigma,t),\chi(\Delta\tau,\Upsilon\sigma,t)*\chi(S\tau,B\sigma,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\Delta\tau,B\sigma,\alpha t),\omega(S\tau,\Upsilon\sigma,t),\omega(S\tau,\Delta\tau,t),\omega(\Upsilon\sigma,B\sigma,t),\omega(\Delta\tau,\Upsilon\sigma,t)\diamond\omega(S\tau,B\sigma,t))} \rho(t)dt \leq 0,$$

$$\int_0^{\psi(\Theta(\Delta\tau,B\sigma,\alpha t),\Theta(S\tau,\Upsilon\sigma,t),\Theta(S\tau,\Delta\tau,t),\Theta(\Upsilon\sigma,B\sigma,t),\Theta(\Delta\tau,\Upsilon\sigma,t)\diamond\Theta(S\tau,B\sigma,t))} \rho(t)dt \leq 0.$$

Where $\rho: \mathbb{R}_+^5 \to \mathbb{R}_+^5$ is a Lebesgue integrable mapping which is non-negative and summable, such that

$$\int_0^{\varepsilon} \rho(t)dt > 0 \text{ for each } \varepsilon > 0,$$

The pairs $(\Delta, S)$ and $(B, \Upsilon)$ share the EA property.

$S(C)$ and $\Upsilon(C)$ are closed subsets $C$.

Moreover, $\Delta, B, S$ and $\Upsilon$ have a unique common FP. So, the pairs $(\Delta, S)$ and $(B, \Upsilon)$ are weakly compatible.

Proof: Since the pair $(\Delta, S)$ and $(B, \Upsilon)$ share the EA property, two sequences $\{\tau_n\}$ and $\{\sigma_n\}$ are exists in $C$. Such that $\lim_{n\to\infty} \Delta\tau_n = \lim_{n\to\infty} S\tau_n = \lim_{n\to\infty} \Upsilon\sigma_n = \sigma$ for some $\sigma \in C$. Then, $S(C)$ is closed subset of $C$, there exists a point $\xi \in C$ such that $\sigma = S\xi$. We say that $\Delta\xi = \sigma$ by (i), we have,

$$\int_0^{\phi(\chi(\Delta\xi,B\sigma_n,\alpha t),\chi(S\xi,\Upsilon\sigma_n,t),\chi(S\xi,\Delta\xi,t)\chi(\Upsilon\sigma_n,B\sigma_n,t),\chi(\Delta\xi,\Upsilon\sigma_n,t)*\chi(S\xi,B\sigma_n,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\Delta\xi,B\sigma_n,\alpha t),\omega(S\xi,\Upsilon\sigma_n,t),\omega(S\xi,\Delta\xi,t),\omega(\Upsilon\sigma_n B\sigma_n,t),\omega(\Delta\xi,\Upsilon\sigma_n,t)\diamond\omega(S\xi,B\sigma_n,t))} \rho(t)dt \leq 0,$$

$$\int_0^{\psi(\Theta(\Delta\xi,B\sigma_n,\alpha t),\Theta(S\xi,\Upsilon\sigma_n,t),\Theta(S\xi,\Delta\xi,t),\Theta(\Upsilon\sigma_n B\sigma_n,t),\Theta(\Delta\xi,\Upsilon\sigma_n,t)\diamond\Theta(S\xi,B\sigma_n,t))} \rho(t)dt \leq 0.$$

Applying, limit as $n \to \infty$,

$$\int_0^{\phi(\chi(\Delta\xi,\sigma,\alpha t),\chi(\sigma,\sigma,t),\chi(\sigma,\Delta\xi,t),\chi(\sigma,\sigma,t),\chi(\Delta\xi,\sigma,t)*\chi(\sigma,\sigma,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\Delta\xi,\sigma,\alpha t),\omega(\sigma,\sigma,t),\omega(\sigma,\Delta\xi,t),\omega(\sigma,\sigma,t),\omega(\Delta\xi,\sigma,t)\diamond\omega(\sigma,\sigma,t))} \rho(t)dt \leq 0,$$

$$\int_0^{\psi(\Theta(\Delta\xi,\sigma,\alpha t),\Theta(\sigma,\sigma,t),\Theta(\sigma,\Delta\xi,t),\Theta(\sigma,\sigma,t),\Theta(\Delta\xi,\sigma,t)\diamond\Theta(\sigma,\sigma,t))} \rho(t)dt \leq 0.$$

