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Complementary Extended Plus Operation of Soft Sets

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ABSTRACT

Soft set theory, introduced by Molodtsov in 1999, offers a versatile framework for handling uncertainty in data analysis and decision-making processes. Unlike traditional set theory, soft sets allow elements to be parametrized, providing a more nuanced representation of uncertainty. In this context, a new kind of soft set operation called complementary extended plus soft set operation, is defined in this paper in order to contribute to the theory. The properties of the operation are examined in detail, along with its distributions over other soft set operations, to establish the relationship between the complementary extended plus operation and other operations.

Keywords: Soft set, soft set operations, Conditional complements, Complementary extended plus operation.

(MSC2020) Codes: 03E72, 03E99

1 | INTRODUCTION

In various real-world scenarios, decision-making processes and data analysis tasks often encounter uncertainty due to imprecision, vagueness, and ambiguity. Traditional set theory provides a powerful framework for organizing and manipulating data, but it falls short in capturing and representing this inherent uncertainty. In response to this limitation, soft set theory emerges as a promising approach to handling uncertain and indeterminate information. Introduced in [1], soft set theory offers a soft and intuitive extension of classical set theory by incorporating the notion of parametrization. Unlike crisp sets, where an element either fully belongs or does not belong to a set, soft sets allow elements to possess varying degrees of membership, reflecting the degree of uncertainty associated with their inclusion. This flexibility enables soft sets to effectively model and manage uncertain information, making them well-suited for applications in diverse fields such as decision support systems, pattern recognition, and information fusion.

Since its introduction, soft set theory has been widely applied across theoretical and practical domains, leading to numerous new studies in the literature. [2] paved the way for new studies in soft set theory by defining the equality of two soft sets, subset and superset of a soft set, complement of a soft set, and soft binary operations such as union and intersection operations for soft sets. [3] redefined the concepts of soft subset and intersection of two soft sets based on set theoretical concepts. Then, [4] proposed some new soft set operations. [5] along with [6] analyzed these soft set operations in detail. Extended difference and extended symmetric difference of soft sets were proposed in [7] and [8], respectively, and their properties were studied in detail in relation to other soft set operations.

When analyzing the studies conducted thus far, it is seen that soft set operations generally proceed under two main headings: restricted soft set operations and extended soft set operations. [9] defined the soft binary piecewise difference operation for soft sets, and examined its properties. [10] later studied the properties of this operation. The definitions of inclusive complement and exclusive complement of sets as new concepts of set theory were proposed in [11], and these concepts were applied to group theory. [12] introduced five new binary complement concepts similar to the binary complement operations in [11]. Inspired by the new set operations defined in this study, [13] proposed many new restricted and extended soft set operations and analyzed their properties. Moreover, the form of soft binary piecewise operation, the pioneer of which is in [9], was slightly modified by taking the complement of the image set in the first row, and thus the complementary soft binary piecewise operations have been studied in detail by various researchers [14-21]. [22] and [23], on the other hand, modified the form of the existing extended soft set operations in the literature, for other applications of soft sets as regards algebraic structures, we refer to the following: [24-34].

The investigation of operations defined on sets, together with the sets themselves, constitutes a fundamental aspect of algebraic structures, aiming to classify them within the realm of algebra. Just as operations in classical set theory form the foundation for understanding sets, soft set operations play a crucial role in the framework of soft set theory.

In this paper, in order to contribute to the theory of soft sets, a new form of soft set operation called complementary extended plus operation is introduced and its properties are discussed in detail. The distributions of complementary extended plus operation over other types of soft set operations are analyzed in order to obtain the relation of the operation with other soft set operations and thus to lead future studies examining the algebraic studies of soft sets with respect to this new operation.

2 | PRELIAMINARIES

2.1. Definition [1]: Let U be the universal set, E be the parameter set, P(U) be the power set of U, and let D ⊆ E. A pair (F, D) is called a soft set on U. Here, F is a function given by F: D → P(U).

The set of all soft sets over U is denoted by $S_E(U)$. Let K be a fixed subset of E, then the set of all soft sets over U with the fixed parameter set K is denoted by $S_K(U)$.

For all undefined concepts regarding null soft set, whole soft set, absolute soft set we refer to [4],[6], for soft subset and equal soft set, we refer to [3], for the soft complement of a soft set, we refer to [4].

Two new complements were introduced proposed in [11]. For two sets D and Y, these binary operations are as D+Y=D'∪Y, DθY=D'∩Y'. [12] examined some new binary operations as follows: D*Y=D'∪Y', DγY= D'∩Y, DλY=D∪Y'.

Let "⍟" be used to represent the set operations, then all types of soft set operations are defined as follows:

2.6. Definition [4,6,13]: Let (F, Ж), (G,Ƀ) ∈ $S_E(U)$. The restricted ⍟ operation of (F, Ж) and (G,Ƀ) is the soft set (Б, K), denoted to be (F, Ж)$⍟_{\Re}$(G,Ƀ) = (H, K), where K=Ж ∩ Ƀ ≠ ∅ and for all ƥ ∈ K, Б (ƥ) = F (ƥ)⍟G (ƥ). Here, if K = Ж ∩Ƀ = ∅, then (F, Ж)$⍟_R$(G,Ƀ)=$\emptyset_\emptyset$.

2.7. Definition [2,4,7,8,13]: Let (F, Ж), (G, Y) ∈ $S_E(U)$. The extended ⍟ operation (F, Ж) and (G,Ƀ) is the soft set (Б, K), denoted by (F, Ж) $⍟_\varepsilon$(G, Ƀ) = (Б, K), where K = Ж ∪ Ƀ and for all ƥ ∈ K,

$$Б(ƥ) = \begin{cases} F(ƥ), & ƥ \in Ж - Ƀ \\ G(ƥ), & ƥ \in Ƀ - Ж \\ F(ƥ)⍟G(ƥ), & ƥ \in Ж \cap Ƀ \end{cases}$$

2.8. Definition [23,24]: Let (F, Ж), (G,Ƀ) ∈ $S_E(U)$. The complementary extended ⍟ operation (F, Ж) and (G,Ƀ) is the soft set (Б, K), denoted by (F, Ж) $\overset{*}{⍟}_\varepsilon$ (G, Ƀ) = (Б, K), where K = Ж ∪ Ƀ and for all ƥ ∈ K,

$$Б(ƥ) = \begin{cases} F'(ж), & ƥ \in Ж - Ƀ \\ G'(ж), & ƥ \in Ƀ - Ж \\ F(ж)⍟G(ж), & ƥ \in Ж \cap Ƀ \end{cases}$$

2.9. Definition [9], [10], [35], [36]**:** Let (F, Ж), (G,Ƀ) ∈ $S_E(U)$. The soft binary piecewise ⍟ of (F, Ж) and (G, Ƀ) is the soft set (Б, Ж), denoted by (F, Ж)$\underset{⍟}{\sim}$(G,Ƀ) = (Б, Ж), where for all ƥ ∈ Ж,

$$Б(ƥ) = \begin{cases} F(ƥ), & ƥ \in Ж - Ƀ \\ F(ƥ)⍟G(ƥ), & ƥ \in Ж \cap Ƀ \end{cases}$$

2.10. Definition [14-22]: Let (F, Ж), (G,Ƀ) ∈ $S_E(U)$. The complementary soft binary piecewise ⍟ of (F, Ж) and (G, Ƀ) is the soft set (Б, Ж), denoted by (F, Ж) $\overset{*}{\underset{⍟}{\sim}}$ (G,Ƀ) = (Б, Ж), where for all ƥ ∈ Ж,

$$Б(ƥ) = \begin{cases} F'(ƥ), & ƥ \in Ж - Ƀ \\ F(ƥ)⍟G(ƥ), & ƥ \in Ж \cap Ƀ \end{cases}$$

For band, semilattice and bounded semilattice, we refer to [37] and for possible applications of graphs and network research concerning soft sets and soft algebraic structures, we refer to [38],[39], [40], [41].

3 | COMPLEMENTARY EXTENDED PLUS OPERATION

In this section, a new soft set operation called complementary extended plus operation of soft sets is introduced with its example, and its full algebraic properties are analyzed.

Definition 3.1: Let (F, Ж) and (G, Ɇ) be soft sets over U. The complementary extended plus operation (+) of (F, Ж) and (G, Ɇ) is the soft set (H, Ş), denoted by (F, Ж) $\overset{*}{+}_{\varepsilon}$ (G,Ɇ) =(H, Ş), where Ş= Ж ∪Ɇ, and

$$H(ж)=\begin{cases} F'(ж), & ж\in Ж-Ɇ \\ G'(ж), & ж\in Ɇ-Ж \\ F(ж)+G(ж), & ж\in Ж\cap Ɇ \end{cases}$$

for all ж∈ Ş=Ж∪Ɇ. Here, F(ж)+G(ж) = F′(ж)∪G(ж).

Example 3.2 Let E= {e_1,e_2,e_3,e_4} be the parameter set, Ж= {e_1, e_3} and Ɇ= {e_2, e_3, e_4} be two subsets of E and U= {h_1,h_2,h_3,h_4,h_5} be the universal set. Assume that (F, Ж) = {(e_1,{h_2,h_5}), (e_3,{h_1,h_2,h_5})} and (G, Ɇ) = {(e_2,{h_1,h_4,h_5}), (e_3,{h_2,h_3,h_4}), (e_4,{h_3,h_5})} be two soft sets over U. Let (F, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ɇ) = (Б, Ж∪Ɇ), where for all ж ∈ Ж∪Ɇ,

$$Б(ж)=\begin{cases} F'(ж), & ж\in Ж-Ɇ \\ G'(ж), & ж\in Ɇ-Ж \\ F'(ж)\cup G(ж), & ж\in Ж\cap Ɇ \end{cases}$$

Here since Ж∪Ɇ={e_1,e_2,e_3,e_4}, Ж-Ɇ={e_1}, Ɇ-Ж={e_2,e_4}, Ж∩Ɇ={e_3} thus, Б(e_1) =F'(e_1)={ h_1,h_3,h_4}, Б(e_2) =G'(e_2)={h_2,h_3},Б(e_4) =G'(e_4)={h_1,h_2, h_4}andБ(e_3)=F'(e_3)∪G(e_3)={h_3,h_4}∪{h_2,h_3,h_4}={h_2,h_3,h_4}. Hence,

(F, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ɇ) = {($e_1$,{$h_1$,$h_3$,$h_4$}), ($e_2$,{$h_2$,$h_3$}), ($e_3$,{$h_2$,$h_3$,$h_4$}), ($e_4$,{$h_1$,$h_2$, h_4})}.

Theorem 3.3: Let (F, Ж), (G, Ɇ), (Б, Ş), (G, Ж), (Б, Ж), (K, Ж) and (L, Ɇ) be soft sets over U. Then,

1) The set SE (U) is closed under $\overset{*}{+}_{\varepsilon}$.

Proof: $\overset{*}{+}_{\varepsilon}$ is a binary operation on SE(U). Indeed,

$$\overset{*}{+}_{\varepsilon}: SE(U)\times SE(U)\rightarrow SE(U)$$

$$((F, Ж), (G, Ɇ)) \rightarrow (F, Ж) \overset{*}{+}_{\varepsilon}(G, Ɇ) = (Б, Ж\cup Ɇ)$$

Similarly

$$\overset{*}{+}_{\varepsilon}: S_Ж(U)\times S_Ж(U)\rightarrow S_Ж(U)$$

$$((F, Ж), (G, Ж)) \rightarrow (F, Ж) \overset{*}{+}_{\varepsilon}(G, Ж)=(T, Ж\cup Ж)=(T,Ж)$$

2) $[(F,Җ) \overset{*}{+}_{\varepsilon} (G, Ƀ)] \overset{*}{+}_{\varepsilon} (Б,Ş) \neq (F,Җ) \overset{*}{+}_{\varepsilon} [(G,Ƀ) \overset{*}{+}_{\varepsilon} (Б,Ş)]$.

Proof: Let $(F,Җ) \overset{*}{+}_{\varepsilon} (G, Ƀ) = (T, Җ\cup Ƀ)$, where for all $җ\in Җ\cup Ƀ$,

$$T(ж)=\begin{cases} F'(ж), & җ\in Җ-Ƀ \\ G'(ж), & җ\in Ƀ-Җ \\ F'(ж)\cup G(ж), & җ\in Җ\cap Ƀ \end{cases}$$

Let $(T, Җ\cup Ƀ) \overset{*}{+}_{\varepsilon} (Б, Ş) = (M, Җ\cup Ƀ\cup Ş)$, where for all $җ \in Җ\cup Ƀ\cup Ş$,

$$M(ж)=\begin{cases} T'(ж), & җ\in (Җ\cup Ƀ)-Ş \\ Б'(ж), & җ\in Ş-(Җ\cup Ƀ) \\ T'(ж)\cup Б(ж), & җ\in (Җ\cup Ƀ)\cap Ş \end{cases}$$

Thus,

$$M(ж)=\begin{cases} F(ж), & җ\in (Җ-Ƀ)-Ş=Җ\cap Ƀ'\cap Ş' \\ G(ж), & җ\in (Ƀ-Җ)-Ş=Җ'\cap Ƀ\cap Ş' \\ F(ж)\cap G'(ж), & җ\in (Җ\cap Ƀ)-Ş=Җ\cap Ƀ\cap Ş' \\ Б'(ж), & җ\in Ş-(Җ\cup Ƀ)=Җ'\cap Ƀ'\cap Ş \\ F(ж)\cup Б(ж), & җ\in (Җ-Ƀ)\cap Ş=Җ\cap Ƀ'\cap Ş \\ G(ж)\cup Б(ж), & җ\in (Ƀ-Җ)\cap Ş=Җ'\cap Ƀ\cap Ş \\ \big(F(ж)\cap G'(ж)\big)\cup Б(ж), & җ\in (Җ\cap Ƀ)\cap Ş=Җ\cap Ƀ\cap Ş \end{cases}$$

Let $(G, Ƀ) \overset{*}{+}_{\varepsilon} (Б, Ş) = (K, Ƀ\cup Ş)$, where for all $җ\in Ƀ\cup Ş$,

$$K(ж)=\begin{cases} G'(ж), & җ\in Ƀ-Ş \\ Б'(ж), & җ\in Ş-Ƀ \\ G'(ж)\cup Б(ж), & җ\in Ƀ\cap Ş \end{cases}$$

Assume that $(F,Җ) \overset{*}{+}_{\varepsilon} (K, Ƀ\cup Ş) = (S, Җ\cup Ƀ\cup Ş)$, where for all $җ\in Җ\cup Ƀ\cup Ş$,

$$S(ж)=\begin{cases} F'(ж), & җ\in Җ-(Ƀ\cup Ş) \\ K'(ж), & җ\in (Ƀ\cup Ş)-Җ \\ F'(ж)\cup K(ж), & җ\in Җ\cap (Ƀ\cup Ş) \end{cases}$$

Thus,

$$S(ж)=\begin{cases} F'(ж), & җ\in Җ-(Ƀ\cup Ş)=Җ\cap Ƀ'\cap Ş' \\ G(ж), & җ\in (Ƀ-Ş)-Җ=Җ'\cap Ƀ\cap Ş' \\ Б(ж), & җ\in (Ş-Ƀ)-Җ=Җ'\cap Ƀ'\cap Ş \\ G(ж)\cap Б'(ж), & җ\in (Ƀ\cap Ş)-Җ=Җ'\cap Ƀ\cap Ş \\ F'(ж)\cup G'(ж), & җ\in Җ\cap (Ƀ-Ş)=Җ\cap Ƀ\cap Ş' \\ F'(ж)\cup Б'(ж), & җ\in Җ\cap (Ş-Ƀ)=Җ\cap Ƀ'\cap Ş \\ F'(ж)\cup (G'(ж)\cup Б(ж)), & җ\in Җ\cap (Ƀ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Thereby, $M\neq S$, implying that $\overset{*}{+}_{\varepsilon}$ is not associative in $S_E(U)$.

