ORIGINAL ARTICLE

An Intelligent Algorithm for Evaluating Martial Arts Teaching Skills Based on Complex Picture Fuzzy Dombi Aggregation Operator

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ABSTRACT

Martial arts teaching skills involve the systematic instruction of physical techniques, mental discipline, and tactical strategies to develop a student's proficiency in self-defense and personal growth. It is relevant when there is a specific analysis of the diverse features involving the technique, versatility, and interaction features. In Multi-Attribute Decision-Making (MADM), conventional approaches cannot provide a framework to capture the uncertainties and vague judgments inevitably associated with global experts' evaluations of such a broad spectrum of skills. These limitations are seen when the evaluations do not consider the dynamism, uncertainty, or subjective nature inherent in the evaluation process, thus having less reliable or less complete decision outcomes. To handle these challenges, the concept of Complex Picture Fuzzy Sets (C-PFS) is utilized, and new advanced Dombi aggregation operators (AO), including Complex Picture Fuzzy Dombi Weighted Averaging (C-PFDWA) and Complex Picture Fuzzy Dombi Weighted Geometric (C-PFDWG) operators are developed. These operators can be effectively utilized to overcome real-life evaluation issues in martial arts skills assessment, serving as intelligent algorithms that facilitate more accurate decision-making processes. To allow the collection of the judgments of different experts over the membership, abstention, and non-membership degrees, offering a comprehensive and detailed picture of an instructor's performance even in cases where there is uncertainty. This comprehensive approach ensures a more reliable and customized assessment, enhancing instructional quality and better student outcomes as instructors receive more targeted feedback on improvement areas. In addition, this study includes detailed simulations and complete results of selected parameters, which demonstrate the practical applicability and robustness of the proposed operators, thus providing strong justification for the research and validating the efficacy of the developed framework.

Keywords: Aggregation operator; complex picture fuzzy Dombi aggregation operators; intelligent algorithm; martial arts teaching evaluation; teaching skills assessment.

(MSC2020) Codes: 03E72, 03E72, 68T20, 97D40, 47S40

1 | INTRODUCTION

In evaluating martial arts teaching skills, instructors are often assessed on multiple factors like technique mastery, student engagement, physical fitness, and professionalism. These assessments involve subjective judgments, which are inherently uncertain and imprecise, making traditional decision-making approaches inadequate. Thus, more complex approaches, such as the Fuzzy sets (FS) theories, are needed for evaluation purposes, as the nature of such problems is multi-criteria and full of uncertainty, which requires a more appropriate analysis of an instructor's skills.

FS [1], affords partial memberships that provide a facilitated way of expressing truth degrees varying from 0 to 1. However, as far as uncertainty is concerned, FS is limited since they only consider membership degrees. This limitation gave rise to intuitionistic fuzzy sets (IFS) [2] to cope with the degree of non-membership. Nevertheless, there is still a lack of more IFS where decision-makers could have neutral or even abstained views about some situations. To overcome these shortcomings mentioned above, a new category called the Pythagorean fuzzy sets (PyFS) [3] was introduced in the literature; that is more flexible as it permits this sum of the square of the membership and non-membership degree to be less than or equal to one. This enlarges the range of uncertainties that it is possible to describe and is more appropriate in cases where accurate decision-making is not feasible in PyFS. Building on this, q-rung orthopair fuzzy sets (q-ROFS) [4], offer more flexibility to the extent that the sum of the q^{th} powers of membership and non-membership degrees can be equal to or less than 1. For more nuanced decision-making, picture fuzzy sets (PFS) [5], incorporate the abstinence (neutral) degree in addition to membership and non-membership degrees. Building further on this, complex picture fuzzy sets (CPFS) [6] use complex grades, which is very suitable for elaborating the complicated connections in martial arts teaching and others, such as subjective evaluations.

The Dombi AO [7] has been successfully extended to other FS environments to improve decision-making within the presence of vagueness. As for FS, it supplied the flexible control of its parameters, which gave the possibility to lead the aggregation. The IFS [8] allows hesitation while handling both membership and non-membership degrees. PyFS [9] and q-ROFS [10] did modify the Dombi AO to fit better the treatment of larger uncertainty spaces more essential for MADM. [11] used Dombi AO in PFS for membership, non-membership, or abstinence, which is also useful for conflicting or neutral responses. Indeed, there are constant successes in using this operator that involve the accretion of voluminous subjective judgments, including policy analysis and decision support systems [12], emergency logistics suppliers selection [13], selection of E-learning websites [14], and selection of cloud services [15].

To improve the quality of assessing the teaching skills in martial arts, we propose C-PFDWA and C-PFDWG operators as the components of the approach. Unlike those traditional fuzzy AOs, which may fail to deal with the compounded aspects of skill evaluation due to the hierarchical or layered nature of information, these operators are indeed effective in processing powerful PF information, resulting in a more accurate assessment of the negative and positive criteria within multi-dimensional criteria. Applying the Dombi operator improves the identification of changes in performance features, including technique and professionalism, making these fuzzy operators most appropriate for use when assessing students' learning martial arts. The use of this innovation enables the proper and efficient combination of data under high uncertainty, which appropriately complements the subjective evaluation of teaching skills often involved in assessment.