Given that both $\phi$ and $\psi$ are non-decreasing from (i), we obtain

$$\int_0^{\phi(\chi(\Delta\xi,\sigma,t),1,\chi(\sigma,\Delta\xi,t),1,\chi(\Delta\xi,\sigma,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\Delta\xi,\sigma,t),0,\omega(\sigma,\Delta\xi,t),0,\omega(\Delta\xi,\sigma,t))} \rho(t)dt \leq 0,$$

and

$$\int_0^{\psi(\Theta(\Delta\xi,\sigma,t),0,\Theta(\sigma,\Delta\xi,t),0,\Theta(\Delta\xi,\sigma,t))} \rho(t)dt \leq 0.$$

Using implicit relations (1), (4) and (7), we have,

$$\chi(\Delta\xi, \sigma, t) \geq 1,$$
$$\omega(\Delta\xi, \sigma, t) \leq 0,$$
$$\Theta(\Delta\xi, \sigma, t) \leq 0.$$

Hence, $\chi(\Delta\xi, \sigma, t) = 1$, $\omega(\Delta\xi, \sigma, t) = 0$ and $\Theta(\Delta\xi, \sigma, t) = 0$. Therefore, $\Delta\xi = \sigma = S\xi$, which shows that $\xi$ is a coincidence point of $(\Delta, S)$. Since $\Upsilon(C)$ is also a closed subset of $C$, therefore, $\lim_{n\to\infty} \Upsilon\sigma_n = \sigma$ in $\Upsilon(C)$ and $\varsigma \in C$ such that $\Upsilon\varsigma = \sigma = \Delta\xi = S\xi$ now, $B\varsigma = \sigma$. by using implicit relation (1), we have,

$$\int_0^{\phi(\chi(\Delta\xi,B\varsigma,\alpha t),\chi(S\xi,\Upsilon\varsigma,t),\chi(S\xi,\Delta\xi,t),\chi(\Upsilon\varsigma,B\varsigma,t),\chi(\Delta\xi,\Upsilon\varsigma,t)*\chi(S\xi,B\varsigma,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\Delta\xi,B\varsigma,\alpha t),\omega(S\xi,\Upsilon\varsigma,t),\omega(S\xi,\Delta\xi,t),\omega(\Upsilon\varsigma,B\varsigma,t),\omega(\Delta\xi,\Upsilon\varsigma,t)\diamond\omega(S\xi,B\varsigma,t))} \rho(t)dt \leq 0,$$

and

$$\int_0^{\psi(\Theta(\Delta\xi,B\varsigma,\alpha t),\Theta(S\xi,\Upsilon\varsigma,t),\Theta(S\xi,\Delta\xi,t),\Theta(\Upsilon\varsigma,B\varsigma,t),\Theta(\Delta\xi,\Upsilon\varsigma,t)\diamond\Theta(S\xi,B\varsigma,t))} \rho(t)dt \leq 0.$$

It follows,

$$\int_0^{\phi(\chi(\sigma,B\varsigma,\alpha t),\chi(\sigma,\sigma,t),\chi(\sigma,\sigma,t),\chi(\sigma,B\varsigma,t),\chi(\sigma,\sigma,t)*\chi(\sigma,B\varsigma,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\sigma,B\varsigma,\alpha t),\omega(\sigma,\sigma,t),\omega(\sigma,\sigma,t),\omega(\sigma,B\varsigma,t),\omega(\sigma,\sigma,t)\diamond\omega(\sigma,B\varsigma,t))} \rho(t)dt \leq 0,$$

and

$$\int_0^{\psi(\Theta(\sigma,B\varsigma,\alpha t),\Theta(\sigma,\sigma,t),\Theta(\sigma,\sigma,t),\Theta(\sigma,B\varsigma,t),\Theta(\sigma,\sigma,t)\diamond\Theta(\sigma,B\varsigma,t))} \rho(t)dt \leq 0.$$