3) $[(F,Җ) \overset{*}{+}_{\varepsilon} (G, Җ)] \overset{*}{+}_{\varepsilon} (Б, Җ) \neq (F,Җ) \overset{*}{+}_{\varepsilon} [(G,Җ) \overset{*}{+}_{\varepsilon} (Б, Җ)]$.

Proof: Let $(F,Җ) \overset{*}{+}_{\varepsilon} (G, Җ) = (T, Җ\cup Җ)$, where for all $җ\in Җ\cup Җ=Җ$,

$$T(ж)=\begin{cases} F'(ж), & җ\in Җ-Җ=\emptyset \\ G'(ж), & җ\in Җ-Җ=\emptyset \\ F'(ж)\cup G(ж), & җ\in Җ\cap Җ=Җ \end{cases}$$

Let (T, Җ) $\overset{*}{+}_{\varepsilon}$ (Б, Ж) = (M, Җ∪Ж), where for all җ∈Ж,

$$M(\text{җ})=\begin{cases} T'(\text{җ}), & \text{җ}\in\text{Ж}-\text{Җ}=\emptyset \\ Б'(\text{җ}), & \text{җ}\in\text{Ж}-\text{Җ}=\emptyset \\ T'(\text{җ})\cup Б(\text{җ}), & \text{җ}\in\text{Җ}\cap\text{Ж}=\text{Ж} \end{cases}$$

Thus,

$$M(\text{җ})=\begin{cases} T'(\text{җ}), & \text{җ}\in\text{Җ}-\text{Ж}=\emptyset \\ Б'(\text{җ}), & \text{җ}\in\text{Ж}-\text{Җ}=\emptyset \\ (F(\text{җ})\cap G'(\text{җ}))\cup Б(\text{җ}), & \text{җ}\in\text{Җ}\cap\text{Ж}=\text{Ж} \end{cases}$$

Let (G, Җ) $\overset{*}{+}_{\varepsilon}$ (Б, Ж) = (L, Җ∪Ж), where for all җ∈Ж,

$$L(\text{җ})=\begin{cases} G'(\text{җ}), & \text{җ}\in\text{Җ}-\text{Ж}=\emptyset \\ Б'(\text{җ}), & \text{җ}\in\text{Ж}-\text{Җ}=\emptyset \\ G'(\text{җ})\cup Б(\text{җ}), & \text{җ}\in\text{Җ}\cap\text{Ж}=\text{Ж} \end{cases}$$

Let (F, Җ) $\overset{*}{+}_{\varepsilon}$ (L, Ж) = (N, Җ∪Ж), where for all җ∈Ж,

$$N(\text{җ})=\begin{cases} F'(\text{җ}), & \text{җ}\in\text{Җ}-\text{Ж}=\emptyset \\ L'(\text{җ}), & \text{җ}\in\text{Ж}-\text{Җ}=\emptyset \\ F'(\text{җ})\cup L(\text{җ}), & \text{җ}\in\text{Җ}\cap\text{Ж}=\text{Ж} \end{cases}$$

Hence,

$$N(\text{җ})=\begin{cases} F'(\text{җ}), & \text{җ}\in\text{Җ}-\text{Ж}=\emptyset \\ L'(\text{җ}), & \text{җ}\in\text{Ж}-\text{Җ}=\emptyset \\ F'(\text{җ})\cup(G'(\text{җ})\cup Б(\text{җ})), & \text{җ}\in\text{Җ}\cap\text{Ж}=\text{Ж} \end{cases}$$

M≠N, implying that $\overset{*}{+}_{\varepsilon}$ is not associative in $S_{\text{Җ}}(U)$, where Җ ⊆ E is a fixed set of parameters.

4) (F, Җ) $\overset{*}{+}_{\varepsilon}$(G, Ƀ)≠(G, Ƀ) $\overset{*}{+}_{\varepsilon}$ (F, Җ).

Proof: Let (F, Җ) $\overset{*}{+}_{\varepsilon}$ (G, Ƀ) = (Б, Җ∪Ƀ), where for all җ∈Җ∪Ƀ,

$$Б(\text{җ})=\begin{cases} F'(\text{җ}), & \text{җ}\in\text{Җ}-\text{Ƀ} \\ G'(\text{җ}), & \text{җ}\in\text{Ƀ}-\text{Җ} \\ F'(\text{җ})\cup G(\text{җ}), & \text{җ}\in\text{Җ}\cap\text{Ƀ} \end{cases}$$

Assume that (G, Ƀ) $\overset{*}{+}_{\varepsilon}$(F, Җ) = (T, Ƀ∪Җ), where for all җ∈Ƀ∪Җ,

$$T(\text{җ})=\begin{cases} G'(\text{җ}), & \text{җ}\in\text{Ƀ}-\text{Җ} \\ F'(\text{җ}), & \text{җ}\in\text{Җ}-\text{Ƀ} \\ G'(\text{җ})\cup F(\text{җ}), & \text{җ}\in\text{Ƀ}\cap\text{Җ} \end{cases}$$

Thus, Б≠T, implying that (F, Җ) $\overset{*}{+}_{\varepsilon}$(G, Җ) ≠ (G, Җ) $\overset{*}{+}_{\varepsilon}$(F, Җ). That is, $\overset{*}{+}_{\varepsilon}$ operation is not commutative in both $S_E(U)$ and $S_{\text{Җ}}(U)$.

5) (F, Җ) $\overset{*}{+}_{\varepsilon}$(F, Җ) = $U_{\text{Җ}}$.

Proof: Let (F, Җ) $\overset{*}{+}_{\varepsilon}$(F, Җ) = (Б, Җ∪Җ), where for all җ∈Җ,

$$Б(ж)=\begin{cases} F^{'}(ж), & ж\in Ж\text{-}Ж=\emptyset \\ F^{'}(ж), & ж\in Ж\text{-}Ж=\emptyset \\ F'(ж)\cup F(ж), & ж\in Ж\cap Ж=Ж \end{cases}$$

Hence, for all ж∈Ж, Б(ж)= F'(ж)∪F(ж)=U and so (Б, Ж) = $U_Ж$.

The operation $\overset{*}{+}_{\varepsilon}$ is not idempotent in $S_E(U)$.

6) (F, Ж) $\overset{*}{+}_{\varepsilon}$ $\emptyset_Ж$= (F, Ж) r.

Proof: Let $\emptyset_Ж$= (S, Ж). Thus, for all ж∈Ж, S(ж)= ∅. Let (F, Ж) $\overset{*}{+}_{\varepsilon}$(S, Ж) = (Б, Ж∪Ж), where for all ж∈Ж,

$$Б(ж)=\begin{cases} F^{'}(ж), & ж\in Ж\text{-}Ж=\emptyset \\ S^{'}(ж), & ж\in Ж\text{-}Ж=\emptyset \\ F'(ж)\cup S(ж), & ж\in Ж\cap Ж=Ж \end{cases}$$

Hence, Б(ж)= F'(ж)∪S(ж)= F'(ж)∪∅=F′(ж), for all ж∈Ж, implying that (Б, Ж) = (F, Ж) r.

7) $\emptyset_Ж$ $\overset{*}{+}_{\varepsilon}$(F, Ж) = $U_Ж$.

Proof: Let $\emptyset_Ж$= (S, Ж). Thus, S(ж)= ∅ for all ж∈Ж. Let (S, Ж) $\overset{*}{+}_{\varepsilon}$(F, Ж) = (Б, Ж∪Ж), where for all ж∈Ж,

$$Б(ж)=\begin{cases} S^{'}(ж), & ж\in Ж\text{-}Ж=\emptyset \\ F^{'}(ж), & ж\in Ж\text{-}Ж=\emptyset \\ S'(ж)\cup F(ж), & ж\in Ж\cap Ж=Ж \end{cases}$$

Hence, Б(ж)= S'(ж)∪F(ж)= U∪F(ж)=U, for all ж∈Ж, implying that (Б, Ж) = $U_Ж$.

8) $\emptyset_E$ $\overset{*}{+}_{\varepsilon}$(F, Ж) =$U_E$.

Proof: Let $\emptyset_E$= (S, E). Thus, S(ж)= ∅, for all ж∈E. Let (S, E) $\overset{*}{+}_{\varepsilon}$(F, Ж) = (Б, E∪Ж=E), where for all ж∈E,

$$Б(ж)=\begin{cases} S^{'}(ж), & ж\in E\text{-}Ж=Ж' \\ F^{'}(ж), & ж\in Ж\text{-}E=\emptyset \\ S'(ж)\cup F(ж), & ж\in Ж\cap E=Ж \end{cases}$$

Since S'(ж)∪F(ж)= U∪F(ж)=U for all ж∈Ж,

$$H(ж)=\begin{cases} U, & ж\in E\text{-}Ж=Ж' \\ F^{'}(ж), & ж\in Ж\text{-}E=\emptyset \\ U, & ж\in Ж\cap E=Ж \end{cases}$$

Thus, for all ж∈E, Б(ж)=U, and so (Б, E) = U_E.

9) (F, Ж) $\overset{*}{+}_{\varepsilon}$ $\emptyset_\emptyset$= $\emptyset_\emptyset$ $\overset{*}{+}_{\varepsilon}$(F, Ж) = (F, Ж) r.

Proof: Let $\emptyset_\emptyset$= (K, ∅) and (F, Ж) $\overset{*}{+}_{\varepsilon}$(K, ∅) = (Q, Ж∪∅) = (Q, Ж), where for all ж∈Ж,

$$Q(ж)=\begin{cases} F^{'}(ж), & ж\in Ж\text{-}\emptyset=Ж \\ K^{'}(ж), & ж\in \emptyset\text{-}Ж=\emptyset \\ F'(ж)\cup K(ж), & ж\in Ж\cap\emptyset=\emptyset \end{cases}$$

Hence, Q(ж)= F'(ж) for all ж∈Ж, and (Q, Ж) = (F, Ж) r.

Similarly, let $\emptyset_\emptyset$ $\overset{*}{+}_{\varepsilon}$(F, Ж) = (W, ∅ ∪ Ж) = (W, Ж), where for all ж∈Ж,

$$W(ж)=\begin{cases} K'(ж), & ж\in\emptyset\text{-}Ж=\emptyset \\ F'(ж), & ж\in Ж\text{-}\emptyset=Ж \\ K'(ж)\cup F(ж), & ж\in\emptyset\cap Ж=\emptyset \end{cases}$$

Hence, W(ж)= F'(ж), for all ж∈Ж, and (W, Ж) = (F, Ж) r.

10) $U_Ж \overset{*}{+}_{\varepsilon}$ (F, Ж) = (F, Ж).

Proof: Let $U_Ж$= (T, Ж), where for all ж∈Ж, T(ж)=U. Let (T, Ж) $\overset{*}{+}_{\varepsilon}$ (F,Ж) = (Б, Ж∪Ж), where for all ж∈Ж,

$$Б(ж)=\begin{cases} T'(ж), & ж\in Ж\text{-}Ж=\emptyset \\ F'(ж), & ж\in Ж\text{-}Ж=\emptyset \\ T'(ж)\cup F(ж), & ж\in Ж\cap Ж=Ж \end{cases}$$

Hence, Б(ж)= T'(ж)∪F(ж)=∅∪F(ж)=F(ж) for all ж∈Ж, and (Б, Ж) = (F, Ж). That is, the left identity element of $\overset{*}{+}_{\varepsilon}$ in $S_Ж(U)$ is the soft set $U_Ж$.

11) (F, Ж) $\overset{*}{+}_{\varepsilon}$ $U_Ж$= $U_Ж$.

Proof: Let $U_Ж$= (T, Ж), where for all ж∈Ж, T(ж)=U. Let (F, Ж) $\overset{*}{+}_{\varepsilon}$ (T,Ж) = (Б, Ж∪Ж), where for all ж∈Ж,

$$Б(ж)=\begin{cases} F'(ж), & ж\in Ж\text{-}Ж=\emptyset \\ T'(ж), & ж\in Ж\text{-}Ж=\emptyset \\ F'(ж)\cup T(ж), & ж\in Ж\cap Ж=Ж \end{cases}$$

Hence, Б(ж)= F'(ж)∪T(ж)= F'(ж)∪U=U for all ж∈Ж, and (Б, Ж) = $U_Ж$. That is, the right absorbing element of $\overset{*}{+}_{\varepsilon}$ in $S_Ж(U)$ is $U_Ж$.

12) (F, Ж) $\overset{*}{+}_{\varepsilon}$ (F, Ж) r = (F, Ж) r.

Proof: Let (F, Ж) r = (Б, Ж), where for all ж∈Ж, Б(ж)=F'(ж). Let (F, Ж) $\overset{*}{+}_{\varepsilon}$ (Б,Ж) =(T, Ж∪Ж), where for all ж∈Ж,

$$T(ж)=\begin{cases} F'(ж), & ж\in Ж\text{-}Ж=\emptyset \\ Б'(ж), & ж\in Ж\text{-}Ж=\emptyset \\ F'(ж)\cup Б(ж), & ж\in Ж\cap Ж=Ж \end{cases}$$

Here, T(ж)= F'(ж)∪Б(ж)= F'(ж)∪F'(ж)= F'(ж) for all ж∈Ж, and thus (T, Ж) = (F, Ж) r . In $S_E(U)$, the complement of every soft set is its right absorbing element for the operation $\overset{*}{+}_{\varepsilon}$.

13) (F, Ж) r $\overset{*}{+}_{\varepsilon}$ (F, Ж) = (F, Ж).

Proof: Let (F,Ж)r = (Б,Ж), where for all ж∈Ж, Б(ж)=F'(ж). Let (Б,Ж) $\overset{*}{+}_{\varepsilon}$ (F,Ж)=(T,Ж∪Ж), where for all ж∈Ж,

$$T(ж)=\begin{cases} Б'(ж), & ж\in Ж\text{-}Ж=\emptyset \\ F'(ж), & ж\in Ж\text{-}Ж=\emptyset \\ Б'(ж)\cup F(ж), & ж\in Ж\cap Ж=Ж \end{cases}$$

Here, T(ж)= Б'(ж)∪F(ж)= F(ж)∪F(ж)= F(ж) for all ж∈Ж, and thus (T,Ж)= (F,Ж). In $S_E(U)$, the complement of every soft set is its left identity element for the operation $\overset{*}{+}_{\varepsilon}$.

14) [(F, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ɓ)] r = (F, Ж) $\setminus_{\varepsilon}$(G, Ɓ).

Proof: Let (F, Ж) $\overset{*}{+}$ (G, Ɓ) = (Б, Ж∪Ɓ), where for all ж∈Ж∪Ɓ,

$$Б(ж)=\begin{cases} F'(ж), & ж\in Ж\text{-}Ɓ \\ G'(ж), & ж\in Ɓ\text{-}Ж \\ F'(ж)\cup G(ж), & ж\in Ж\cap Ɓ \end{cases}$$

Let (Б, Ж∪Ɓ) r = (T, Ж∪Ɓ), where for all ж∈Ж∪Ɓ,

$$T(ж)=\begin{cases} F(ж), & ж\in Ж\text{-}Ɓ \\ G(ж), & ж\in Ɓ\text{-}Ж \\ F(ж)\cap G'(ж), & ж\in Ж\cap Ɓ \end{cases}$$

Hence, (T, Ж∪Ɓ) =(F, Ж) $\setminus_{\varepsilon}$(G,Ɓ).

15) (F, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ж)= $\emptyset_Ж$ ⇔(F, Ж) = $U_Ж$ and (G,Ж) = $\emptyset_Ж$.