1.1 | Research Gap and Motivations

Thus, in MADM problems, traditional fuzzy AOs, including IFS and PyFS, cannot depict the abstinence degree well, which is significant when facing uncertainty and incompleteness. However, there remain certain problems when applying these methods, specifically due to several interconnections between the criteria, which can be critical in actual practice with an essential interpretation of experts' opinions about the rated object, for instance, the skills in teaching martial arts. Other approaches can also not acknowledge enhanced fuzzy environments, such as C-PFS, even in assessing membership, non-membership, or abstinence. Furthermore, there is no strong mathematical basis for efficient AOs that can address the dynamic characteristics of a decision-making environment well, where decisions may be based on uncertain, interdependent, or ambiguous data.

The motivation for this work is to develop improved AOs that neutralize the effects of conventional fuzzy approaches. Specifically, the research is driven by the following motivations:

- It is crucial to design AOs considering uncertainty, absence, and intricate interconnections between decision criteria for subjective evaluation, such as martial arts teaching assessments.
- Several studies have been done to develop the new fuzzy models, but they do not possess high elasticity on abstinence degrees and multiple decision-making levels. C-PFDWA and C-PFDWG operators present a solution to this limitation by enabling additional flexibility and operations on decision parameters.
- This paper's examples, like scoring martial arts instructors, highlight that decision-makers look for a mechanism that can harness collective judgment and that tends to be complex and vague.
- Since the proposed operators make it possible to produce a more accurate and complete evaluation of teaching abilities more comprehensively, such innovative approaches

contributed to the motivation for creating new methods. By including new AOs that deal with abstinence degrees and using complex fuzzy logic, the present paper extends the advancement of MADM plans for solving real-world problems by enhancing flexibility and applicability.

1.2 | Objectives and Contributions

The objectives of this study are summarized as follows:

- This paper's objective is to advance and suggest the C-PFDWA and C-PFDWG operators to improve decision-making under conditions of complexity and ambiguity.
- To cover the uncertainty and subjectivity of MADM by presenting more practical cases, such as comparing the teaching skills of martial arts.
- Use these novelties to evaluate martial arts trainers in a specific case, demonstrating the operators' flexibility to deal with subjective estimates and different criteria.
- Suggest a more sophisticated approach, which can become more expansive according to different degrees of membership, non-membership, and abstinence.

The contributions of this study are outlined below:

- In this paper, C-PFDWA and C-PFDWG, two new AOs, are proposed to overcome the shortcomings of standard AOs using abstinence degrees and complex fuzzy logic.
- The proposed operators eliminate the problems with conventional operators, which cannot handle abstinence or operate properly within fuzzy spaces.
- To this end, the study demonstrates the proposed operators' application to analyze the teaching performance of four martial arts instructors regarding technique proficiency, students' attention, physical endurance, and business-like attitude. Thus, the results from the example show that the proposed operators can perform effective subjective and uncertain evaluations.
- The study expands the existing work on MADM by proposing a potentially more flexible model that can accommodate decisional contexts characterized by vagueness and imprecision.

1.3 | Organization of Study

The organization of this study is as follows: Section 2 discusses the literature review of the study. Section 3 provides the background on C-PFS and Dombi AOs. Section 4 introduces the proposed C-PFDWA and C-PFDWG operators, which manage uncertainty and abstinence in decision-making. Section 5 outlines the steps to solve the martial arts instructor assessment problem, including constructing the decision matrix, applying the operators, and ranking the instructors.

Section 6 solves the example, demonstrating the application of the proposed methods in detail. Section 7 provides a comparison with traditional methods, highlighting the advantages of the proposed approach. Section 8 concludes the study, summarizing the contributions, limitations, and future research possibilities.

2 | LITERATURE REVIEW

Subsequently, in this section, we will explore two key areas: the evaluation of martial arts teaching skills and the role of AOs in MADM.