The functions $\phi, \psi$ are non-decreasing and by (i), we have

$$\int_0^{\phi(\chi(\sigma,B\varsigma,t),1,1,\chi(\sigma,B\varsigma,t),\chi(\sigma,B\varsigma,t))} \rho(t)dt \geq 0,$$

and

$$\int_0^{\psi(\omega(\sigma,B\varsigma,t),0,0,\omega(\sigma,B\varsigma,t),\omega(\sigma,B\varsigma,t))} \rho(t)dt \leq 0,$$

$$\int_0^{\psi(\Theta(\sigma,B\varsigma,t),0,0,\Theta(\sigma,B\varsigma,t),\Theta(\sigma,B\varsigma,t))} \rho(t)dt \leq 0.$$

Using implicit relation (2) and (7), we get,

$$\chi(\sigma, B\varsigma, t) \geq 1,$$
$$\omega(\sigma, B\varsigma, t) \leq 0,$$
$$\Theta(\sigma, B\varsigma, t) \leq 0.$$

Hence $\chi(\sigma, B\varsigma, t) = 1$, $\omega(\sigma, B\varsigma, t) = 0$ and $\Theta(\sigma, B\varsigma, t) = 0$. so, $B\varsigma = \sigma = \Upsilon\varsigma$, which shows that $\varsigma$ is the point $(B, \Upsilon)$ coincidence. Then, the pairs $(\Delta, S)$ and $(B, \Upsilon)$ are weakly compatible and $\Delta\xi = S\xi, B\varsigma = \Upsilon\varsigma$. Therefore, $\Delta\sigma = \Delta S\xi = S\Delta\xi = S\sigma, B\sigma = B\Upsilon\varsigma = \Upsilon B\varsigma = \Upsilon\sigma$. Next, we say that $\Delta\sigma = \sigma$ for showing the existence of a FP of $\Delta$. With implicit relation (1), we obtain

$$\int_0^{\phi(\chi(\Delta\sigma,B\varsigma,\alpha t),\chi(S\sigma,\Upsilon\varsigma,t),\chi(S\sigma,\Delta\sigma,t),\chi(\Upsilon\varsigma,B\varsigma,t),\chi(\Delta\sigma,\Upsilon\varsigma,t)*\chi(S\sigma,B\varsigma,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\Delta\sigma,B\varsigma,\alpha t),\omega(S\sigma,\Upsilon\varsigma,t),\omega(S\sigma,\Delta\sigma,t),\omega(\Upsilon\varsigma,B\varsigma,t),\omega(\Delta\sigma,\Upsilon\varsigma,t)\circ\omega(S\sigma,B\varsigma,t))} \rho(t)dt \leq 0,$$

and

$$\int_0^{\psi(\Theta(\Delta\sigma,B\varsigma,\alpha t),\Theta(S\sigma,\Upsilon\varsigma,t),\Theta(S\sigma,\Delta\sigma,t),\Theta(\Upsilon\varsigma,B\varsigma,t),\Theta(\Delta\sigma,\Upsilon\varsigma,t)\circ\Theta(S\sigma,B\varsigma,t))} \rho(t)dt \leq 0.$$

It follows that,

$$\int_0^{\phi(\chi(\Delta\sigma,\sigma,\alpha t),\chi(\Delta\sigma,\sigma,t),\chi(\Delta\sigma,\Delta\sigma,t),\chi(\sigma,\sigma,t),\chi(\Delta\sigma,\sigma,t)*\chi(\Delta\sigma,\sigma,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\Delta\sigma,\sigma,\alpha t),\omega(\Delta\sigma,\sigma,t),\omega(\Delta\sigma,\Delta\sigma,t),\omega(\sigma,\sigma,t),\omega(\Delta\sigma,\sigma,t)\circ\omega(\Delta\sigma,\sigma,t))} \rho(t)dt \leq 0,$$

and

$$\int_0^{\psi(\Theta(\Delta\sigma,\sigma,\alpha t),\Theta(\Delta\sigma,\sigma,t),\Theta(\Delta\sigma,\Delta\sigma,t),\Theta(\sigma,\sigma,t),\Theta(\Delta\sigma,\sigma,t)\circ\Theta(\Delta\sigma,\sigma,t))} \rho(t)dt \leq 0.$$