Proof: Let (F, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ж) = (T, Ж∪Ж), where for all ж∈Ж,

$$T(ж)=\begin{cases} F'(ж), & ж\in Ж\text{-}Ж=\emptyset \\ G'(ж), & ж\in Ж\text{-}Ж=\emptyset \\ F'(ж)\cup G(ж), & ж\in Ж\cap Ж=Ж \end{cases}$$

Since (T,Ж)= $\emptyset_Ж$, T(ж)=∅ for all ж∈Ж. Thus, for all ж∈Ж, F'(ж)∪G(ж)=∅. So, for all ж ∈ Ж, F'(ж)=∅ and G(ж)=∅. Hence, (F, Ж = $U_Ж$ and (G,Ж) = $\emptyset_Ж$.

16) $\emptyset_Ж$ $\tilde{\subseteq}$(F, Ж) $\overset{*}{+}_{\varepsilon}$(G,Ɓ), $\emptyset_Ɓ$ $\tilde{\subseteq}$(F, Ж) $\overset{*}{+}_{\varepsilon}$(G,Ɓ), $\emptyset_{Ж\cup Ɓ}$ $\tilde{\subseteq}$(F, Ж) $\overset{*}{+}_{\varepsilon}$(G,Ɓ), (F, Ж) $\overset{*}{+}_{\varepsilon}$(G,Ɓ) $\tilde{\subseteq}$ $U_{Ж\cup Ɓ}$.

17) (F,Ж)r$\tilde{\subseteq}$ (F, Ж) $\overset{*}{+}_{\varepsilon}$(G,Ɓ) and (G,Ж) $\tilde{\subseteq}$ (F, Ж) $\overset{*}{+}_{\varepsilon}$(G,Ж).

Proof: Let (F, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ɓ)=(Б, Ж ∪ Ɓ), where for all ж∈Ж∪Ɓ,

$$Б(ж)=\begin{cases} F'(ж), & ж\in Ж\text{-}Ɓ \\ G'(ж), & ж\in Ɓ\text{-}Ж \\ F'(ж)\cup G(ж), & ж\in Ж\cap Ɓ \end{cases}$$

Thereby, Б(ж)=F′(ж) ⊆ F′(ж), for all ж∈Ж-Ɓ and Б(ж)=F′(ж) ⊆ F′(ж) ∪ G(ж), for all ж∈ Ж∩Ɓ. Thus, (F, Ж) r$\tilde{\subseteq}$ (F, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ɓ). Similarly, G(ж) ⊆ F′(ж) ∪ G(ж), for all ж∈Ж. Thus, (G, Ж) $\tilde{\subseteq}$ (F, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ж).

18) If (F, Ж) $\tilde{\subseteq}$(G, Ж), then (Б, Ж) $\overset{*}{+}_{\varepsilon}$(F, Ж) $\tilde{\subseteq}$ (Б, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ж).

Proof: Let (F, Ж) $\tilde{\subseteq}$ (G, Ж), where for all ж∈Ж, F(ж)⊆G(ж). Let (Б,Ж) $\overset{*}{+}_{\varepsilon}$(F,Ж)= (W, Ж), where for all ж∈Ж,

$$W(ж)=\begin{cases} Б^{'}(ж), & ж\in Ж-Ж=\emptyset \\ F^{'}(ж), & ж\in Ж-Ж=\emptyset \\ Б'(ж)\cup F(ж), & ж\in Ж\cap Ж=Ж \end{cases}$$

Let (Б,Ж) $\overset{*}{+}_{\varepsilon}$(G,Ж) = (L, Ж), where for all ж∈Ж,

$$L(ж)=\begin{cases} Б^{'}(ж), & ж\in Ж-Ж=\emptyset \\ G^{'}(ж), & ж\in Ж-Ж=\emptyset \\ Б'(ж)\cup G(ж), & ж\in Ж\cap Ж=Ж \end{cases}$$

Thus, for all ж∈Ж, W (ж)=Б'(ж)∪F(ж) ⊆ Б'(ж)∪G(ж)=L (ж). Hence (Б, Ж) $\overset{*}{+}_{\varepsilon}$(F, Ж) $\widetilde{\subseteq}$ (Б, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ж).

19) If (Б, Ж) $\overset{*}{+}_{\varepsilon}$(F, Ж) $\widetilde{\subseteq}$ (Б, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ж), then (F, Ж) $\widetilde{\subseteq}$(G, Ж) needs not be true.

Proof: Let E= $\{e_1,e_2,e_3,e_4,e_5\}$ be the parameter set, Ж= $\{e_1,e_3\}$ be the subset of E, U= $\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set. Let (F, Ж) = $\{(e_1,\{h_2,h_5\}),\ (e_3,\{h_1,h_2,h_5\})\}$, (G, Ж) =$\{(e_1,\{h_2\}),(e_3,\{h_1,h_2\})\}$, (Б,Ж)=$\{(e_1,\emptyset),(e_3,\emptyset)\}$be soft sets over U. Let (Б,Ж) $\overset{*}{+}_{\varepsilon}$(F,Ж)= (L, Ж). Then, (L, Ж) =$\{(e_1,U), (e_3,U)\}$ and let (Б, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ж)= (K, Ж), thus (K, Ж) =$\{(e_1,U),(e_3,U)\}$. Hence (Б, Ж) $\overset{*}{+}_{\varepsilon}$(F, Ж) $\widetilde{\subseteq}$ (Б, Ж) $\overset{*}{+}_{\varepsilon}$(G, Ж), but (F, Ж) is not soft subset of (G, Ж).

20) If (G, Ж) $\widetilde{\subseteq}$ (F, Ƀ) and (K, Ж) $\widetilde{\subseteq}$ (L, Ƀ), then (F, Ж) $\overset{*}{+}_{\varepsilon}$(K, Ж) $\widetilde{\subseteq}$(G, Ƀ) $\overset{*}{+}_{\varepsilon}$(L, Ƀ).

Proof: Let (G, Ж) $\widetilde{\subseteq}$ (F, Ƀ) and (K, Ж) $\widetilde{\subseteq}$ (L, Ƀ). Hence, Ж ⊆ Ƀ and for all ж∈Ж, G (ж)⊆ F(ж) and K (ж)⊆ L(ж). Let (F, Ж) $\overset{*}{+}_{\varepsilon}$(K,Ж) = (W, Ж). Thus,

$$W(ж)=\begin{cases} F^{'}(ж), & ж\in Ж-Ж=\emptyset \\ K^{'}(ж), & ж\in Ж-Ж=\emptyset \\ F'(ж)\cup K(ж), & ж\in Ж\cap Ж=Ж \end{cases}$$

for all ж∈Ж. Let (G,Ƀ) $\overset{*}{+}_{\varepsilon}$(L, Ƀ) = (S,Ƀ). Thus,

$$S(ж)=\begin{cases} G^{'}(ж), & ж\in Ƀ-Ƀ=\emptyset \\ L^{'}(ж), & ж\in Ƀ-Ƀ=\emptyset \\ G'(ж)\cup L(ж), & ж\in Ƀ\cap Ƀ=Ƀ \end{cases}$$

for all ж∈Ƀ. Hence, since W(ж)=F'(ж)∪K(ж) ⊆ G'(ж)∪L(ж)=S(ж), for all ж∈Ж, (F, Ж) $\overset{*}{+}_{\varepsilon}$(K,Ж) $\widetilde{\subseteq}$(G,Ƀ) $\overset{*}{+}_{\varepsilon}$(L,Ƀ).

Theorem 3.4. Let (F, Ж), (G, Ƀ), (Б, Ş) be soft sets over U. Then,

1) If Ж'∩Ƀ∩Ş=∅, then (F, Ж)$\overset{*}{+}_{\varepsilon}$ [(G, Ƀ)$\cap_R$(Б, Ş)] = [(F, Ж)$\overset{*}{+}_{\varepsilon}$(G, Ƀ)]$\cap_R$[(F, Ж)$\overset{*}{+}_{\varepsilon}$(Б, Ş)].

2) If Ж'∩Ƀ∩Ş=Ж∩Ƀ∩Ş=∅, then (F, Ж)$\overset{*}{+}_{\varepsilon}$ [(G, Ƀ)$\cup_R$(Б, Ş)] = [(F, Ж)$\overset{*}{+}_{\varepsilon}$(G, Ƀ)]$\cap_R$[(F, Ж)$\overset{*}{+}_{\varepsilon}$(Б, Ş)].

3) If Ж'∩Ƀ∩Ş= Ж∩Ƀ∩Ş=∅, then (F, Ж)$\overset{*}{+}_{\varepsilon}$ [(G, Ƀ) $*_R$(Б, Ş)] = [(F, Ж) $\overset{*}{*}_{\varepsilon}$(G, Ƀ)$\cap_R$[(F, Ж) $\overset{*}{*}_{\varepsilon}$(Б, Ş)].

4) If Җ'∩Ƀ∩Ş= Җ∩Ƀ∩Ş=∅, then $(F, Җ) \overset{*}{+}_{\varepsilon} [(G, Ƀ)\theta_R(Б, Ş)] = [(F, Җ) \overset{*}{*}_{\varepsilon}(G, Ƀ)]\cup_R[(F, Җ) \overset{*}{*}_{\varepsilon}(Б, Ş)]$.

Proof: (1) Let $(G, Ƀ)\cap_R(Б, Ş) = (M, Ƀ\cap Ş)$, where $M(җ) = G(җ) \cap Б(җ)$, for all $җ\in Ƀ \cap Ş$. Let $(F, Җ)\overset{*}{+}_{\varepsilon} (M,Ƀ \cap Ş)=(N, Җ\cup (Ƀ \cap Ş)))$, where

$$N(җ)=\begin{cases} F'(җ), & җ\in Җ-(Ƀ\cap Ş) \\ M'(җ), & җ\in(Ƀ\cap Ş)-Җ \\ F'(җ)\cup M(җ), & җ\in Җ\cap(Ƀ\cap Ş) \end{cases}$$

for all $җ\in Җ\cup (Ƀ \cap Ş)$. Thus,

$$N(җ)=\begin{cases} F'(җ), & җ\in Җ-(Ƀ\cap Ş)=Җ-(Ƀ\cap Ş) \\ G'(җ)\cup Б'(җ), & җ\in(Ƀ\cap Ş)-Җ=Җ'\cap Ƀ\cap Ş \\ F'(җ)\cup(G(җ)\cap Б(җ)), & җ\in Җ\cap(Ƀ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Let $(F, Җ)\overset{*}{+}_{\varepsilon}(G, Ƀ) = (V, Җ\cup Ƀ)$, where

$$V(җ)=\begin{cases} F'(җ), & җ\in Җ-Ƀ \\ G'(җ), & җ\in Ƀ-Җ \\ F'(җ)\cup G(җ), & җ\in Җ\cap Ƀ \end{cases}$$

for all $җ\in Җ\cup Ƀ$. Assume that $(F, Җ) \overset{*}{+}_{\varepsilon}(Б, Ş) = (W, Җ\cup Ş)$, where

$$W(җ)=\begin{cases} F'(җ), & җ\in Җ-Ş \\ Б'(җ), & җ\in Ş-Җ \\ F'(җ)\cup Б(җ), & җ\in Җ\cap Ş \end{cases}$$

for all $җ\in Җ\cup Ş$, Let $(V, Җ\cup Ƀ)\cap_R(W, Җ\cup Ş) =(T, (Җ\cup Ƀ)\cap (Җ \cup Ş)))$, where $T(җ)=V(җ)\cap W(җ)$ for all $җ\in Җ\cup(Ƀ \cap Ş)$. Hence,

$$T(җ)=\begin{cases} F'(җ)\cap F'(җ), & җ\in(Җ-Ƀ)\cap(Җ-Ş)=Җ\cap Ƀ'\cap Ş' \\ F'(җ)\cap Б'(җ), & җ\in(Җ-Ƀ)\cap(Ş-Җ)=\emptyset \\ F'(җ)\cap(F'(җ)\cup Б(җ)) & җ\in(Җ-Ƀ)\cap(Җ\cap Ş)=Җ\cap Ƀ'\cap Ş \\ G'(җ)\cap F'(җ), & җ\in(Ƀ-Җ)\cap(Җ-Ş)=\emptyset \\ G'(җ)\cap Б'(җ), & җ\in(Ƀ-Җ)\cap(Ş-Җ)=Җ'\cap Ƀ\cap Ş \\ G'(җ)\cap\left(F'(җ)\cup Б(җ)\right), & җ\in(Ƀ-Җ)\cap(Җ\cap Ş)=\emptyset \\ (F'(җ)\cup G(җ))\cap F'(җ), & җ\in(Җ\cap Ƀ)\cap(Җ-Ş)=Җ\cap Ƀ\cap Ş' \\ (F'(җ)\cup G(җ))\cap Б'(җ), & җ\in(Җ\cap Ƀ)\cap(Ş-Җ)=\emptyset \\ (F'(җ\cup G(җ))\cap(F'(җ)\cup Б(җ)), & җ\in(Җ\cap Ƀ)\cap(Җ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Thus,

$$T(җ)=\begin{cases} F'(җ), & җ\in(Җ-Ƀ)\cap(Җ-Ş)=Җ\cap Ƀ'\cap Ş' \\ F'(җ) & җ\in(Җ-Ƀ)\cap(Җ\cap Ş)=Җ\cap Ƀ'\cap Ş \\ G'(җ)\cap Б'(җ), & җ\in(Ƀ-Җ)\cap(Ş-Җ)=Җ'\cap Ƀ\cap Ş \\ F'(җ) & җ\in(Җ\cap Ƀ)\cap(Җ-Ş)=Җ\cap Ƀ\cap Ş' \\ (F'(җ)\cup G(җ))\cap(F'(җ)\cup Б(җ)), & җ\in(Җ\cap Ƀ)\cap(Җ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Here, when considering Җ-(Ƀ∩Ş) in the function N, since Җ-(Ƀ∩Ş) =Җ∩(Ƀ∩Ş)', if an element is in the complement of (Ƀ∩Ş), then it is either in Ƀ-Ş, in Ş-Ƀ, or in $(Ƀ \cup Ş)'$. Thus, if $җ \in Җ -(Ƀ\cap Ş)$, then $җ \in Җ \cap Ƀ\cap Ş'$ or $җ \in Җ \cap Ƀ'\cap Ş$ or $җ \in Җ \cap Ƀ'\cap Ş'$. Hence, N=T under Җ'∩Ƀ∩Ş =∅.

Theorem 3.5. Let (F, Ж), (G, Ƀ), (Б, Ş) be soft sets over U. Then,

1) If $(Җ\Delta Ƀ)\cap Ş=\emptyset$, then $[(F,Ж)\cup_R(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş) = [(F,Ж)\overset{*}{+}_{\varepsilon}(Б,Ş)]\cap_R[(G,Ƀ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

2) If $(Җ\Delta Ƀ)\cap Ş=\emptyset$, then $[(F,Ж)\cap_R(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş) = [(F,Ж)\overset{*}{+}_{\varepsilon}(Б,Ş)]\cup_R[(G,Ƀ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

3) If $Җ\cap Ƀ\cap Ş'=(Җ\Delta Ƀ)\cap Ş=\emptyset$, then $[(F,Ж)\theta_R(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş) = [(F,Ж)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]\cup_R[(G,Ƀ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]$.

4) If $Җ\cap Ƀ\cap Ş'=(Җ\Delta Ƀ)\cap Ş=\emptyset$, then $[(F,Ж) * _R(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş) =[(F,Ж)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]\cap_R[(G,Ƀ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]$.