2.1 | Martial Arts Teaching Skills in MADM

Martial Arts Teaching Skills are defined as the specifics of technical competence, subject matter content knowledge, physical fitness, and interpersonal communication skills involved in the teaching of martial arts [16]. These skills encompass the ability to show the use of the techniques as well as learning facilitation and motivational support to students, besides showing how the students can safely progress in learning one or the other martial arts forms. In the last few years, these teaching skills have been assessed and prioritized with the help of MADM techniques, which enable systematic comparison of such characteristics as the mastery of the specified technique, students' interest, or the ability to control a class. For instance, MADM methods like the TOPSIS and the MARCOS are quite helpful in martial arts because of the reasons iterated above; this is because the ranking of martial arts instructors and their training as per a given assessment often requires determination of the weight of various factors tangible and intangible both alike. Techniques like PFS show, therefore, that it is possible to gain greater flexibility to address the uncertain and vague nature of the fuzzy environment. For example, in martial arts, some of the assessments can be hermaphrodite (for example, evaluating an instructor's capability to encourage or teach), and fuzzy logic is beneficial [17]. Based on fuzzy marked data and certain fuzzy AOs such as Dombi or Bonferroni operators, the evaluation is enhanced by the fact that the multiple criteria are aggregated more realistically. These fuzzy environments have proven useful in improving decision-making models through the incorporation of human factors, particularly which is paramount when assessing a constantly evolving and highly technical profession such as martial arts instruction.

2.2 | Aggregation Operators in MADM

Aggregation Operator (AO) [18] is a mathematical function that takes one or many data or inputs to produce an output. [19] categorize AO, which is used in MADM to combine many inputs into a single and meaningful production. Two classes of operators, Ordered Weighted Averaging (OWA) [20] and Ordered Weighted Geometric (OWG) [21], rank all values in a set and provide rankings-based weights to those values. For IF data, the Weighted Averaging (WA) operator [22], and the geometric AO were introduced in [23]. [24] used Einstein operations, while [25] proposed modified interactive aggregation techniques in an environment of Intuitionistic Fuzzy Values (IFVs). [26] proposed T-norm and t-conorm operations on IFVs, and [27] developed hybrid geometric and averaging operators to solve MADM problems containing IFS data. After the launch of Hamacher AO [28], others generalized these operators; [29], [30], and [31] extended them into IFS. [32] also gave additional results for PFS. Similarly [33] introduced other operators in the context of PFS. [34] initiated Einstein operations on PFS, and [35] incorporated Hamacher operators on PFS.

3 | PRELIMINARIES

The idea of a PFS is proposed in [5]. In addition to other related ideas, this section briefly overviews the literature on current topics relevant to PFS, C-PFS, DTN, and DTCN.

Definition 1: A PFS, represented as $\mathfrak{C}$ on universal set X, is defined as

$$\mathfrak{C} = \{(x, m_{\mathfrak{C}}(x), a_{\mathfrak{C}}(x), \dot{n}_{\mathfrak{C}}(x)) : x \in X\},$$

where $m_{\mathfrak{C}}(x), \dot{n}_{\mathfrak{C}}(x), a_{\mathfrak{C}}(x) \in [0,1]$represent the membership, abstinence, and non-membership degrees, respectively. These degrees satisfy the following conditions:

$$0 \leq m_{\mathfrak{C}}(x) + a_{\mathfrak{C}}(x) + \dot{n}_{\mathfrak{C}}(x) \leq 1, \forall\, x \in X.$$

Furthermore, $r_{\mathfrak{C}}(x) \in [0,1]$ represents the refusal degree calculated as

$$r_{\mathfrak{C}}(x) = 1 - m_{\mathfrak{C}}(x) - a_{\mathfrak{C}}(x) - \dot{n}_{\mathfrak{C}}(x).$$

Definition 2: [36] Let X be a universal set, and $\mathfrak{C} \subseteq X$. A C-PFS on $\mathfrak{C}$ is defined as

$$\mathfrak{C}_a = \left\{\begin{pmatrix} x, (m_{\mathfrak{C}}(x), m'_{\mathfrak{C}}(x)), (a_{\mathfrak{C}}(x), a'_{\mathfrak{C}}(x)), \\ (\dot{n}_{\mathfrak{C}}(x), \dot{n}'_{\mathfrak{C}}(x)) : x \in X \end{pmatrix}\right\},$$

where, $(m_{\mathfrak{C}}, m'_{\mathfrak{C}}), (a_{\mathfrak{C}}, a'_{\mathfrak{C}}), (\dot{n}_{\mathfrak{C}}, \dot{n}'_{\mathfrak{C}}) \in [0,1]$ represent the membership, abstinence, and non-membership degrees of the element x in $\mathfrak{C}$, respectively. These degrees satisfy the following conditions:

$$0 \leq m_{\mathfrak{C}}(x) + a_{\mathfrak{C}}(x) + \dot{n}_{\mathfrak{C}}(x) \leq 1,$$

$$0 \leq m'_{\mathfrak{C}}(x) + a'_{\mathfrak{C}}(x) + \dot{n}'_{\mathfrak{C}}(x) \leq 1.$$

Furthermore, the $(r_{\mathfrak{C}}, r'_{\mathfrak{C}}) : X \to [0, 1]$ represents the refusal degree:

$$r_{\mathfrak{C}}(x) = 1 - m_{\mathfrak{C}}(x) - a_{\mathfrak{C}}(x) - \dot{n}_{\mathfrak{C}}(x),$$

$$r'_{\mathfrak{C}}(x) = 1 - m'_{\mathfrak{C}}(x) - a'_{\mathfrak{C}}(x) - \dot{n}'_{\mathfrak{C}}(x).$$