The functions $\phi$ and $\psi$ are non-decreasing, then we have

$$\int_0^{\phi(\chi(\Delta\sigma,\sigma,t),\chi(\Delta\sigma,\sigma,t),1,1,\chi(\Delta\sigma,\sigma,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\Delta\sigma,\sigma,t),\omega(\Delta\sigma,\sigma,t),0,0,\omega(\Delta\sigma,\sigma,t))} \rho(t)dt \leq 0,$$

and

$$\int_0^{\psi(\omega(\Delta\sigma,\sigma,t),\omega(\Delta\sigma,\sigma,t),0,0,\omega(\Delta\sigma,\sigma,t))} \rho(t)dt \leq 0.$$

By using implicit relations (3), (6) and (9), we obtain,

$$\chi(\Delta\sigma,\sigma,t) \geq 1, \omega(\Delta\sigma,\sigma,t) \leq 0, \Theta(\Delta\sigma,\sigma,t) \leq 0.$$

Hence,

$$\chi(\Delta\sigma,\sigma,t) = 1, \qquad \omega(\Delta\sigma,\sigma,t) = 0, \Theta(\Delta\sigma,\sigma,t) = 0.$$

So, $\Delta\sigma = \sigma = S\sigma$. Similarly, we have to prove that $B\sigma = \Upsilon\sigma = \sigma$. Hence, $\Delta\sigma = B\sigma = S\sigma = \Upsilon\sigma = \sigma$, which implies that $\sigma$ is a common FP of $\Delta, B, S$ and $\Upsilon$.

Uniqueness: Let $\upsilon$ be other common FP of $\Delta, B, S$ and $\Upsilon$ by (i)

$$\int_0^{\phi(\chi(\Delta\sigma,B\upsilon,\alpha t),\chi(S\sigma,\Upsilon\upsilon,t),\chi(S\sigma,\Delta\sigma,t),\chi(\Upsilon\upsilon,B\upsilon,t),\chi(\Delta\sigma,\Upsilon\upsilon,t)*\chi(S\sigma,B\upsilon,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\Delta\sigma,B\upsilon,\alpha t),\omega(S\sigma,\Upsilon\upsilon,t),\omega(S\sigma,\Delta\sigma,t),\omega(\Upsilon\upsilon,B\upsilon,t),\omega(\Delta\sigma,\Upsilon\upsilon,t)\circ\omega(S\sigma,B\upsilon,t))} \rho(t)dt \leq 0,$$

and

$$\int_0^{\psi(\Theta(\Delta\sigma,Bv,\alpha t),\Theta(S\sigma,\Upsilon v,t),\Theta(S\sigma,\Delta\sigma,t),\Theta(\Upsilon v,Bv,t),\Theta(\Delta\sigma,\Upsilon v,t)\diamond\Theta(S\sigma,Bv,t))} \rho(t)dt \leq 0.$$

It follows that,

$$\int_0^{\phi(\chi(\sigma,v,\alpha t),\chi(\sigma,v,t),\chi(\sigma,\sigma,t),\chi(v,v,t),\chi(\sigma,v,t)*\chi(\sigma,v,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\sigma,v,\alpha t),\omega(\sigma,v,t),\omega(\sigma,\sigma,t),\omega(v,v,t),\omega(\sigma,v,t)\diamond\omega(\sigma,v,t))} \rho(t)dt \leq 0,$$

and

$$\int_0^{\psi(\Theta(\sigma,v,\alpha t),\Theta(\sigma,v,t),\Theta(\sigma,\sigma,t),\Theta(v,v,t),\Theta(\sigma,v,t)\diamond\omega(\sigma,v,t))} \rho(t)dt \leq 0.$$

the functions $\phi$ and $\psi$ are non-decreasing, so we have

$$\int_0^{\phi(\chi(\sigma,v,t),\chi(\sigma,v,t),1,1,\chi(\sigma,v,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\sigma,v,\alpha t),\omega(\sigma,v,t),0,0,\omega(\sigma,v,t))} \rho(t)dt \leq 0,$$

and

$$\int_0^{\psi(\Theta(\sigma,v,\alpha t),\Theta(\sigma,v,t),0,0,\Theta(\sigma,v,t))} \rho(t)dt \leq 0.$$

Using implicit relation (3) (6), and (9), we have,

$$\chi(\sigma,v,t) \geq 1, \omega(\sigma,v,t) \leq 0, \Theta(\sigma,v,t) \leq 0.$$

Hence,

$$\chi(\sigma,v,t) = 1, \omega(\sigma,v,t) = 0, \Theta(\sigma,v,t) = 0.$$

So, $\sigma = v$, i.e., mappings $\Delta, B, S$ and $\Upsilon$ have a unique common FP.