Proof: (1) Consider first LHS. Let $(F,Ж)\cup_R(G,Ƀ) = (M, Җ\cap Ƀ)$, where $M(җ)=F(җ)\cup G(җ)$ for all $җ\in Җ\cap Ƀ$. Let $(M, Җ\cap Ƀ)\overset{*}{+}_{\varepsilon}(Б,Ş) = (N, (Җ\cap Ƀ)\cup Ş)$, where for all $җ\in(Җ\cap Ƀ)\cup Ş$,

$$N(җ)=\begin{cases} M'(җ), & җ\in(Җ\cap Ƀ)-Ş \\ Б'(җ), & җ\in Ş-(Җ\cap Ƀ) \\ M'(җ)\cup Б(җ), & җ\in Җ\cap(Ƀ\cap Ş) \end{cases}$$

Thus,

$$N(җ)=\begin{cases} F'(җ)\cap G'(җ), & җ\in(Җ\cap Ƀ)-Ş=Җ\cap Ƀ\cap Ş' \\ Б'(җ), & җ\in Ş-(Җ\cap Ƀ)=Ş-(Җ\cap Ƀ) \\ (F'(җ)\cap G'(җ))\cup Б(җ), & җ\in Җ\cap(Ƀ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Let $(F,Ж)\overset{*}{+}_{\varepsilon}(Б,Ş) = (V, Җ\cup Ş)$, where

$$V(җ)=\begin{cases} F'(җ), & җ\in Җ-Ş \\ Б'(җ), & җ\in Ş-Җ \\ F'(җ)\cup Б(җ), & җ\in Җ\cap Ş \end{cases}$$

for all $җ\in Җ\cup Ş$. Now, let $(G,Ƀ)\overset{*}{+}_{\varepsilon}(Б,Ş) = (W, Ƀ\cup Ş)$,

$$W(җ)=\begin{cases} G'(җ), & җ\in Ƀ-Ş \\ Б'(җ), & җ\in Ş-Ƀ \\ G'(җ)\cup Б(җ), & җ\in Ƀ\cap Ş \end{cases}$$

where for all $җ\in Ƀ\cup Ş$. Let $(V, Җ\cup Ş)\cap_R(W, Ƀ\cup Ş) = (T, (Җ\cup Ş)\cap(Ƀ\cup Ş))$. Here, $T(җ)=V(җ)\cap W(җ)$ for all $җ\in (Җ\cup Ş)\cap(Ƀ\cup Ş)$. Thus,

$$T(җ)=\begin{cases} F'(җ)\cap G'(җ), & җ\in(Җ-Ş)\cap(Ƀ-Ş)=Җ\cap Ƀ\cap Ş' \\ F'(җ)\cap Б'(җ), & җ\in(Җ-Ş)\cap(Ş-Ƀ)=\emptyset \\ F'(җ)\cap\big(G'(җ)\cup Б(җ)\big), & җ\in(Җ-Ş)\cap(Ƀ\cap Ş)=\emptyset \\ Б'(җ)\cap G'(җ), & җ\in(Ş-Җ)\cap(Ƀ-Ş)=\emptyset \\ Б'(җ)\cap Б'(җ), & җ\in(Ş-Җ)\cap(Ş-Ƀ)=Җ'\cap Ƀ'\cap Ş \\ Б'(җ)\cap(G'(җ)\cup Б(җ)), & җ\in(Ş-Җ)\cap(Ƀ\cap Ş)=Җ'\cap Ƀ\cap Ş \\ \big(F'(җ)\cup Б(җ)\big)\cap G'(җ), & җ\in(Җ\cap Ş)\cap(Ƀ-Ş)=\emptyset \\ \big(F'(җ)\cup Б(җ)\big)\cap Б'(җ) & җ\in(Җ\cap Ş)\cap(Ş-Ƀ)=Җ\cap Ƀ'\cap Ş \\ (F'(җ)\cup Б(җ))\cap(G'(җ)\cup Б(җ)), & җ\in(Җ\cap Ş)\cap(Ƀ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Thus,

$$T(җ)=\begin{cases} F'(җ)\cap G'(җ), & җ\in(Җ-Ş)\cap(Ƀ-Ş)=Җ\capɃ\cap Ş' \\ Б'(җ) & җ\in(Ş-Җ)\cap(Ş-Ƀ)=Җ'\capɃ'\cap Ş \\ Б'(җ)\cap G'(җ) & җ\in(Ş-Җ)\cap(Ƀ\cap Ş)=Җ'\capɃ\cap Ş \\ F'(җ)\cap Б'(җ) & җ\in(Җ\cap Ş)\cap(Ş-Ƀ)=Җ\capɃ'\cap Ş \\ (F'(җ)\cup Б(җ))\cap(G'(җ)\cup Б(җ)), & җ\in(Җ\capɃ)\cap(Җ\cap Ş)=Җ\capɃ\cap Ş \end{cases}$$

Here, when considering $Ş-(Җ\capɃ)$ in the function N, since $Җ-(Ƀ\cap Ş)=Җ\cap(Ƀ\cap Ş)'$, if an element is in the complement of $(Ƀ\cap Ş)$, then it is either in $Ƀ-Ş$, in $Ş-Ƀ$, or in $(Ƀ\cup Ş)'$. Thus, if $җ\in Җ-(Ƀ\cap Ş)$, then $җ\in Җ\cap Ƀ\cap Ş'$ or $җ\in Җ\cap Ƀ'\cap Ş$ or $җ\in Җ\cap Ƀ'\cap Ş'$. Hence, N=T, where $Җ'\capɃ\cap Ş=Җ\capɃ'\cap Ş=\emptyset$. The condition $Җ'\capɃ\cap Ş=Җ\capɃ'\cap Ş=\emptyset$ implies that $(Җ\DeltaɃ)\cap Ş=\emptyset$.

Theorem 3.6. Let $(F,Җ)$, $(G,Ƀ)$, $(Б,Ş)$ be soft sets over U. Then,

1) If $Җ\capɃ\cap Ş'=(Җ\DeltaɃ)\cap Ş=\emptyset$, then $(F,Җ)\overset{*}{+}_{\varepsilon}[(G,Ƀ)\cap_{\varepsilon}(Б,Ş)]=[(F,Җ)\overset{*}{+}_{\varepsilon}(G,Ƀ)]\cap_{\varepsilon}[(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$

2) If $Җ\capɃ\cap Ş'=(Җ\DeltaɃ)\cap Ş=\emptyset$, then $(F,Җ)\overset{*}{+}_{\varepsilon}[(G,Ƀ)\cup_{\varepsilon}(Б,Ş)]=[(F,Җ)\overset{*}{+}_{\varepsilon}(G,Ƀ)]\cup_{\varepsilon}[(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

3) If $Җ\capɃ\cap Ş'=(Җ\DeltaɃ)\cap Ş=\emptyset$, then $(F,Җ)\overset{*}{+}_{\varepsilon}[(G,Ƀ)*_{\varepsilon}(Б,Ş)]=[(F,Җ)\overset{*}{*}_{\varepsilon}(G,Ƀ)]\cup_{\varepsilon}[(F,Җ)\overset{*}{*}_{\varepsilon}(Б,Ş)]$.

4) If $Җ\capɃ\cap Ş'=(Җ\DeltaɃ)\cap Ş=\emptyset$, then $(F,Җ)\overset{*}{+}_{\varepsilon}[(G,Ƀ)\theta_{\varepsilon}(Б,Ş)]=[(F,Җ)\overset{*}{*}_{\varepsilon}(G,Ƀ)]\cap_{\varepsilon}[(F,Җ)\overset{*}{*}_{\varepsilon}(Б,Ş)]$.

Proof: (1) Let $(G,Ƀ)\cap_{\varepsilon}(Б,Ş)=(M,Ƀ\cup Ş)$, where

$$M(җ)=\begin{cases} G(җ), & җ\inɃ-Ş \\ Б(җ), & җ\in Ş-Ƀ \\ G(җ)\cap Б(җ), & җ\inɃ\cap Ş \end{cases}$$

for all $җ\inɃ\cup Ş$. Let $(F,Җ)\overset{*}{+}_{\varepsilon}(M,Ƀ\cup Ş)=(N,Җ\cup(Ƀ\cup Ş))$, where

$$N(җ)=\begin{cases} F'(җ), & җ\in Җ-(Ƀ\cup Ş) \\ M'(җ), & җ\in(Ƀ\cup Ş)-Җ \\ F'(җ)\cup M(җ), & җ\in Җ\cap(Ƀ\cup Ş) \end{cases}$$

for all $җ\in Җ\cupɃ\cup Ş$. Thus,

$$N(җ)=\begin{cases} F'(җ), & җ\in Җ-(Ƀ\cup Ş)=Җ\capɃ'\cap Ş' \\ G'(җ), & җ\in(Ƀ-Ş)-Җ=Җ'\capɃ\cap Ş' \\ Б'(җ), & җ\in(Ş-Ƀ)-Җ=Җ'\capɃ'\cap Ş \\ G'(җ)\cup Б'(җ), & җ\in(Ƀ\cap Ş)-Җ=Җ'\capɃ\cap Ş \\ F'(җ)\cup G(җ), & җ\in Җ\cap(Ƀ-Ş)=Җ\capɃ\cap Ş' \\ F'(җ)\cup Б(җ), & җ\in Җ\cap(Ş-Ƀ)=Җ\capɃ'\cap Ş \\ F'(җ)\cup(G(җ)\cap Б(җ)), & җ\in Җ\cap(Ƀ\cap Ş)=Җ\capɃ\cap Ş \end{cases}$$

Let $(F,Җ)\overset{*}{+}_{\varepsilon}(G,Ƀ)=(V,Җ\cupɃ)$, where

$$V(җ)=\begin{cases} F'(җ), & җ\in Җ-Ƀ \\ G'(җ), & җ\inɃ-Җ \\ F'(җ)\cup G(җ), & җ\in Җ\capɃ \end{cases}$$

for all $җ\in Җ\cupɃ$. Let $(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)=(W,Җ\cup Ş)$, where

$$W(җ)=\begin{cases} F'(җ), & җ\in Җ-Ş \\ Б'(җ), & җ\in Ş-Җ \\ F'(җ)\cup Б(җ), & җ\in Җ\cap Ş \end{cases}$$

for all җ∈Җ∪Ş. Let (V, Җ∪Ƀ) $\cap_{\varepsilon}$(W, Җ∪Ş) =(T, (Җ∪Ƀ)∪Ş), where

$$\text{T(җ)}=\begin{cases} \text{V(җ)}, & \text{җ∈(Җ∪Ƀ)-(Җ∪Ş)} \\ \text{W(җ)}, & \text{җ∈(Җ∪Ş)-(Җ∪Ƀ)} \\ \text{V(җ)∩W(җ)}, & \text{җ∈(Җ∪Ƀ)∩(Җ∪Ş)} \end{cases}$$

for all җ∈Җ∪Ƀ∪Ş. Thus,

$$\text{T(җ)}=\begin{cases} \text{F}'\text{(җ)}, & \text{җ∈(Җ-Ƀ)-(Җ∪Ş)=∅} \\ \text{G}'\text{(җ)}, & \text{җ∈(Ƀ-Җ)-(Җ∪Ş)=Җ}'\text{∩Ƀ∩Ş}' \\ \text{F}'\text{(җ)∪G(җ)}, & \text{җ∈(Җ∩Ƀ)-(Җ∪Ş)=∅} \\ \text{F}'\text{(җ)}, & \text{җ∈(Җ-Ş)-(Җ∪Ƀ)=∅} \\ \text{Б}'\text{(җ)}, & \text{җ∈(Ş-Җ)-(Җ∪Ƀ)=Җ}'\text{∩Ƀ}'\text{∩Ş} \\ \text{F}'\text{(җ)∪Б(җ)}, & \text{җ∈(Җ∩Ş)-(Җ∪Ƀ)=∅} \\ \text{F}'\text{(җ)∩F}'\text{(җ)}, & \text{җ∈(Җ-Ƀ)∩(Җ-Ş)=Җ∩Ƀ}'\text{∩Ş}' \\ \text{F}'\text{(җ)∩Б}'\text{(җ)}, & \text{җ∈(Җ-Ƀ)∩(Ş-Җ)=∅} \\ \text{F}'\text{(җ)∩(F}'\text{(җ)∪Б(җ))}, & \text{җ∈(Җ-Ƀ)∩(Җ∩Ş)=Җ∩Ƀ}'\text{∩Ş} \\ \text{G}'\text{(җ)∩F}'\text{(җ)}, & \text{җ∈(Ƀ-Җ)∩(Җ-Ş)=∅} \\ \text{G}'\text{(җ)∩Б}'\text{(җ)}, & \text{җ∈(Ƀ-Җ)∩(Ş-Җ)=Җ}'\text{∩Ƀ∩Ş} \\ \text{G}'\text{(җ)∩(F}'\text{(җ)∪Б(җ))}, & \text{җ∈(Ƀ-Җ)∩(Җ∩Ş)=∅} \\ \text{(F}'\text{(җ)∪G(җ))∩F}'\text{(җ)}, & \text{җ∈(Җ∩Ƀ)∩(Җ-Ş)=Җ∩Ƀ∩Ş}' \\ \text{(F}'\text{(җ)∪G(җ))∩Б}'\text{(җ)}, & \text{җ∈(Җ∩Ƀ)∩(Ş-Җ)=∅} \\ \text{(F}'\text{(җ)∪G(җ))∩(F}'\text{(җ)∪Б(җ))}, & \text{җ∈(Җ∩Ƀ)∩(Җ∩Ş)=Җ∩Ƀ∩Ş} \end{cases}$$

Hence,

$$\text{T(җ)}=\begin{cases} \text{G}'\text{(җ)}, & \text{җ∈(Ƀ-Җ)-(Җ∪Ş)=Җ}'\text{∩Ƀ∩Ş}' \\ \text{Б}'\text{(җ)}, & \text{җ∈(Ş-Җ)-(Җ∪Ƀ)=Җ}'\text{∩Ƀ}'\text{∩Ş} \\ \text{F}'\text{(җ)}, & \text{җ∈(Җ-Ƀ)∩(Җ-Ş)=Җ∩Ƀ}'\text{∩Ş}' \\ \text{F}'\text{(җ)}, & \text{җ∈(Җ-Ƀ)∩(Җ∩Ş)=Җ∩Ƀ}'\text{∩Ş} \\ \text{G}'\text{(җ)∩Б}'\text{(җ)}, & \text{җ∈(Ƀ-Җ)∩(Ş-Җ)=Җ}'\text{∩Ƀ∩Ş} \\ \text{F}'\text{(җ)}, & \text{җ∈(Җ∩Ƀ)∩(Җ-Ş)=Җ∩Ƀ∩Ş}' \\ \text{(F}'\text{(җ)∪G(җ))∩(F}'\text{(җ)∪ Б(җ))}, & \text{җ∈(Җ∩Ƀ)∩(Җ∩Ş)=Җ∩Ƀ∩Ş} \end{cases}$$

It is seen that N=T, where Җ'∩Ƀ∩Ş=Җ∩Ƀ'∩Ş=Җ∩Ƀ∩Ş'=∅. Here, the condition Җ'∩Ƀ∩Ş=Җ∩Ƀ'∩Ş=∅ is equal to (ҖΔɃ)∩Ş= ∅.

Theorem 3.7. Let (F, Җ), (G, Ƀ), (Б, Ş) be soft sets over U. Then,

1) If (ҖΔɃ)∩ Ş=∅, then [(F, Җ) $\cup_{\varepsilon}$(G, Ƀ)] $\overset{*}{+}_{\varepsilon}$ (Б, Ş) = [(F, Җ) $\overset{*}{+}_{\varepsilon}$(Б, Ş)]$\cap_{\varepsilon}$[(G, Ƀ) $\overset{*}{+}_{\varepsilon}$(Б, Ş)].

2) If (ҖΔɃ)∩ Ş=∅, then [(F, Җ) $\cap_{\varepsilon}$(G, Ƀ)]$\overset{*}{+}_{\varepsilon}$ (Б, Ş) = [(F, Җ) $\overset{*}{+}_{\varepsilon}$(Б, Ş)]$\cup_{\varepsilon}$[(G, Ƀ) $\overset{*}{+}_{\varepsilon}$(Б, Ş)].