Definition 3: [36] The following score and accuracy functions are defined for any Complex Picture Fuzzy Values (C-PFVs) represented by $\mathfrak{C}_i = ((m_{\mathfrak{C}_i}, m'_{\mathfrak{C}_i}), (a_{\mathfrak{C}_i}, a'_{\mathfrak{C}_i}), (\dot{n}_{\mathfrak{C}_i}, \dot{n}'_{\mathfrak{C}_i}))$:

$$Ƶ(\mathfrak{C}_i) = \left(\frac{1}{2}\right) \times \begin{pmatrix} (m_{\mathfrak{C}_i} + m'_{\mathfrak{C}_i}) \\ -(a_{\mathfrak{C}_i} + a'_{\mathfrak{C}_i}) - (\dot{n}_{\mathfrak{C}_i} + \dot{n}'_{\mathfrak{C}_i}) \end{pmatrix}, \quad (1)$$

where $Ƶ(\mathfrak{C}_i) \in [-1,1]$,

$$Ⱥ(\mathfrak{C}_i) = \left(\frac{1}{2}\right) \times \begin{pmatrix} (m_{\mathfrak{C}_i} + m'_{\mathfrak{C}_i}) \\ +(a_{\mathfrak{C}_i} + a'_{\mathfrak{C}_i}) + (\dot{n}_{\mathfrak{C}_i} + \dot{n}'_{\mathfrak{C}_i}) \end{pmatrix}, \quad (2)$$

where $Ⱥ(\mathfrak{C}_i) \in [-1,1]$.

Definition 4: [36] Let $\mathfrak{C}_i = \left\{\left(x, ṃ_{\mathfrak{C}_i}(x), ạ_{\mathfrak{C}_i}(x), ṅ_{\mathfrak{C}_i}(x)\right) : x \in X\right\}$ and $\mathfrak{C}_j = \left\{\left(x, ṃ_{\mathfrak{C}_j}(x), ạ_{\mathfrak{C}_j}(x), ṅ_{\mathfrak{C}_j}(x)\right) : x \in X\right\}$ be any two C-PFVs. Then

$$\mathfrak{C}_i \cup \mathfrak{C}_j = \left\{\begin{matrix} x, \begin{pmatrix} \max\left(ṃ_{\mathfrak{C}_i}(x), ṃ_{\mathfrak{C}_j}(x)\right), \\ \max\left(ṃ'_{\mathfrak{C}_i}(x), ṃ'_{\mathfrak{C}_j}(x)\right) \end{pmatrix}, \\ \begin{pmatrix} \min\left(ạ_{\mathfrak{C}_i}(x), ạ_{\mathfrak{C}_j}(x)\right), \\ \min\left(ạ'_{\mathfrak{C}_i}(x), ạ'_{\mathfrak{C}_j}(x)\right) \end{pmatrix}, \\ \begin{pmatrix} \min\left(ṅ_{\mathfrak{C}_i}(x), ṅ_{\mathfrak{C}_j}(x)\right), \\ \min\left(ṅ'_{\mathfrak{C}_i}(x), ṅ'_{\mathfrak{C}_j}(x)\right) \end{pmatrix} \end{matrix}\right\}.$$

$$\mathfrak{C}_i \cap \mathfrak{C}_j = \left\{\begin{matrix} x, \begin{pmatrix} \min\left(ṃ_{\mathfrak{C}_i}(x), ṃ_{\mathfrak{C}_j}(x)\right), \\ \min\left(ṃ'_{\mathfrak{C}_i}(x), ṃ'_{\mathfrak{C}_j}(x)\right) \end{pmatrix}, \\ \begin{pmatrix} \min\left(ạ_{\mathfrak{C}_i}(x), ạ_{\mathfrak{C}_j}(x)\right), \\ \min\left(ạ'_{\mathfrak{C}_i}(x), ạ'_{\mathfrak{C}_j}(x)\right) \end{pmatrix}, \\ \begin{pmatrix} \max\left(ṅ_{\mathfrak{C}_i}(x), ṅ_{\mathfrak{C}_j}(x)\right), \\ \max\left(ṅ'_{\mathfrak{C}_i}(x), ṅ'_{\mathfrak{C}_j}(x)\right) \end{pmatrix} \end{matrix}\right\}.$$

$$\mathfrak{C}_i^c = \{(ṅ_{\mathfrak{C}}, ṅ'_{\mathfrak{C}}), (ạ_{\mathfrak{C}}, ạ'_{\mathfrak{C}}), (ṃ_{\mathfrak{C}}, ṃ'_{\mathfrak{C}}) : x \in X\}.$$

Definition 5: [37] Consider two real numbers ş and ţ, then DTN and DTCN are defined as follows

$$DTN(ş, ţ) = \frac{1}{1 + \left\{\left(\frac{1-ş}{ş}\right)^{n} + \left(\frac{1-ţ}{ţ}\right)^{n}\right\}^{\frac{1}{n}}}. \tag{3}$$

$$DTCN'(ş, ţ) = 1 - \frac{1}{1 + \left\{\left(\frac{ş}{1-ş}\right)^{n} + \left(\frac{ţ}{1-ţ}\right)^{n}\right\}^{\frac{1}{n}}}. \tag{4}$$

Here, $n \geq 1$ and $(ş, ţ) \in [0,1] \times [0,1]$.