**Example 4.2**: Suppose that $(C,\chi,\omega,\Theta,*,\diamond)$ be an NMS $C = [0,\infty)$ with CTN $a * b = \min\{a,b\}$ and CTCN $a \diamond b = \max\{a,b\}$. $\chi, \omega$, and $\Theta$ are NSs on $C^2 \times (0,\infty)$ defined by,

$$\chi(\tau,\sigma,t) = \frac{t}{t + d(\tau,\sigma)},$$

$$\omega(\tau,\sigma,t) = \frac{d(\tau,\sigma)}{t + d(\tau,\sigma)},$$

$$\Theta(\tau,\sigma,t) = \frac{d(\tau,\sigma)}{t}.$$

Where $d(\tau,\sigma) = |\tau - \sigma|$, $\forall \tau,\sigma \in C, t > 0$, and $\rho(t) = e^{-t}$.Define $\Delta, B, S, \Upsilon : C \to C$ by,

$$\Delta(\tau) = \begin{cases} 1 & \text{if} \quad \tau = 1, \\ 3 & \text{if} \quad 1 < \tau < 7. \end{cases}$$

$$B(\tau) = \begin{cases} 1 & \text{if} \quad \tau = 1, \\ 4 & \text{if} \quad 1 < \tau < 7. \end{cases}$$

$$S(\tau) = \begin{cases} 1 & \text{if} \quad \tau = 1, \\ 4 & \text{if} \quad 1 < \tau \leq 5, \\ \tau - 5, & \text{if} \quad 5 < \tau \leq 7. \end{cases}$$

$$\Upsilon(\tau) = \begin{cases} 1 & \text{if} \quad \tau = 1, \\ \tau - 1 & \text{if} \quad 1 < \tau \leq 5, \\ 3 & \text{if} \quad 5 < \tau \leq 7. \end{cases}$$

Suppose that sequence$\{\tau_n\}$ and $\{\sigma_n\}$, define by,

$$\tau_n = \frac{1}{2n}, \quad n \geq 1.$$

Clearly, $(\Delta, B)$, and $(S, \Upsilon)$ are satisfied the EA property. Then, we have Then, we have

$$\lim_{n\to\infty} \Delta\tau_n = \lim_{n\to\infty} B\tau_n = \lim_{n\to\infty} S\tau_n = \lim_{n\to\infty} \Upsilon\tau_n = 0.$$

All the conditions of Theorem 4.1 are satisfied $\Delta, B, S$, and $\Upsilon$, have a unique FP which is 1.

**Corollary 4.1**: Suppose that $\Delta$ and $S$ be self-mappings of NMSs $(C, \chi, \omega, \Theta, *, \diamond)$ fulfill the following condition:

The pair $(\Delta, S)$ share the EA property.

For any $\tau, \sigma \in C, \phi, \psi \in \mathbb{R}_+^5$ and for all $t > 0$, there exists $k \in (0,1)$ such that

$$\int_0^{\phi(\chi(\Delta\tau,\Delta\sigma,\alpha t),\chi(S\tau,S\sigma,t),\chi(S\tau,\Delta\tau,t),\chi(S\sigma,\Delta\sigma,t),\chi(\Delta\tau,S\sigma,t)*\chi(S\tau,\Delta\sigma,t))} \rho(t)dt \geq 0,$$

$$\int_0^{\psi(\omega(\Delta\tau,\Delta\sigma,\alpha t),\omega(S\tau,S\sigma,t),\omega(S\tau,S\tau,t),\omega(S\sigma,\Delta\sigma,t),\omega(\Delta\tau,S\sigma,t)\diamond\omega(S\tau,\Delta\sigma,t))} \rho(t)dt \leq 0,$$

and

$$\int_0^{\psi(\Theta(\Delta\tau,\Delta\sigma,\alpha t),\Theta(S\tau,S\sigma,t),\Theta(S\tau,S\tau,t),\Theta(S\sigma,\Delta\sigma,t),\Theta(\Delta\tau,S\sigma,t)\diamond\Theta(S\tau,\Delta\sigma,t))} \rho(t)dt \leq 0.$$