3) If (ҖΔɃ)∩ Ş=Җ∩Ƀ∩Ş'=∅, then [(F, Җ)θ_{ε}(G, Ƀ)]$\overset{*}{+}_{\varepsilon}$ (Б, Ş) = [(F, Җ) $\overset{*}{\cup}_{\varepsilon}$(Б, Ş)]$\cup_{\varepsilon}$[(G, Ƀ) $\overset{*}{\cup}_{\varepsilon}$ (Б, Ş)].

4) If (ҖΔɃ)∩ Ş=Җ∩Ƀ∩Ş'=∅, then [(F, Җ) $*_{\varepsilon}$(G, Ƀ)]$\overset{*}{+}_{\varepsilon}$ (Б, Ş) = [(F, Җ) $\overset{*}{\cup}_{\varepsilon}$(Б, Ş)]$\cap_{\varepsilon}$[(G, Ƀ) $\overset{*}{\cup}_{\varepsilon}$(Б, Ş)].

Proof: (1) Let (F, Җ) $\cup_{\varepsilon}$(G, Ƀ) = (M, Җ∪Ƀ), where

$$\text{M(җ)}=\begin{cases} \text{F(җ)}, & \text{җ∈Җ-Ƀ} \\ \text{G(җ)}, & \text{җ∈Ƀ-Җ} \\ \text{F(җ)∪G(җ)}, & \text{җ∈Җ∩Ƀ} \end{cases}$$

for all ж∈Җ∪Ƀ. Let (M, Җ∪Ƀ) $\overset{*}{+}_{\varepsilon}$ (Б, Ş) = (N, (Җ∪Ƀ)∪Ş, where

$$\text{N(ж)}=\begin{cases} \text{M}'(\text{ж}), & \text{ж}\in(\text{Җ}\cup\text{Ƀ})-\text{Ş} \\ \text{Б}'(\text{ж}), & \text{ж}\in\text{Ş}-(\text{Җ}\cup\text{Ƀ}) \\ \text{M}'(\text{ж})\cup\text{Б}(\text{ж}), & \text{ж}\in(\text{Җ}\cup\text{Ƀ})\cap\text{Ş} \end{cases}$$

for all ж∈Җ∪Ƀ∪Ş. Thus,

$$\text{N(ж)}=\begin{cases} \text{F}'(\text{ж}), & \text{ж}\in(\text{Җ}-\text{Ƀ})-\text{Ş}=\text{Җ}\cap\text{Ƀ}'\cap\text{Ş}' \\ \text{G}'(\text{ж}), & \text{ж}\in(\text{Ƀ}-\text{Җ})-\text{Ş}=\text{Җ}'\cap\text{Ƀ}\cap\text{Ş}' \\ \text{F}'(\text{ж})\cap\text{G}'(\text{ж}), & \text{ж}\in(\text{Җ}\cap\text{Ƀ})-\text{Ş}=\text{Җ}\cap\text{Ƀ}\cap\text{Ş}' \\ \text{Б}'(\text{ж}), & \text{ж}\in\text{Ş}-(\text{Җ}\cup\text{Ƀ})=\text{Җ}'\cap\text{Ƀ}'\cap\text{Ş} \\ \text{F}'(\text{ж})\cup\text{Б}(\text{ж}), & \text{ж}\in(\text{Җ}-\text{Ƀ})\cap\text{Ş}=\text{Җ}\cap\text{Ƀ}'\cap\text{Ş} \\ \text{G}'(\text{ж})\cup\text{Б}(\text{ж}), & \text{ж}\in(\text{Ƀ}-\text{Җ})\cap\text{Ş}=\text{Җ}'\cap\text{Ƀ}\cap\text{Ş} \\ (\text{F}'(\text{ж})\cap\text{G}'(\text{ж}))\cup\text{Б}(\text{ж}), & \text{ж}\in(\text{Җ}\cap\text{Ƀ})\cap\text{Ş}=\text{Җ}\cap\text{Ƀ}\cap\text{Ş} \end{cases}$$

Let (F, Җ) $\overset{*}{+}_{\varepsilon}$ (Б, Ş) =(V, Җ∪Ş), where

$$\text{V(ж)}=\begin{cases} \text{F}'(\text{ж}), & \text{ж}\in\text{Җ}-\text{Ş} \\ \text{Б}'(\text{ж}), & \text{ж}\in\text{Ş}-\text{Җ} \\ \text{F}'(\text{ж})\cup\text{Б}(\text{ж}), & \text{ж}\in\text{Җ}\cap\text{Ş} \end{cases}$$

for all ж∈Җ∪Ş. Let (G, Ƀ) $\overset{*}{+}_{\varepsilon}$ (Б, Ş) =(W,Ƀ∪Ş), where

$$\text{W(ж)}=\begin{cases} \text{G}'(\text{ж}), & \text{ж}\in\text{Ƀ}-\text{Ş} \\ \text{Б}'(\text{ж}), & \text{ж}\in\text{Ş}-\text{Ƀ} \\ \text{G}'(\text{ж})\cup\text{Б}(\text{ж}), & \text{ж}\in\text{Ƀ}\cap\text{Ş} \end{cases}$$

for all ж∈Ƀ∪Ş. Let (V, Җ∪Ş) $\cap_{\varepsilon}$ (W, Ƀ∪Ş) =(T,Җ∪Ƀ∪Ş), where

$$\text{T(ж)}=\begin{cases} \text{V}(\text{ж}), & \text{ж}\in(\text{Җ}\cup\text{Ş})-(\text{Ƀ}\cup\text{Ş}) \\ \text{W}(\text{ж}), & \text{ж}\in(\text{Ƀ}\cup\text{Ş})-(\text{Җ}\cup\text{Ş}) \\ \text{V}(\text{ж})\cap\text{W}(\text{ж}), & \text{ж}\in(\text{Җ}\cup\text{Ş})\cap(\text{Ƀ}\cup\text{Ş}) \end{cases}$$

for all ж∈Җ∪Ƀ∪Ş. Thus,

$$\text{T(ж)}=\begin{cases} \text{F}'(\text{ж}), & \text{ж}\in(\text{Җ}-\text{Ş})-(\text{Ƀ}\cup\text{Ş})=\text{Җ}\cap\text{Ƀ}'\cap\text{Ş}' \\ \text{Б}'(\text{ж}), & \text{ж}\in(\text{Ş}-\text{Җ})-(\text{Ƀ}\cup\text{Ş})=\emptyset \\ \text{F}'(\text{ж})\cup\text{Б}(\text{ж}), & \text{ж}\in(\text{Җ}\cap\text{Ş})-(\text{Ƀ}\cup\text{Ş})=\emptyset \\ \text{G}'(\text{ж}), & \text{ж}\in(\text{Ƀ}-\text{Ş})-(\text{Җ}\cup\text{Ş})=\text{Җ}'\cap\text{Ƀ}\cap\text{Ş}' \\ \text{Б}'(\text{ж}), & \text{ж}\in(\text{Ş}-\text{Ƀ})-(\text{Җ}\cup\text{Ş})=\emptyset \\ \text{G}'(\text{ж})\cup\text{Б}(\text{ж}), & \text{ж}\in(\text{Ƀ}\cap\text{Ş})-(\text{Җ}\cup\text{Ş})=\emptyset \\ \text{F}'(\text{ж})\cap\text{G}'(\text{ж}), & \text{ж}\in(\text{Җ}-\text{Ş})\cap(\text{Ƀ}-\text{Ş})=\text{Җ}\cap\text{Ƀ}\cap\text{Ş}' \\ \text{F}'(\text{ж})\cap\text{Б}'(\text{ж}), & \text{ж}\in(\text{Җ}-\text{Ş})\cap(\text{Ş}-\text{Ƀ})=\emptyset \\ \text{F}'(\text{ж})\cap(\text{G}'(\text{ж})\cup\text{Б}(\text{ж})), & \text{ж}\in(\text{Җ}-\text{Ş})\cap(\text{Ƀ}\cap\text{Ş})=\emptyset \\ \text{Б}'(\text{ж})\cap\text{G}'(\text{ж}), & \text{ж}\in(\text{Ş}-\text{Җ})\cap(\text{Ƀ}-\text{Ş})=\emptyset \\ \text{Б}'(\text{ж})\cap\text{Б}'(\text{ж}), & \text{ж}\in(\text{Ş}-\text{Җ})\cap(\text{Ş}-\text{Ƀ})=\text{Җ}'\cap\text{Ƀ}'\cap\text{Ş} \\ \text{Б}'(\text{ж})\cap(\text{G}'(\text{ж})\cup\text{Б}(\text{ж})) & \text{ж}\in(\text{Ş}-\text{Җ})\cap(\text{Ƀ}\cap\text{Ş})=\text{Җ}'\cap\text{Ƀ}\cap\text{Ş} \\ (\text{F}'(\text{ж})\cup\text{Б}(\text{ж}))\cap\text{G}'(\text{ж}), & \text{ж}\in(\text{Җ}\cap\text{Ş})\cap(\text{Ƀ}-\text{Ş})=\emptyset \\ (\text{F}'(\text{ж})\cup\text{Б}(\text{ж}))\cap\text{Б}'(\text{ж}) & \text{ж}\in(\text{Җ}\cap\text{Ş})\cap(\text{Ş}-\text{Ƀ})=\text{Җ}\cap\text{Ƀ}'\cap\text{Ş} \\ (\text{F}'(\text{ж})\cup\text{Б}(\text{ж}))\cap(\text{G}'(\text{ж})\cup\text{Б}(\text{ж})) & \text{ж}\in(\text{Җ}\cap\text{Ş})\cap(\text{Ƀ}\cap\text{Ş})=\text{Җ}\cap\text{Ƀ}\cap\text{Ş} \end{cases}$$

Thereby,

$$T(ж)=\begin{cases} F'(ж), & җ\in(Җ-Ş)-(Ƀ\cup Ş)=Җ\cap Ƀ'\cap Ş' \\ G'(ж), & җ\in(Ƀ-Ş)-(Җ\cup Ş)=Җ'\cap Ƀ\cap Ş' \\ F'(ж)\cap G'(ж), & җ\in(Җ-Ş)\cap(Ƀ-Ş)=Җ\cap Ƀ\cap Ş' \\ Б'(ж), & җ\in(Ş-Җ)\cap(Ş-Ƀ)=Җ'\cap Ƀ'\cap Ş \\ Б'(ж)\cap G'(ж), & җ\in(Ş-Җ)\cap(Ƀ\cap Ş)=Җ'\cap Ƀ\cap Ş \\ F'(ж)\cap Б'(ж) & җ\in(Җ\cap Ş)\cap(Ş-Ƀ)=Җ\cap Ƀ'\cap Ş \\ (F'(ж)\cup Б(ж))\cap(G'(ж)\cup Б(ж)) & җ\in(Җ\cap Ş)\cap(Ƀ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Hence, under the condition $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ'\cap Ş=\emptyset$, N=T is satisfied. The condition $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ'\cap Ş=\emptyset$ is equivalent to $(Җ\Delta Ƀ)\cap Ş=\emptyset$.

Theorem 3.8. Let (F, Җ), (G, Ƀ), (Б, Ş) be soft sets over U. Then,

1) If $(Җ\Delta Ƀ)\cap Ş=Җ\cap Ƀ\cap Ş'=\emptyset$, then $(F,Җ)\overset{*}{+}_{\varepsilon}[(G,Ƀ)\overset{*}{\cap}_{\varepsilon}(Б,Ş)]=[(F,Җ)\overset{*}{+}_{\varepsilon}(G,Ƀ)]\overset{*}{\cap}_{\varepsilon}[(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

2) If $(Җ\Delta Ƀ)\cap Ş=Җ\cap Ƀ\cap Ş'=\emptyset$, then $(F,Җ)\overset{*}{+}_{\varepsilon}[(G,Ƀ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]=[(F,Җ)\overset{*}{+}_{\varepsilon}(G,Ƀ)]\overset{*}{\cup}_{\varepsilon}[(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

3) If $Җ'\cap Ƀ\cap Ş=\emptyset$, then $(F,Җ)\overset{*}{+}_{\varepsilon}[(G,Ƀ)\overset{*}{*}_{\varepsilon}(Б,Ş)]=[(F,Җ)\overset{*}{*}_{\varepsilon}(G,Ƀ)]\overset{*}{\cup}_{\varepsilon}[(F,Җ)\overset{*}{*}_{\varepsilon}(Б,Ş)]$.

4) If $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ\cap Ş=\emptyset$, then $(F,Җ)\overset{*}{+}_{\varepsilon}[(G,Ƀ)\overset{*}{\theta}_{\varepsilon}(Б,Ş)]=[(F,Җ)\overset{*}{*}_{\varepsilon}(G,Ƀ)]\overset{*}{\cup}_{\varepsilon}[(F,Җ)\overset{*}{*}_{\varepsilon}(Б,Ş)]$.

Proof: (1) Let $(G,Ƀ)\overset{*}{\cap}_{\varepsilon}((Б,Ş)=(M,Ƀ\cup Ş)$, where

$$M(ж)=\begin{cases} G'(ж), & җ\in Ƀ-Ş \\ Б'(ж), & җ\in Ş-Ƀ \\ G(ж)\cap Б(ж), & җ\in Ƀ\cap Ş \end{cases}$$

for all $җ\in Ƀ\cup Ş$. Let $(F,Җ)\overset{*}{+}_{\varepsilon}(M,Ƀ\cup Ş)=(N,Җ\cup(Ƀ\cup Ş))$, where

$$N(ж)=\begin{cases} F'(ж), & җ\in Җ-(Ƀ\cup Ş) \\ M'(ж), & җ\in(Ƀ\cup Ş)-Җ \\ F'(ж)\cup M(ж), & җ\in Җ\cap(Ƀ\cup Ş) \end{cases}$$

for all $җ\in Җ\cup Ƀ\cup Ş$. Thus,

$$N(ж)=\begin{cases} F'(ж), & җ\in Җ-(Ƀ\cup Ş)=Җ\cap Ƀ'\cap Ş' \\ G(ж), & җ\in(Ƀ-Ş)-Җ=Җ'\cap Ƀ\cap Ş' \\ Б(ж), & җ\in(Ş-Ƀ)-Җ=Җ'\cap Ƀ'\cap Ş \\ G'(ж)\cup Б'(ж), & җ\in(Ƀ\cap Ş)-Җ=Җ'\cap Ƀ\cap Ş \\ F'(ж)\cup G'(ж), & җ\in Җ\cap(Ƀ-Ş)=Җ\cap Ƀ\cap Ş' \\ F'(ж)\cup Б'(ж), & җ\in Җ\cap(Ş-Ƀ)=Җ\cap Ƀ'\cap Ş \\ F'(ж)\cup(G(ж)\cap Б(ж)), & җ\in Җ\cap(Ƀ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Let $(F,Җ)\overset{*}{+}_{\varepsilon}(G,Ƀ)=(V,Җ\cup Ƀ)$, where

$$V(ж)=\begin{cases} F'(ж), & җ\in Җ-Ƀ \\ G'(ж), & җ\in Ƀ-Җ \\ F'(ж)\cup G(ж), & җ\in Җ\cap Ƀ \end{cases}$$

for all $җ\in Җ\cup Ƀ$. Let $(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)=(W,Җ\cup Ş)$, where

$$W(ж)=\begin{cases} F'(ж), & җ\in Җ-Ş \\ Б'(ж), & җ\in Ş-Җ \\ F'(ж)\cup Б(ж), & җ\in Җ\cap Ş \end{cases}$$

for all $җ\in Җ\cup Ş$. Let $(V,Җ\cup Ƀ)\overset{*}{\cap}_{\varepsilon}(W,Җ\cup Ş)=(T,(Җ\cup Ƀ)\cup Ş)$, where

$$T(ж)=\begin{cases} V'(ж), & җ\in(Җ\cup Ƀ)-(Җ\cup Ş)\\ W'(ж), & җ\in(Җ\cup Ş)-(Җ\cup Ƀ)\\ V(ж)\cap W(ж), & җ\in(Җ\cup Ƀ)\cap(Җ\cup Ş)\end{cases}$$