4 | COMPLEX PICTURE FUZZY DOMBI AGGREGATION OPERATOR

We will explain Dombi operations in the context of C-PFVs, specifically concentrating on DTN and DTCN. Additionally, proposed AOs such as C-PFDWG and C-PFDWA.

Definition 6: Consider two C-PFVs denoted as

$$\mathfrak{C}_i = \left(\left(ṃ_{\mathfrak{C}_i}, ṃ'_{\mathfrak{C}_i}\right), \left(ạ_{\mathfrak{C}_i}, ạ'_{\mathfrak{C}_i}\right), \left(ṅ_{\mathfrak{C}_i}, ṅ'_{\mathfrak{C}_i}\right)\right),$$

$$\mathfrak{C}_j = \left(\left(ṃ_{\mathfrak{C}_j}, ṃ'_{\mathfrak{C}_j}\right), \left(ạ_{\mathfrak{C}_j}, ạ'_{\mathfrak{C}_j}\right), \left(ṅ_{\mathfrak{C}_j}, ṅ'_{\mathfrak{C}_j}\right)\right),$$

where $n \geq \mathbf{1}$, $\gamma > \mathbf{0}$.

The Complex-Dombi T-Conorm (C-DTCN) and Complex-Dombi T-Norm (C-DTN) operations for C-PFVs are defined as follows.

$$\mathfrak{C}_i\oplus\mathfrak{C}_j=\left(\begin{matrix}\begin{pmatrix}1-\dfrac{1}{1+\left\{\left(\dfrac{\underset{.}{m}_{\mathfrak{C}_i}}{1-\underset{.}{m}_{\mathfrak{C}_i}}\right)^{\eta}+\left(\dfrac{\underset{.}{m}_{\mathfrak{C}_j}}{1-\underset{.}{m}_{\mathfrak{C}_j}}\right)^{\eta}\right\}^{\frac{1}{\eta}}},\\ 1-\dfrac{1}{1+\left\{\left(\dfrac{\underset{.}{m}'_{\mathfrak{C}_i}}{1-\underset{.}{m}'_{\mathfrak{C}_i}}\right)^{\eta}+\left(\dfrac{\underset{.}{m}'_{\mathfrak{C}_j}}{1-\underset{.}{m}'_{\mathfrak{C}_j}}\right)^{\eta}\right\}^{\frac{1}{\eta}}}\end{pmatrix},\begin{pmatrix}1-\dfrac{1}{1+\left\{\left(\dfrac{\underset{.}{a}_{\mathfrak{C}_i}}{1-\underset{.}{a}_{\mathfrak{C}_i}}\right)^{\eta}+\left(\dfrac{\underset{.}{a}_{\mathfrak{C}_j}}{1-\underset{.}{a}_{\mathfrak{C}_j}}\right)^{\eta}\right\}^{\frac{1}{\eta}}},\\ 1-\dfrac{1}{1+\left\{\left(\dfrac{\underset{.}{a}_{\mathfrak{C}_i}}{1-\underset{.}{a}_{\mathfrak{C}_i}}\right)^{\eta}+\left(\dfrac{\underset{.}{a}_{\mathfrak{C}_j}}{1-\underset{.}{a}_{\mathfrak{C}_j}}\right)^{\eta}\right\}^{\frac{1}{\eta}}}\end{pmatrix},\\ \begin{pmatrix}\dfrac{1}{1+\left\{\left(\dfrac{1-\dot{n}_{\mathfrak{C}_i}}{\dot{n}_{\mathfrak{C}_i}}\right)^{\eta}+\left(\dfrac{1-\dot{n}_{\mathfrak{C}_j}}{\dot{n}_{\mathfrak{C}_j}}\right)^{\eta}\right\}^{\frac{1}{\eta}}},\\ \dfrac{1}{1+\left\{\left(\dfrac{1-\dot{n}_{\mathfrak{C}_i}}{\dot{n}_{\mathfrak{C}_i}}\right)^{\eta}+\left(\dfrac{1-\dot{n}_{\mathfrak{C}_j}}{\dot{n}_{\mathfrak{C}_j}}\right)^{\eta}\right\}^{\frac{1}{\eta}}}\end{pmatrix}\end{matrix}\right)$$