Where $\rho: \mathbb{R}_+^5 \to \mathbb{R}_+^5$ is a Lebesgue integrable mapping which is non-negative and summable, such that$\int_0^{\varepsilon} \rho(t)dt > 0$ for each $\varepsilon > 0$. $S(C)$ is a closed subset of $C$.

**Corollary 4.2**: Suppose that $\Delta, B, S$ and $\Upsilon$ be self-mappings of an NMS $(C, \chi, \omega, \Theta, *, \diamond)$ fulfill the following conditions:

For any $\tau, \sigma \in C, \phi, \psi \in \chi^5$ and for all $t > 0$, there exists $k \in (0,1)$ such that

$$\phi\big(\chi(\Delta\tau, B\sigma, \alpha t), \chi(S\tau, \Upsilon\sigma, t), \chi(S\tau, \Delta\tau, t), \chi(\Upsilon\sigma, B\sigma, t), \chi(\Delta\tau, \Upsilon\sigma, t) * \chi(S\tau, B\sigma, t)\big) \geq 0,$$

$$\psi\big(\omega(\Delta\tau, B\sigma, \alpha t), \omega(S\tau, \Upsilon\sigma, t), \omega(S\tau, \Delta\tau, t), \omega(\Upsilon\sigma, B\sigma, t), \omega(\Delta\tau, \Upsilon\sigma, t) \diamond \omega(S\tau, B\sigma, t)\big) \leq 0,$$

and

$$\psi\big(\Theta(\Delta\tau, B\sigma, \alpha t), \Theta(S\tau, \Upsilon\sigma, t), \Theta(S\tau, \Delta\tau, t), \Theta(\Upsilon\sigma, B\sigma, t), \Theta(\Delta\tau, \Upsilon\sigma, t) \diamond \Theta(S\tau, B\sigma, t)\big) \leq 0.$$

$S(C)$ and $\Upsilon(C)$ are closed subsets of $C$.

The pair $(\Delta, S)$ and $(B, \Upsilon)$ share the EA property.

Then the pair of mappings $(\Delta, S)$ and $(B, \Upsilon)$ have a coincidence point. Additionally, $\Delta, S$ $B$, and $\Upsilon$ have the common unique FP both provided pairs $(\Delta, S)$ and $(B, \Upsilon)$ are weakly compatible.

Proof: The outcome of Theorem 4.2 follows if we assume that $\rho(t) = 1$.

## 4 | CONCLUSION

In this manuscript, we investigated some new type FPT in the setting of NMS. We apply the common EA property to examine the existences and uniqueness of fixed and common FP for weakly compatible mapping within NMS. This work uses the concept of implicit relation to establish a number of common FP results of integral type in NMS. Moreover, we have provided some nontrivial examples to demonstrate the viability of the proposed methods. Since our results is more general than the class of FMS and IFMSs. This study can be easily generalized in the structure of neutrosophic cone MS, neutrosophic triple partial MS, neutrosophic triple V-generalized MS, neutrosophic triple partial bipolar MS, and neutrosophic triple partial g-MS.

**Acknowledgment**: We are thankful to the editors and anonymous reviewers for their constructive suggestions.

**Author Contributions:** Conceptualization, K.A.; methodology, K.A., F.A; software, F.A., H.A.S.; validation, K.A., S.Y; investigation, K.A., H.A.S; data curation, H.A.S., F.A; writing—original draft preparation, K.A., F.A; writing—review and editing, S.K. Author has read and agreed to the published version of the manuscript.

**Declaration of Conflicting Interest**: The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.



**Funding**: The author(s) received no financial support for the research, authorship, and/or publication of this article.

**Data Availability Statement**: Data that supports the findings of this study are available on request from the corresponding author.

**Plagiarism Statement:** This article was scanned by the plagiarism program. No plagiarism was detected.

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