for all $җ\in Җ\cup Ƀ\cup Ş$. Thus,

$$T(ж)=\begin{cases} F(ж), & җ\in(Җ-Ƀ)-(Җ\cup Ş)=\emptyset\\ G(ж), & җ\in(Ƀ-Җ)-(Җ\cup Ş)=Җ^{'}\cap Ƀ\cap Ş^{'}\\ F(ж)\cap G'(ж), & җ\in(Җ\cap Ƀ)-(Җ\cup Ş)=\emptyset\\ F(ж), & җ\in(Җ-Ş)-(Җ\cup Ƀ)=\emptyset\\ Б(ж), & җ\in(Ş-Җ)-(Җ\cup Ƀ)=Җ^{'}\cap Ƀ^{'}\cap Ş\\ F(ж)\cap Б'(ж), & җ\in(Җ\cap Ş)-(Җ\cup Ƀ)=\emptyset\\ F^{'}(ж)\cap F^{'}(ж), & җ\in(Җ-Ƀ)\cap(Җ-Ş)=Җ\cap Ƀ^{'}\cap Ş^{'}\\ F^{'}(ж)\cap Б^{'}(ж), & җ\in(Җ-Ƀ)\cap(Ş-Җ)=\emptyset\\ F^{'}(ж)\cap(F^{'}(ж)\cup Б(ж)), & җ\in(Җ-Ƀ)\cap(Җ\cap Ş)=Җ\cap Ƀ^{'}\cap Ş\\ G^{'}(ж)\cap F^{'}(ж), & җ\in(Ƀ-Җ)\cap(Җ-Ş)=\emptyset\\ G^{'}(ж)\cap Б^{'}(ж), & җ\in(Ƀ-Җ)\cap(Ş-Җ)=Җ^{'}\cap Ƀ\cap Ş\\ G^{'}(ж)\cap(F^{'}(ж)\cup Б(ж)), & җ\in(Ƀ-Җ)\cap(Җ\cap Ş)=\emptyset\\ (F^{'}(ж)\cup G(ж))\cap F^{'}(ж), & җ\in(Җ\cap Ƀ)\cap(Җ-Ş)=Җ\cap Ƀ\cap Ş^{'}\\ (F^{'}(ж)\cup G(ж))\cap Б^{'}(ж), & җ\in(Җ\cap Ƀ)\cap(Ş-Җ)=\emptyset\\ (F^{'}(ж)\cup G(ж))\cap(F^{'}(ж)\cup Б(ж)), & җ\in(Җ\cap Ƀ)\cap(Җ\cap Ş)=Җ\cap Ƀ\cap Ş\end{cases}$$

Hence,

$$T(ж)=\begin{cases} G(ж), & җ\in(Ƀ-Җ)-(Җ\cup Ş)=Җ^{'}\cap Ƀ\cap Ş^{'}\\ Б(ж), & җ\in(Ş-Җ)-(Җ\cup Ƀ)=Җ^{'}\cap Ƀ^{'}\cap Ş\\ F^{'}(ж) & җ\in(Җ-Ƀ)\cap(Җ-Ş)=Җ\cap Ƀ^{'}\cap Ş^{'}\\ F^{'}(ж), & җ\in(Җ-Ƀ)\cap(Җ\cap Ş)=Җ\cap Ƀ^{'}\cap Ş\\ G^{'}(ж)\cap Б^{'}(ж), & җ\in(Ƀ-Җ)\cap(Ş-Җ)=Җ^{'}\cap Ƀ\cap Ş\\ F^{'}(ж) & җ\in(Җ\cap Ƀ)\cap(Җ-Ş)=Җ\cap Ƀ\cap Ş^{'}\\ (F^{'}(ж)\cup G(ж))\cap(F^{'}(ж)\cup Б(ж)), & җ\in(Җ\cap Ƀ)\cap(Җ\cap Ş)=Җ\cap Ƀ\cap Ş\end{cases}$$

N=T is satisfied where $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ'\cap Ş=Җ\cap Ƀ\cap Ş'=\emptyset$. The condition $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ'\cap Ş=\emptyset$ is equivalent to $(Җ\Delta Ƀ)\cap Ş=\emptyset$.

Theorem 3.9. Let (F, Җ), (G,Ƀ), (Б,Ş) be soft sets over U. Then,

1) If $(Җ\Delta Ƀ)\cap Ş=\emptyset$, then $[(F,Җ)\overset{*}{\cup}_{\varepsilon}(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş)=[(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)]\overset{*}{\cap}_{\varepsilon}[(G,Ƀ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

2) If $(Җ\Delta Ƀ)\cap Ş=\emptyset$, then $[(F,Җ)\overset{*}{\cap}_{\varepsilon}(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş)=[(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)]\overset{*}{\cap}_{\varepsilon}[(G,Ƀ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

3) If $(Җ\Delta Ƀ)\cap Ş=Җ\cap Ƀ\cap Ş'=\emptyset$, then $[(F,Җ)\overset{*}{\theta}_{\varepsilon}(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş)=[(F,Җ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]\overset{*}{\cup}_{\varepsilon}[(G,Ƀ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]$.

4) If $(Җ\Delta Ƀ)\cap Ş=Җ\cap Ƀ\cap Ş'=\emptyset$, then $[(F,Җ)\overset{*}{*}_{\varepsilon}(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş)=[(F,Җ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]\overset{*}{\cap}_{\varepsilon}[(G,Ƀ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]$.

Proof: (1) Let $(F,Җ)\overset{*}{\cup}_{\varepsilon}(G,Ƀ)=(M,Җ\cup Ƀ)$, where

$$M(\text{җ})=\begin{cases} F'(\text{җ}), & \text{җ}\in \text{Җ}-\text{Ƀ} \\ G'(\text{җ}), & \text{җ}\in \text{Ƀ}-\text{Җ} \\ F(\text{җ})\cup G(\text{җ}), & \text{җ}\in \text{Җ}\cap\text{Ƀ} \end{cases}$$

for all $\text{җ}\in\text{Җ}\cup\text{Ƀ}$. Let $(M,\text{Җ}\cup\text{Ƀ})\overset{*}{+}_{\varepsilon}(\text{Б},\text{Ş})=(N,(\text{Җ}\cup\text{Ƀ})\cup\text{Ş})$, where for all $\text{җ}\in\text{Җ}\cup\text{Ƀ}\cup\text{Ş}$,

$$N(\text{җ})=\begin{cases} M'(\text{җ}), & \text{җ}\in(\text{Җ}\cup\text{Ƀ})-\text{Ş} \\ \text{Б}'(\text{җ}), & \text{җ}\in\text{Ş}-(\text{Җ}\cup\text{Ƀ}) \\ M'(\text{җ})\cup\text{Б}(\text{җ}), & \text{җ}\in(\text{Җ}\cup\text{Ƀ})\cap\text{Ş} \end{cases}$$

Thus,

$$N(\text{җ})=\begin{cases} F(\text{җ}), & \text{җ}\in(\text{Җ}-\text{Ƀ})-\text{Ş}=\text{Җ}\cap\text{Ƀ}'\cap\text{Ş}' \\ G(\text{җ}), & \text{җ}\in(\text{Ƀ}-\text{Җ})-\text{Ş}=\text{Җ}'\cap\text{Ƀ}\cap\text{Ş}' \\ F'(\text{җ})\cap G'(\text{җ}), & \text{җ}\in(\text{Җ}\cap\text{Ƀ})-\text{Ş}=\text{Җ}\cap\text{Ƀ}\cap\text{Ş}' \\ \text{Б}'(\text{җ}), & \text{җ}\in\text{Ş}-(\text{Җ}\cup\text{Ƀ})=\text{Җ}'\cap\text{Ƀ}'\cap\text{Ş} \\ F(\text{җ})\cup\text{Б}(\text{җ}), & \text{җ}\in(\text{Җ}-\text{Ƀ})\cap\text{Ş}=\text{Җ}\cap\text{Ƀ}'\cap\text{Ş} \\ G(\text{җ})\cup\text{Б}(\text{җ}), & \text{җ}\in(\text{Ƀ}-\text{Җ})\cap\text{Ş}=\text{Җ}'\cap\text{Ƀ}\cap\text{Ş} \\ (F'(\text{җ})\cap G'(\text{җ}))\cup\text{Б}(\text{җ}), & \text{җ}\in(\text{Җ}\cap\text{Ƀ})\cap\text{Ş}=\text{Җ}\cap\text{Ƀ}\cap\text{Ş} \end{cases}$$

Let $(F,\text{Җ})\overset{*}{+}_{\varepsilon}(\text{Б},\text{Ş})=(V,\text{Җ}\cup\text{Ş})$, where

$$V(\text{җ})=\begin{cases} F'(\text{җ}), & \text{җ}\in\text{Җ}-\text{Ş} \\ \text{Б}'(\text{җ}), & \text{җ}\in\text{Ş}-\text{Җ} \\ F'(\text{җ})\cup\text{Б}(\text{җ}), & \text{җ}\in\text{Җ}\cap\text{Ş} \end{cases}$$

for all $\text{җ}\in\text{Җ}\cup\text{Ş}$. Let $(G,\text{Ƀ})\overset{*}{+}_{\varepsilon}(\text{Б},\text{Ş})=(W,\text{Ƀ}\cup\text{Ş})$, where

$$W(\text{җ})=\begin{cases} G'(\text{җ}), & \text{җ}\in\text{Ƀ}-\text{Ş} \\ \text{Б}'(\text{җ}), & \text{җ}\in\text{Ş}-\text{Ƀ} \\ G'(\text{җ})\cup\text{Б}(\text{җ}), & \text{җ}\in\text{Ƀ}\cap\text{Ş} \end{cases}$$

for all $\text{җ}\in\text{Ƀ}\cup\text{Ş}$. Let $(V,\text{Җ}\cup\text{Ş})\overset{*}{\cap}_{\varepsilon}(W,\text{Ƀ}\cup\text{Ş})=(T,\text{Җ}\cup\text{Ƀ}\cup\text{Ş})$, where

$$T(\text{җ})=\begin{cases} V'(\text{җ}), & \text{җ}\in(\text{Җ}\cup\text{Ş})-(\text{Ƀ}\cup\text{Ş}) \\ W'(\text{җ}), & \text{җ}\in(\text{Ƀ}\cup\text{Ş})-(\text{Җ}\cup\text{Ş}) \\ V(\text{җ})\cap W(\text{җ}), & \text{җ}\in(\text{Җ}\cup\text{Ş})\cap(\text{Ƀ}\cup\text{Ş}) \end{cases}$$

for all $\text{җ}\in\text{Җ}\cup\text{Ƀ}\cup\text{Ş}$. Thereby,

$$T(\text{җ})=\begin{cases} F(\text{җ}), & \text{җ}\in(\text{Җ}-\text{Ş})-(\text{Ƀ}\cup\text{Ş})=\text{Җ}\cap\text{Ƀ}'\cap\text{Ş}' \\ \text{Б}(\text{җ}), & \text{җ}\in(\text{Ş}-\text{Җ})-(\text{Ƀ}\cup\text{Ş})=\emptyset \\ F(\text{җ})\cap\text{Б}'(\text{җ}), & \text{җ}\in(\text{Җ}\cap\text{Ş})-(\text{Ƀ}\cup\text{Ş})=\emptyset \\ G(\text{җ}), & \text{җ}\in(\text{Ƀ}-\text{Ş})-(\text{Җ}\cup\text{Ş})=\text{Җ}'\cap\text{Ƀ}\cap\text{Ş}' \\ \text{Б}(\text{җ}), & \text{җ}\in(\text{Ş}-\text{Ƀ})-(\text{Җ}\cup\text{Ş})=\emptyset \\ G(\text{җ})\cap\text{Б}'(\text{җ}), & \text{җ}\in(\text{Ƀ}\cap\text{Ş})-(\text{Җ}\cup\text{Ş})=\emptyset \\ F'(\text{җ})\cap G'(\text{җ}), & \text{җ}\in(\text{Җ}-\text{Ş})\cap(\text{Ƀ}-\text{Ş})=\text{Җ}\cap\text{Ƀ}\cap\text{Ş}' \\ F'(\text{җ})\cap\text{Б}'(\text{җ}), & \text{җ}\in(\text{Җ}-\text{Ş})\cap(\text{Ş}-\text{Ƀ})=\emptyset \\ F'(\text{җ})\cap(G'(\text{җ})\cup\text{Б}(\text{җ})), & \text{җ}\in(\text{Җ}-\text{Ş})\cap(\text{Ƀ}\cap\text{Ş})=\emptyset \\ \text{Б}'(\text{җ})\cap G'(\text{җ}), & \text{җ}\in(\text{Ş}-\text{Җ})\cap(\text{Ƀ}-\text{Ş})=\emptyset \\ \text{Б}'(\text{җ})\cap\text{Б}'(\text{җ}), & \text{җ}\in(\text{Ş}-\text{Җ})\cap(\text{Ş}-\text{Ƀ})=\text{Җ}'\cap\text{Ƀ}'\cap\text{Ş} \\ \text{Б}'(\text{җ})\cap\big(G'(\text{җ})\cup\text{Б}(\text{җ})\big) & \text{җ}\in(\text{Ş}-\text{Җ})\cap(\text{Ƀ}\cap\text{Ş})=\text{Җ}'\cap\text{Ƀ}\cap\text{Ş} \\ (F'(\text{җ})\cup\text{Б}(\text{җ}))\cap G'(\text{җ}), & \text{җ}\in(\text{Җ}\cap\text{Ş})\cap(\text{Ƀ}-\text{Ş})=\emptyset \\ \big(F'(\text{җ})\cup\text{Б}(\text{җ})\big)\cap\text{Б}'(\text{җ}) & \text{җ}\in(\text{Җ}\cap\text{Ş})\cap(\text{Ş}-\text{Ƀ})=\text{Җ}\cap\text{Ƀ}'\cap\text{Ş} \\ (F'(\text{җ})\cup\text{Б}(\text{җ}))\cap(G'(\text{җ})\cup\text{Б}(\text{җ})) & \text{җ}\in(\text{Җ}\cap\text{Ş})\cap(\text{Ƀ}\cap\text{Ş})=\text{Җ}\cap\text{Ƀ}\cap\text{Ş} \end{cases}$$

Therefore,

$$T(җ)=\begin{cases} F(җ), & җ\in(Җ-Ş)-(Ƀ\cup Ş)=Җ\cap Ƀ'\cap Ş' \\ G(җ), & җ\in(Ƀ-Ş)-(Җ\cup Ş)=Җ'\cap Ƀ\cap Ş' \\ F'(җ)\cap G'(җ), & җ\in(Җ-Ş)\cap(Ƀ-Ş)=Җ\cap Ƀ\cap Ş' \\ Б'(җ), & җ\in(Ş-Җ)\cap(Ş-Ƀ)=Җ'\cap Ƀ'\cap Ş \\ Б'(җ)\cap G'(җ), & җ\in(Ş-Җ)\cap(Ƀ\cap Ş)=Җ'\cap Ƀ\cap Ş \\ F'(җ)\cap Б'(җ), & җ\in(Җ\cap Ş)\cap(Ş-Ƀ)=Җ\cap Ƀ'\cap Ş \\ (F'(җ)\cap Б(җ))\cap(G'(җ)\cup Б(җ)) & җ\in(Җ\cap Ş)\cap(Ƀ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

N=T under the condition $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ'\cap Ş=\emptyset$. Here, $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ'\cap Ş=\emptyset$ is equivalent to the condition $(Җ\Delta Ƀ)\cap Ş=\emptyset$.