$$\mathfrak{C}_i\otimes\mathfrak{C}_j=\left(\begin{matrix}\begin{pmatrix}\dfrac{1}{1+\left\{\left(\dfrac{1-\left(\underset{.}{m}_{\mathfrak{C}_i}\right)}{\left(\underset{.}{m}_{\mathfrak{C}_i}\right)}\right)^{\eta}+\left(\dfrac{1-\left(\underset{.}{m}_{\mathfrak{C}_j}\right)}{\left(\underset{.}{m}_{\mathfrak{C}_j}\right)}\right)^{\eta}\right\}^{\frac{1}{\eta}}},\\ \dfrac{1}{1+\left\{\left(\dfrac{1-\left(\underset{.}{m}'_{\mathfrak{C}_i}\right)}{\left(\underset{.}{m}'_{\mathfrak{C}_i}\right)}\right)^{\eta}+\left(\dfrac{1-\left(\underset{.}{m}'_{\mathfrak{C}_j}\right)}{\left(\underset{.}{m}'_{\mathfrak{C}_j}\right)}\right)^{\eta}\right\}^{\frac{1}{\eta}}}\end{pmatrix},\begin{pmatrix}\dfrac{1}{1+\left\{\left(\dfrac{1-\left(\underset{.}{a}_{\mathfrak{C}_i}\right)}{\left(\underset{.}{a}_{\mathfrak{C}_i}\right)}\right)^{\eta}+\left(\dfrac{1-\left(\underset{.}{a}_{\mathfrak{C}_j}\right)}{\left(\underset{.}{a}_{\mathfrak{C}_j}\right)}\right)^{\eta}\right\}^{\frac{1}{\eta}}},\\ \dfrac{1}{1+\left\{\left(\dfrac{1-\left(\underset{.}{a}'_{\mathfrak{C}_i}\right)}{\left(\underset{.}{a}'_{\mathfrak{C}_i}\right)}\right)^{\eta}+\left(\dfrac{1-\left(\underset{.}{a}'_{\mathfrak{C}_j}\right)}{\left(\underset{.}{a}'_{\mathfrak{C}_j}\right)}\right)^{\eta}\right\}^{\frac{1}{\eta}}}\end{pmatrix},\\ \begin{pmatrix}1-\dfrac{1}{1+\left\{\left(\dfrac{1-\left(\dot{n}_{\mathfrak{C}_i}\right)}{\left(\dot{n}_{\mathfrak{C}_i}\right)}\right)^{\eta}+\left(\dfrac{1-\left(\dot{n}_{\mathfrak{C}_j}\right)}{\left(\dot{n}_{\mathfrak{C}_j}\right)}\right)^{\eta}\right\}^{\frac{1}{\eta}}},\\ 1-\dfrac{1}{1+\left\{\left(\dfrac{1-\left(\dot{n}'_{\mathfrak{C}_i}\right)}{\left(\dot{n}'_{\mathfrak{C}_i}\right)}\right)^{\eta}+\left(\dfrac{1-\left(\dot{n}'_{\mathfrak{C}_j}\right)}{\left(\dot{n}'_{\mathfrak{C}_j}\right)}\right)^{\eta}\right\}^{\frac{1}{\eta}}}\end{pmatrix}\end{matrix}\right)$$

$$\gamma\mathfrak{C}_i=\left(\begin{pmatrix}1-\dfrac{1}{1+\left\{\gamma\left(\dfrac{\left(\underset{.}{m}_{\mathfrak{C}_i}\right)}{1-\left(\underset{.}{m}_{\mathfrak{C}_i}\right)}\right)^{\eta}\right\}^{\frac{1}{\eta}}},\\ 1-\dfrac{1}{1+\left\{\gamma\left(\dfrac{\left(\underset{.}{m}'_{\mathfrak{C}_i}\right)}{1-\left(\underset{.}{m}'_{\mathfrak{C}_i}\right)}\right)^{\eta}\right\}^{\frac{1}{\eta}}}\end{pmatrix},\begin{pmatrix}1-\dfrac{1}{1+\left\{\gamma\left(\dfrac{\left(\underset{.}{a}_{\mathfrak{C}_i}\right)}{1-\left(\underset{.}{a}_{\mathfrak{C}_i}\right)}\right)^{\eta}\right\}^{\frac{1}{\eta}}},\\ 1-\dfrac{1}{1+\left\{\gamma\left(\dfrac{\left(\underset{.}{a}'_{\mathfrak{C}_i}\right)}{1-\left(\underset{.}{a}'_{\mathfrak{C}_i}\right)}\right)^{\eta}\right\}^{\frac{1}{\eta}}}\end{pmatrix},\begin{pmatrix}\dfrac{1}{1+\left\{\gamma\left(\dfrac{1-\left(\dot{n}_{\mathfrak{C}_i}\right)}{\left(\dot{n}_{\mathfrak{C}_i}\right)}\right)^{\eta}\right\}^{\frac{1}{\eta}}},\\ \dfrac{1}{1+\left\{\gamma\left(\dfrac{1-\left(\dot{n}'_{\mathfrak{C}_i}\right)}{\left(\dot{n}'_{\mathfrak{C}_i}\right)}\right)^{\eta}\right\}^{\frac{1}{\eta}}}\end{pmatrix}\right)$$