Theorem 3.10. Let (F, Җ), (G,Ƀ), (Б,Ş) be soft sets over U. Then,

1) If $Җ\cap Ƀ'\cap Ş=Җ\cap Ƀ\cap Ş=\emptyset$, then $(F,Җ)\overset{*}{+}_{\varepsilon}[(G,Ƀ)\tilde{\cap}(Б,Ş)]=[(F,Җ)\overset{*}{+}_{\varepsilon}(G,Ƀ)]\tilde{\cap}[(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

2) If $(Җ\Delta Ƀ)\cap Ş=\emptyset$, then $(F,Җ)\overset{*}{+}_{\varepsilon}[(G,Ƀ)\tilde{\cup}(Б,Ş)]=[(F,Җ)\overset{*}{+}_{\varepsilon}(G,Ƀ)]\tilde{\cup}[(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

3) If $(Җ\Delta Ƀ)\cap Ş=Җ\cap Ƀ\cap Ş'=\emptyset$, then $(F,Җ)\overset{*}{+}_{\varepsilon}[(G,Ƀ)\tilde{*}(Б,Ş)]=[(F,Җ)\overset{*}{*}_{\varepsilon}(G,Ƀ)]\tilde{\cup}[(F,Җ)\overset{*}{*}_{\varepsilon}(Б,Ş)]$.

4) If $(Җ\Delta Ƀ)\cap Ş=Җ\cap Ƀ'\cap Ş=\emptyset$, then $(F,Җ)\overset{*}{+}_{\varepsilon}[(G,Ƀ)\tilde{\theta}(Б,Ş)]=[(F,Җ)\overset{*}{*}_{\varepsilon}(G,Ƀ)]\tilde{\cap}[(F,Җ)\overset{*}{*}_{\varepsilon}(Б,Ş)]$.

Proof: (1) Let $(G,Ƀ)\tilde{\cap}(Б,Ş)=(M,Ƀ)$, where

$$M(җ)=\begin{cases} G(җ), & җ\in Ƀ-Ş \\ G(җ)\cap Б(җ), & җ\in Ƀ\cap Ş \end{cases}$$

for all $җ\in Ƀ$. Let $(F,Җ)\overset{*}{+}_{\varepsilon}(M,Ƀ)=(N,Җ\cup Ƀ)$, where

$$N(җ)=\begin{cases} F'(җ), & җ\in Җ-Ƀ \\ M'(җ), & җ\in Ƀ-Җ \\ F'(җ)\cup M(җ), & җ\in Җ\cap Ƀ \end{cases}$$

for all $җ\in Җ\cup Ƀ$. Thus,

$$N(җ)=\begin{cases} F'(җ), & җ\in Җ-Ƀ \\ G'(җ), & җ\in(Ƀ-Ş)-Җ=Җ'\cap Ƀ\cap Ş' \\ G'(җ)\cup Б'(җ), & җ\in(Ƀ\cap Ş)-Җ=Җ'\cap Ƀ\cap Ş \\ F'(җ)\cup G(җ), & җ\in Җ\cap(Ƀ-Ş)=Җ\cap Ƀ\cap Ş' \\ F'(җ)\cup(G(җ)\cap Б(җ)), & җ\in Җ\cap(Ƀ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Let $(F,Җ)\overset{*}{+}_{\varepsilon}(G,Ƀ)=(V,Җ\cup Ƀ)$, where

$$V(җ)=\begin{cases} F'(җ), & җ\in Җ-Ƀ \\ G'(җ), & җ\in Ƀ-Җ \\ F'(җ)\cup G(җ), & җ\in Җ\cap Ƀ \end{cases}$$

for all $җ\in Җ\cup Ƀ$. Let $(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)=(W,Җ\cup Ş)$, where

$$W(җ)=\begin{cases} F'(җ), & җ\in Җ-Ş \\ Б'(җ), & җ\in Ş-Җ \\ F'(җ)\cup Б(җ), & җ\in Җ\cap Ş \end{cases}$$

for all $җ\in Җ\cup Ş$. Let $(V,Җ\cup Ƀ)\tilde{\cap}(W,Җ\cup Ş)=(T,(Җ\cup Ƀ))$, where

$$T(ж)=\begin{cases} V(ж), & ж\in(Ж\cupɃ)-(Ж\cupŞ) \\ V(ж)\cap W(ж), & ж\in(Ж\cupɃ)\cap(Ж\cupŞ) \end{cases}$$

for all $ж\in Ж\cupɃ$. Thus,

$$T(ж)=\begin{cases} F'(ж), & ж\in(Ж-Ƀ)-(Ж\cupŞ)=\emptyset \\ G'(ж), & ж\in(Ƀ-Ж)-(Ж\cupŞ)=Ж'\capɃ\capŞ' \\ F'(ж)\cup G(ж), & ж\in(Ж\capɃ)-(Ж\cupŞ)=\emptyset \\ F'(ж)\cup F'(ж), & ж\in(Ж-Ƀ)\cap(Ж-Ş)=Ж\capɃ'\capŞ' \\ F'(ж)\cup Б'(ж), & ж\in(Ж-Ƀ)\cap(Ş-Ж)=\emptyset \\ F'(ж)\cup(F'(ж)\cup Б(ж)), & ж\in(Ж-Ƀ)\cap(Ж\capŞ)=Ж\capɃ'\capŞ \\ G'(ж)\cup F'(ж), & ж\in(Ƀ-Ж)\cap(Ж-Ş)=\emptyset \\ G'(ж)\cup Б'(ж), & ж\in(Ƀ-Ж)\cap(Ş-Ж)=Ж'\capɃ\capŞ \\ G'(ж)\cup(F'(ж)\cup Б(ж)), & ж\in(Ƀ-Ж)\cap(Ж\capŞ)=\emptyset \\ (F'(ж)\cup G(ж))\cup F'(ж), & ж\in(Ж\capɃ)\cap(Ж-Ş)=Ж\capɃ\capŞ' \\ (F'(ж)\cup G(ж))\cup Б'(ж), & ж\in(Ж\capɃ)\cap(Ş-Ж)=\emptyset \\ (F'(ж)\cup G(ж))\cup(F'(ж)\cup Б(ж)), & ж\in(Ж\capɃ)\cap(Ж\capŞ)=Ж\capɃ\capŞ \end{cases}$$

Therefore,

$$T(ж)=\begin{cases} G'(ж), & ж\in(Ƀ-Ж)-(Ж\cupŞ)=Ж'\capɃ\capŞ' \\ F'(ж), & ж\in(Ж-Ƀ)\cap(Ж-Ş)=Ж\capɃ'\capŞ' \\ F'(ж)\cup Б(ж), & ж\in(Ж-Ƀ)\cap(Ж\capŞ)=Ж\capɃ'\capŞ \\ G'(ж)\cup Б'(ж), & ж\in(Ƀ-Ж)\cap(Ş-Ж)=Ж'\capɃ\capŞ \\ F'(ж)\cup G(ж), & ж\in(Ж\capɃ)\cap(Ж-Ş)=Ж\capɃ\capŞ' \\ (F'(ж)\cup G(ж))\cup(F'(ж)\cup Б(ж)), & ж\in(Ж\capɃ)\cap(Ж\capŞ)=Ж\capɃ\capŞ \end{cases}$$

Here, if we consider $Ж-Ƀ$ in the function N, since $Ж-Ƀ= Ж\capɃ'$, if an element is in the complement of Ƀ, then it is either in $Ş-Ƀ$ or $(Ƀ \cup Ş)'$. Thus, if $ж \in Ж-Ƀ$, then $ж \in Ж\capɃ'\capŞ$ or $ж \in Ж\capɃ'\capŞ'$. Thus, it is seen that N=T, where $Ж\capɃ'\capŞ=Ж\capɃ\capŞ=\emptyset$.

Theorem 3.11. Let (F, Ж), (G,Ƀ), (Б,Ş) be soft sets over U. Then,

1) If $(Ж\DeltaɃ)\capŞ=\emptyset$, then $[(F,Ж)\tilde{\cup}(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş)=[(F,Ж)\overset{*}{+}_{\varepsilon}(Б,Ş)]\tilde{\cap}[(G,Ƀ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

2) If $(Ж\DeltaɃ)\capŞ=\emptyset$, then $[(F,Ж)\tilde{\cap}(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş)=[(F,Ж)\overset{*}{+}_{\varepsilon}(Б,Ş)]\tilde{\cup}[(G,Ƀ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

3) If $(Ж\DeltaɃ)\capŞ=Ж\capɃ\capŞ'=\emptyset$, then $[(F,Ж)\tilde{\theta}(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş)=[(F,Ж)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]\tilde{\cup}[(G,Ƀ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]$.

4) If $(Ж\DeltaɃ)\capŞ=Ж\capɃ\capŞ'=\emptyset$, then $[(F,Ж)\tilde{*}(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş)=[(F,Ж)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]\tilde{\cap}[(G,Ƀ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]$.

Proof: (1) Let $(F,Ж)\tilde{\cup}(G,Ƀ)=(M,Ж)$, where

$$M(ж)=\begin{cases} F(ж), & ж\in Ж-Ƀ \\ F(ж)\cup G(ж), & ж\in Ж\capɃ \end{cases}$$

for all $ж\in Ж$. Let $M,Ж)\overset{*}{+}_{\varepsilon}(Б,Ş)=(N,Ж\cupŞ)$, where

$$N(ж)=\begin{cases} M'(ж), & ж\in Ж-Ş \\ Б'(ж), & ж\in Ş-Ж \\ M'(ж)\cup Б(ж), & ж\in Ж\capŞ \end{cases}$$

for all $җ \in Җ \cup Ş$, Thus,

$$N(җ)=\begin{cases} F'(җ), & җ \in (Җ-Ƀ)-Ş=Җ\cap Ƀ'\cap Ş' \\ F'(җ)\cap G'(җ), & җ \in (Җ\cap Ƀ)-Ş=Җ\cap Ƀ\cap Ş' \\ Б'(җ), & җ \in Ş-Җ \\ F'(җ)\cup Б(җ), & җ \in (Җ-Ƀ)\cap Ş=Җ\cap Ƀ'\cap Ş \\ (F'(җ)\cap G'(җ))\cup Б(җ), & җ \in (Җ\cap Ƀ)\cap Ş=Җ\cap Ƀ\cap Ş \end{cases}$$

Let $(F, Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)=(V,Җ\cup Ş)$, where

$$V(җ)=\begin{cases} F'(җ), & җ \in Җ-Ş \\ Б'(җ), & җ \in Ş-Җ \\ F'(җ)\cup Б(җ), & җ \in Җ\cap Ş \end{cases}$$

for all $җ \in Җ\cup Ş$. Now let $(G,Ƀ)\overset{*}{+}_{\varepsilon}(Б,Ş)=(W,Ƀ\cup Ş)$, where

$$W(җ)=\begin{cases} G'(җ), & җ \in Ƀ-Ş \\ Б'(җ), & җ \in Ş-Ƀ \\ G'(җ)\cup Б(җ), & җ \in Ƀ\cap Ş \end{cases}$$

for all $җ \in Ƀ\cup Ş$. Let $(V,Җ\cup Ş)\,\tilde{\cap}\,(W,Ƀ\cup Ş)=(T,(Җ\cup Ş))$, where

$$T(җ)=\begin{cases} V(җ), & җ \in (Җ\cup Ş)-(Ƀ\cup Ş) \\ V(җ)\cap W(җ), & җ \in (Җ\cup Ş)\cap(Ƀ\cup Ş) \end{cases}$$

for all $җ \in Җ\cup Ş$. Thereby,

$$T(җ)=\begin{cases} F'(җ), & җ \in (Җ-Ş)-(Ƀ\cup Ş)=Җ\cap Ƀ'\cap Ş' \\ Б'(җ), & җ \in (Ş-Җ)-(Ƀ\cup Ş)=\emptyset \\ F'(җ)\cup Б(җ), & җ \in (Җ\cap Ş)-(Ƀ\cup Ş)=\emptyset \\ F'(җ)\cap G'(җ), & җ \in (Җ-Ş)\cap(Ƀ-Ş)=Җ\cap Ƀ\cap Ş' \\ F'(җ)\cap Б'(җ), & җ \in (Җ-Ş)\cap(Ş-Ƀ)=\emptyset \\ F'(җ)\cap\big(G'(җ)\cup Б(җ)\big), & җ \in (Җ-Ş)\cap(Ƀ\cap Ş)=\emptyset \\ Б'(җ)\cap G'(җ), & җ \in (Ş-Җ)\cap(Ƀ-Ş)=\emptyset \\ Б'(җ)\cap Б'(җ), & җ \in (Ş-Җ)\cap(Ş-Ƀ)=Җ'\cap Ƀ'\cap Ş \\ Б'(җ)\cap\big(G'(җ)\cup Б(җ)\big) & җ \in (Ş-Җ)\cap(Ƀ\cap Ş)=Җ'\cap Ƀ\cap Ş \\ \big(F'(җ)\cup Б(җ)\big)\cap G'(җ), & җ \in (Җ\cap Ş)\cap(Ƀ-Ş)=\emptyset \\ \big(F'(җ)\cup Б(җ)\big)\cap Б'(җ) & җ \in (Җ\cap Ş)\cap(Ş-Ƀ)=Җ\cap Ƀ'\cap Ş \\ \big(F'(җ)\cup Б(җ)\big)\cap\big(G'(җ)\cup Б(җ)\big), & җ \in (Җ\cap Ş)\cap(Ƀ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Hence,

$$T(җ)=\begin{cases} F'(җ), & җ \in (Җ-Ş)-(Ƀ\cup Ş)=Җ\cap Ƀ'\cap Ş' \\ F'(җ)\cap G'(җ), & җ \in (Җ-Ş)\cap(Ƀ-Ş)=Җ\cap Ƀ\cap Ş' \\ Б'(җ), & җ \in (Ş-Җ)\cap(Ş-Ƀ)=Җ'\cap Ƀ'\cap Ş \\ Б'(җ)\cap G'(җ), & җ \in (Ş-Җ)\cap(Ƀ\cap Ş)=Җ'\cap Ƀ\cap Ş \\ F'(җ)\cap Б'(җ), & җ \in (Җ\cap Ş)\cap(Ş-Ƀ)=Җ\cap Ƀ'\cap Ş \\ \big(F'(җ)\cup Б(җ)\big)\cap\big(G'(җ)\cup Б(җ)\big), & җ \in (Җ\cap Ş)\cap(Ƀ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Under the condition $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ'\cap Ş=\emptyset$, N=T. The condition $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ'\cap Ş=\emptyset$ is equivalent to $(Җ\Delta Ƀ)\cap Ş=\emptyset$.

Theorem 3.12. Let (F, Җ), (G,Ƀ), (Б,Ş) be soft sets over U. Then,

1) If $(\text{Җ}\Delta\text{Ş})\cap\text{Ƀ}=\emptyset$, then $(F,\text{Җ})\overset{*}{+}_{\varepsilon}[(G,\text{Ƀ})\underset{\cap}{\overset{*}{\sim}}(\text{Б},\text{Ş})]=[(F,\text{Җ})\overset{*}{+}_{\varepsilon}(G,\text{Ƀ})]\underset{\cap}{\overset{*}{\sim}}[(F,\text{Җ})\overset{*}{+}_{\varepsilon}(\text{Б},\text{Ş})]$.

2) If $(\text{Җ}\Delta\text{Ƀ})\cap\text{Ş}=\text{Җ}\cap\text{Ƀ}\cap\text{Ş}'=\emptyset$, then $(F,\text{Җ})\overset{*}{+}_{\varepsilon}[(G,\text{Ƀ})\underset{\cup}{\overset{*}{\sim}}(\text{Б},\text{Ş})]=[(F,\text{Җ})\overset{*}{+}_{\varepsilon}(G,\text{Ƀ})]\underset{\cup}{\overset{*}{\sim}}[(F,\text{Җ})\overset{*}{+}_{\varepsilon}(\text{Б},\text{Ş})]$.