$$\mathfrak{C}_i^{\mathrm{Y}} = \left(\left(\frac{1}{1+\left\{ \mathrm{Y}\left(\frac{1-(\mathrm{m}_{\mathfrak{C}_i})}{(\mathrm{m}_{\mathfrak{C}_i})} \right)^{\mathrm{n}} \right\}^{\frac{1}{\mathrm{n}}}}, \frac{1}{1+\left\{ \mathrm{Y}\left(\frac{1-(\mathrm{m}'_{\mathfrak{C}_i})}{(\mathrm{m}'_{\mathfrak{C}_i})} \right)^{\mathrm{n}} \right\}^{\frac{1}{\mathrm{n}}}} \right), \left(\frac{1}{1+\left\{ \mathrm{Y}\left(\frac{1-(\mathrm{a}_{\mathfrak{C}_i})}{(\mathrm{a}_{\mathfrak{C}_i})} \right)^{\mathrm{n}} \right\}^{\frac{1}{\mathrm{n}}}}, \frac{1}{1+\left\{ \mathrm{Y}\left(\frac{1-(\mathrm{a}'_{\mathfrak{C}_i})}{(\mathrm{a}'_{\mathfrak{C}_i})} \right)^{\mathrm{n}} \right\}^{\frac{1}{\mathrm{n}}}} \right), \left(\frac{1}{1+\left\{ \mathrm{Y}\left(\frac{\dot{\mathrm{n}}_{\mathfrak{C}_i}}{1-\dot{\mathrm{n}}_{\mathfrak{C}_i}} \right)^{\mathrm{n}} \right\}^{\frac{1}{\mathrm{n}}}}, \frac{1}{1+\left\{ \mathrm{Y}\left(\frac{\dot{\mathrm{n}}'_{\mathfrak{C}_i}}{1-\dot{\mathrm{n}}'_{\mathfrak{C}_i}} \right)^{\mathrm{n}} \right\}^{\frac{1}{\mathrm{n}}}} \right) \right)$$

Theorem 1: The following relationship can be established for any two C-PFVs defined as $\mathfrak{C}_i = \left((\mathrm{m}_{\mathfrak{C}_i}, \mathrm{m}'_{\mathfrak{C}_i}), (\mathrm{a}_{\mathfrak{C}_i}, \mathrm{a}'_{\mathfrak{C}_i}), (\dot{\mathrm{n}}_{\mathfrak{C}_i}, \dot{\mathrm{n}}'_{\mathfrak{C}_i})\right)$ and $\mathfrak{C}_j = \left((\mathrm{m}_{\mathfrak{C}_j}, \mathrm{m}'_{\mathfrak{C}_j}), (\mathrm{a}_{\mathfrak{C}_j}, \mathrm{a}'_{\mathfrak{C}_j}), (\dot{\mathrm{n}}_{\mathfrak{C}_j}, \dot{\mathrm{n}}'_{\mathfrak{C}_j})\right)$, with $\mathrm{Y}, \mathrm{Y}_1, \mathrm{Y}_2$ any three positive real numbers:

- $\mathfrak{C}_i \oplus \mathfrak{C}_j = \mathfrak{C}_j \oplus \mathfrak{C}_i$
- $\mathfrak{C}_i \otimes \mathfrak{C}_j = \mathfrak{C}_j \otimes \mathfrak{C}_i$
- $\mathrm{Y}(\mathfrak{C}_i \oplus \mathfrak{C}_j) = \mathrm{Y}\mathfrak{C}_i \oplus \mathrm{Y}\mathfrak{C}_j$
- $(\mathrm{Y}_1 + \mathrm{Y}_2)\mathfrak{C}_i = (\mathrm{Y}_1\mathfrak{C}_i \oplus \mathrm{Y}_2\mathfrak{C}_i)$
- $(\mathfrak{C}_i \oplus \mathfrak{C}_j)^{\mathrm{Y}} = \mathfrak{C}_i^{\mathrm{Y}} \otimes \mathfrak{C}_j^{\mathrm{Y}}$
- $\mathfrak{C}_1^{\mathrm{Y}} \otimes \mathfrak{C}_2^{\mathrm{Y}} = \mathfrak{C}^{\mathrm{Y}_1+\mathrm{Y}_2}$

Proof: Straightforward.

4.1 | Complex-Picture Fuzzy DOMBI Weighted Averaging (C-PFDWA) Operator

Introducing the C-PFDWA operator aims to boost decision-making in scenarios where data is complicated and unclear.