3) If $(\text{Җ}\Delta\text{Ƀ})\cap\text{Ş}=\emptyset$, then $(F,\text{Җ})\overset{*}{+}_{\varepsilon}[(G,\text{Ƀ})\underset{*}{\overset{*}{\sim}}(\text{Б},\text{Ş})]=[(F,\text{Җ})\overset{*}{*}_{\varepsilon}(G,\text{Ƀ})]\underset{\cup}{\overset{*}{\sim}}[(F,\text{Җ})\overset{*}{*}_{\varepsilon}(\text{Б},\text{Ş})]$.

4) If $(\text{Җ}\Delta\text{Ƀ})\cap\text{Ş}=\text{Җ}\cap\text{Ƀ}\cap\text{Ş}'=\emptyset$, then $(F,\text{Җ})\overset{*}{+}_{\varepsilon}[(G,\text{Ƀ})\underset{\theta}{\overset{*}{\sim}}(\text{Б},\text{Ş})]=[(F,\text{Җ})\overset{*}{*}_{\varepsilon}(G,\text{Ƀ})]\underset{\cap}{\overset{*}{\sim}}[(F,\text{Җ})\overset{*}{*}_{\varepsilon}(\text{Б},\text{Ş})]$.

Proof: (1) Let $(G,\text{Ƀ})\underset{\cap}{\overset{*}{\sim}}(\text{Б},\text{Ş})=(M,\text{Ƀ})$, where

$$M(\text{җ})=\begin{cases} G'(\text{җ}), & \text{җ}\in\text{Ƀ}-\text{Ş} \\ G(\text{җ})\cap\text{Б}(\text{җ}), & \text{җ}\in\text{Ƀ}\cap\text{Ş} \end{cases}$$

for all җ∈Ƀ. Let $(F,\text{Җ})\overset{*}{+}_{\varepsilon}(M,\text{Ƀ})=(N,\text{Җ}\cup\text{Ƀ})$, where

$$N(\text{җ})=\begin{cases} F'(\text{җ}), & \text{җ}\in\text{Җ}-\text{Ƀ} \\ M'(\text{җ}), & \text{җ}\in\text{Ƀ}-\text{Җ} \\ F'(\text{җ})\cup M(\text{җ}), & \text{җ}\in\text{Җ}\cap\text{Ƀ} \end{cases}$$

for all җ∈Җ∪Ƀ. Hence,

$$N(\text{җ})=\begin{cases} F'(\text{җ}), & \text{җ}\in\text{Җ}-\text{Ƀ} \\ G(\text{җ}), & \text{җ}\in(\text{Ƀ}-\text{Ş})-\text{Җ}=\text{Җ}'\cap\text{Ƀ}\cap\text{Ş}' \\ G'(\text{җ})\cup\text{Б}'(\text{җ}), & \text{җ}\in(\text{Ƀ}\cap\text{Ş})-\text{Җ}=\text{Җ}'\cap\text{Ƀ}\cap\text{Ş} \\ F'(\text{җ})\cup G'(\text{җ}), & \text{җ}\in\text{Җ}\cap(\text{Ƀ}-\text{Ş})=\text{Җ}\cap\text{Ƀ}\cap\text{Ş}' \\ F'(\text{җ})\cup(G(\text{җ})\cap\text{Б}(\text{җ})), & \text{җ}\in\text{Җ}\cap(\text{Ƀ}\cap\text{Ş})=\text{Җ}\cap\text{Ƀ}\cap\text{Ş} \end{cases}$$

Let $(F,\text{Җ})\overset{*}{+}_{\varepsilon}(G,\text{Ƀ})=(V,\text{Җ}\cup\text{Ƀ})$, where

$$V(\text{җ})=\begin{cases} F'(\text{җ}), & \text{җ}\in\text{Җ}-\text{Ƀ} \\ G'(\text{җ}), & \text{җ}\in\text{Ƀ}-\text{Җ} \\ F'(\text{җ})\cup G(\text{җ}), & \text{җ}\in\text{Җ}\cap\text{Ƀ} \end{cases}$$

for all җ∈Җ∪Ƀ. Let $(F,\text{Җ})\overset{*}{+}_{\varepsilon}(\text{Б},\text{Ş})=(W,\text{Җ}\cup\text{Ş})$, where

$$W(\text{җ})=\begin{cases} F'(\text{җ}), & \text{җ}\in\text{Җ}-\text{Ş} \\ \text{Б}'(\text{җ}), & \text{җ}\in\text{Ş}-\text{Җ} \\ F'(\text{җ})\cup\text{Б}(\text{җ}), & \text{җ}\in\text{Җ}\cap\text{Ş} \end{cases}$$

for all җ∈Җ∪Ş. Let $(V,\text{Җ}\cup\text{Ƀ})\underset{\cap}{\overset{*}{\sim}}(W,\text{Җ}\cup\text{Ş})=(T,(\text{Җ}\cup\text{Ƀ}))$, where

$$T(\text{җ})=\begin{cases} V'(\text{җ}), & \text{җ}\in(\text{Җ}\cup\text{Ƀ})-(\text{Җ}\cup\text{Ş}) \\ V(\text{җ})\cap W(\text{җ}), & \text{җ}\in(\text{Җ}\cup\text{Ƀ})\cap(\text{Җ}\cup\text{Ş}) \end{cases}$$

for all җ∈Җ∪Ƀ. Thus,

$$T(җ)=\begin{cases} F(җ), & җ∈(Җ-Ƀ)-(Җ\cup Ş)=\emptyset \\ G(җ), & җ∈(Ƀ-Җ)-(Җ\cup Ş)=Җ'\cap Ƀ\cap Ş' \\ F(җ)\cap G'(җ), & җ∈(Җ\cap Ƀ)-(Җ\cup Ş)=\emptyset \\ F'(җ)\cap F'(җ), & җ∈(Җ-Ƀ)\cap(Җ-Ş)=Җ\cap Ƀ'\cap Ş' \\ F'(җ)\cap Б'(җ), & җ∈(Җ-Ƀ)\cap(Ş-Җ)=\emptyset \\ F'(җ)\cap(F'(җ)\cup Б(җ)), & җ∈(Җ-Ƀ)\cap(Җ\cap Ş)=Җ\cap Ƀ'\cap Ş \\ G'(җ)\cap F'(җ), & җ∈(Ƀ-Җ)\cap(Җ-Ş)=\emptyset \\ G'(җ)\cap Б'(җ), & җ∈(Ƀ-Җ)\cap(Ş-Җ)=Җ'\cap Ƀ\cap Ş \\ G'(җ)\cap(F'(җ)\cup Б(җ)), & җ∈(Ƀ-Җ)\cap(Җ\cap Ş)=\emptyset \\ (F'(җ)\cup G(җ))\cap F'(җ), & җ∈(Җ\cap Ƀ)\cap(Җ-Ş)=Җ\cap Ƀ\cap Ş' \\ (F'(җ)\cup G(җ))\cap Б'(җ), & җ∈(Җ\cap Ƀ)\cap(Ş-Җ)=\emptyset \\ (F'(җ)\cup G(җ))\cap(F'(җ)\cup Б(җ)), & җ∈(Җ\cap Ƀ)\cap(Җ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Hence,

$$T(җ)=\begin{cases} G(җ), & җ∈(Ƀ-Җ)-(Җ\cup Ş)=Җ'\cap Ƀ\cap Ş' \\ F'(җ), & җ∈(Җ-Ƀ)\cap(Җ-Ş)=Җ\cap Ƀ'\cap Ş' \\ F'(җ), & җ∈(Җ-Ƀ)\cap(Җ\cap Ş)=Җ\cap Ƀ'\cap Ş \\ G'(җ)\cap Б'(җ), & җ∈(Ƀ-Җ)\cap(Ş-Җ)=Җ'\cap Ƀ\cap Ş \\ F'(җ), & җ∈(Җ\cap Ƀ)\cap(Җ-Ş)=Җ\cap Ƀ\cap Ş' \\ (F'(җ)\cup G(җ))\cap(F'(җ)\cup Б(җ)), & җ∈(Җ\cap Ƀ)\cap(Җ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Under the condition $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ\cap Ş'=\emptyset$, N=T. The condition $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ\cap Ş'=\emptyset$ is equivalent to $(Җ\Delta Ş)\cap Ƀ=\emptyset$.

Theorem 3.13. Let (F, Җ), (G,Ƀ), (Б,Ş) be soft sets over U. Then,

1) If $(Җ\Delta Ƀ)\cap Ş=\emptyset$, then $[(F,Җ)\overset{*}{\underset{\cup}{\sim}}(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş)=[(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)]\overset{*}{\underset{\cap}{\sim}}[(G,Ƀ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

2) If $(Җ\Delta Ƀ)\cap Ş=\emptyset$, then $[(F,Җ)\overset{*}{\underset{\cap}{\sim}}(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş)=[(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)]\overset{*}{\underset{\cup}{\sim}}[(G,Ƀ)\overset{*}{+}_{\varepsilon}(Б,Ş)]$.

3) If $(Җ\Delta Ƀ)\cap Ş=Җ\cap Ƀ\cap Ş'=\emptyset$, then $[(F,Җ)\overset{*}{\underset{\theta}{\sim}}(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş)=[(F,Җ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]\overset{*}{\underset{\cup}{\sim}}[(G,Ƀ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]$.

4) If $(Җ\Delta Ƀ)\cap Ş=Җ\cap Ƀ\cap Ş'=\emptyset$ then $[(F,Җ)\overset{*}{\underset{*}{\sim}}(G,Ƀ)]\overset{*}{+}_{\varepsilon}(Б,Ş)=[(F,Җ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]\overset{*}{\underset{\cap}{\sim}}[(G,Ƀ)\overset{*}{\cup}_{\varepsilon}(Б,Ş)]$.

Proof: (1) Let $(F,Җ)\overset{*}{\underset{\cup}{\sim}}(G,Ƀ)=(M,Җ)$, where

$$M(җ)=\begin{cases} F'(җ), & җ∈Җ-Ƀ \\ F(җ)\cup G(җ), & җ∈Җ\cap Ƀ \end{cases}$$

for all $җ∈Җ$. Let $(M,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)=(N,Җ\cup Ş)$, where

$$N(җ)=\begin{cases} M'(җ), & җ∈Җ-Ş \\ Б'(җ), & җ∈Ş-Җ \\ M'(җ)\cup Б(җ), & җ∈Җ\cap Ş \end{cases}$$

for all $җ∈Җ\cup Ş$. Thus,

$$N(җ)=\begin{cases} F(җ), & җ\in(Җ-Ƀ)-Ş=Җ\cap Ƀ'\cap Ş' \\ F'(җ)\cap G'(җ), & җ\in(Җ\cap Ƀ)-Ş=Җ\cap Ƀ\cap Ş' \\ Б'(җ), & җ\in Ş-Җ \\ F(җ)\cup Б(җ), & җ\in(Җ-Ƀ)\cap Ş=Җ\cap Ƀ'\cap Ş \\ (F'(җ)\cap G'(җ))\cup Б(җ), & җ\in(Җ\cap Ƀ)\cap Ş=Җ\cap Ƀ\cap Ş \end{cases}$$

Let $(F,Җ)\overset{*}{+}_{\varepsilon}(Б,Ş)=(V,Җ\cup Ş)$, where

$$V(җ)=\begin{cases} F'(җ), & җ\in Җ-Ş \\ Б'(җ), & җ\in Ş-Җ \\ F'(җ)\cup Б(җ), & җ\in Җ\cap Ş \end{cases}$$

for all $җ\in Җ\cup Ş$. Let $(G,Ƀ)\overset{*}{+}_{\varepsilon}(Б,Ş)=(W,Ƀ\cup Ş)$, where

$$W(җ)=\begin{cases} G'(җ), & җ\in Ƀ-Ş \\ Б'(җ), & җ\in Ş-Ƀ \\ G'(җ)\cup Б(җ), & җ\in Ƀ\cap Ş \end{cases}$$

for all $җ\in Ƀ\cup Ş$. Let $(V,Җ\cup Ş)\underset{\cap}{\overset{*}{\sim}}(W,Ƀ\cup Ş)=(T,(Җ\cup Ş))$, where

$$T(җ)=\begin{cases} V'(җ), & җ\in(Җ\cup Ş)-(Ƀ\cup Ş) \\ V(җ)\cap W(җ), & җ\in(Җ\cup Ş)\cap(Ƀ\cup Ş) \end{cases}$$

for all $җ\in Җ\cup Ş$. Thus,

$$T(җ)=\begin{cases} F(җ), & җ\in(Җ-Ş)-(Ƀ\cup Ş)=Җ\cap Ƀ'\cap Ş' \\ Б(җ), & җ\in(Ş-Җ)-(Ƀ\cup Ş)=\emptyset \\ F(җ)\cap Б'(җ), & җ\in(Җ\cap Ş)-(Ƀ\cup Ş)=\emptyset \\ F'(җ)\cap G'(җ), & җ\in(Җ-Ş)\cap(Ƀ-Ş)=Җ\cap Ƀ\cap Ş' \\ F'(җ)\cap Б'(җ), & җ\in(Җ-Ş)\cap(Ş-Ƀ)=\emptyset \\ F'(җ)\cap\big(G'(җ)\cup Б(җ)\big), & җ\in(Җ-Ş)\cap(Ƀ\cap Ş)=\emptyset \\ Б'(җ)\cap G'(җ), & җ\in(Ş-Җ)\cap(Ƀ-Ş)=\emptyset \\ Б'(җ)\cap Б'(җ), & җ\in(Ş-Җ)\cap(Ş-Ƀ)=Җ'\cap Ƀ'\cap Ş \\ Б'(җ)\cap\big(G'(җ)\cup Б(җ)\big) & җ\in(Ş-Җ)\cap(Ƀ\cap Ş)=Җ'\cap Ƀ\cap Ş \\ \big(F'(җ)\cup Б(җ)\big)\cap G'(җ), & җ\in(Җ\cap Ş)\cap(Ƀ-Ş)=\emptyset \\ \big(F'(җ)\cup Б(җ)\big)\cap Б'(җ) & җ\in(Җ\cap Ş)\cap(Ş-Ƀ)=Җ\cap Ƀ'\cap Ş \\ \big(F'(җ)\cup Б(җ)\big)\cap\big(G'(җ)\cup Б(җ)\big), & җ\in(Җ\cap Ş)\cap(Ƀ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Therefore,

$$T(җ)=\begin{cases} F(җ), & җ\in(Җ-Ş)-(Ƀ\cup Ş)=Җ\cap Ƀ'\cap Ş' \\ F'(җ)\cap G'(җ), & җ\in(Җ-Ş)\cap(Ƀ-Ş)=Җ\cap Ƀ\cap Ş' \\ Б'(җ), & җ\in(Ş-Җ)\cap(Ş-Ƀ)=Җ'\cap Ƀ'\cap Ş \\ Б'(җ)\cap G'(җ) & җ\in(Ş-Җ)\cap(Ƀ\cap Ş)=Җ'\cap Ƀ\cap Ş \\ F'(җ)\cap Б'(җ) & җ\in(Җ\cap Ş)\cap(Ş-Ƀ)=Җ\cap Ƀ'\cap Ş \\ \big(F'(җ)\cup Б(җ)\big)\cap\big(G'(җ)\cup Б(җ)\big), & җ\in(Җ\cap Ş)\cap(Ƀ\cap Ş)=Җ\cap Ƀ\cap Ş \end{cases}$$

Under the condition $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ'\cap Ş=\emptyset$, N=T. The condition $Җ'\cap Ƀ\cap Ş=Җ\cap Ƀ'\cap Ş=\emptyset$ is equivalent to $(Җ\Delta Ƀ)\cap Ş=\emptyset$.

4 | CONCLUSION

Soft set operations are fundamental in soft set theory, providing a framework for managing uncertainty in data analysis and decision-making. In this context, we introduce a novel soft set operation called the complementary extended plus operation and explore its algebraic properties. This study examines the distributions of the complementary extended plus operation over various other soft set operations. Future research may investigate different forms of complementary extended soft set operations, their distributions, and their characteristics to identify the algebraic structures they generate within the collection of soft sets.

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