Definition 7: Take into consideration the collection of C-PFVs shown as $\mathfrak{C}_{\mathfrak{z}} = \left((\mathrm{m}_{\mathfrak{z}}, \mathrm{m}'_{\mathfrak{z}}), (\mathrm{a}_{\mathfrak{z}}, \mathrm{a}'_{\mathfrak{z}}), (\dot{\mathrm{n}}_{\mathfrak{z}}, \dot{\mathrm{n}}'_{\mathfrak{z}})\right)$, where ($\mathfrak{z} = 1,2, \dots \dots . . \mathfrak{y}$).

The C-PFDWA operator is a function from $\mathfrak{C}_i^{\mathfrak{y}} \rightarrow \mathfrak{C}_i$ defined as $C - PFDWA(\mathfrak{C}_1, \mathfrak{C}_2, \dots, \mathfrak{C}_{\mathfrak{y}}) = \oplus\, \vartheta_{\mathfrak{z}}\mathfrak{C}_{i_{\mathfrak{z}}}$

$$
=\left(\begin{array}{c}
\left(\begin{array}{c}1-\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\left(\frac{\left(\text{ṃ}_{\mathfrak{C}_{i_{\mathfrak{z}}}}\right)}{1-\left(\text{ṃ}_{\mathfrak{C}_{i_{\mathfrak{z}}}}\right)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}},\\ 1-\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\left(\frac{\left(\text{ṃ}'_{\mathfrak{C}_{i_{\mathfrak{z}}}}\right)}{1-\left(\text{ṃ}'_{\mathfrak{C}_{i_{\mathfrak{z}}}}\right)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}}\end{array}\right),
\left(\begin{array}{c}1-\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\left(\frac{\left(\text{ạ}_{\mathfrak{C}_{i_{\mathfrak{z}}}}\right)}{1-\left(\text{ạ}_{\mathfrak{C}_{i_{\mathfrak{z}}}}\right)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}},\\ 1-\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\left(\frac{\left(\text{ạ}'_{\mathfrak{C}_{i_{\mathfrak{z}}}}\right)}{1-\left(\text{ạ}'_{\mathfrak{C}_{i_{\mathfrak{z}}}}\right)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}}\end{array}\right),\\
\left(\begin{array}{c}\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\left(\frac{1-\left(\text{ṅ}_{\mathfrak{C}_{i_{\mathfrak{z}}}}\right)}{\left(\text{ṅ}_{\mathfrak{C}_{i_{\mathfrak{z}}}}\right)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}},\\ \dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\left(\frac{1-\left(\text{ṅ}'_{\mathfrak{C}_{i_{\mathfrak{z}}}}\right)}{\left(\text{ṅ}'_{\mathfrak{C}_{i_{\mathfrak{z}}}}\right)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}}\end{array}\right)
\end{array}\right). \tag{5}
$$

Theorem 2: (**Idempotency property**) Take into consideration the collection of C-PFVs shown as

$$\mathfrak{C}_{\mathfrak{z}}=\left((\text{ṃ}_{\mathfrak{z}},\text{ṃ}'_{\mathfrak{z}}),(\text{ạ}_{\mathfrak{z}},\text{ạ}'_{\mathfrak{z}}),(\text{ṅ}_{\mathfrak{z}},\text{ṅ}'_{\mathfrak{z}})\right),\text{ where }(\mathfrak{z}=1,2,\ldots\ldots..\mathfrak{y}).$$

Suppose they all are identical i.e., $\mathfrak{C}_{i_{\mathfrak{z}}}=\mathfrak{C}_i,\forall\,\mathfrak{z}$, where $\mathfrak{C}_i=\left((\text{ṃ},\text{ṃ}'),(\text{ạ},\text{ạ}'),(\text{ṅ},\text{ṅ}')\right)$.

Consequently, $C-PFDWA(\mathfrak{C}_1,\mathfrak{C}_2,\ldots,\mathfrak{C}_{\mathfrak{y}})=\mathfrak{C}_i$

Proof: There is

$$
C-PFDWA(\mathfrak{C}_1,\mathfrak{C}_2,\ldots,\mathfrak{C}_{\mathfrak{y}})=\left(\begin{array}{c}
\left(\begin{array}{c}1-\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\left(\frac{(\text{ṃ}_{\mathfrak{z}})}{1-(\text{ṃ}_{\mathfrak{z}})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}},\\ 1-\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\left(\frac{(\text{ṃ}'_{\mathfrak{z}})}{1-(\text{ṃ}'_{\mathfrak{z}})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}}\end{array}\right),\\
\left(\begin{array}{c}1-\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\left(\frac{(\text{ạ}_{\mathfrak{z}})}{1-(\text{ạ}_{\mathfrak{z}})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}},\\ 1-\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\left(\frac{(\text{ạ}'_{\mathfrak{z}})}{1-(\text{ạ}'_{\mathfrak{z}})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}}\end{array}\right),\\
\left(\begin{array}{c}\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\left(\frac{1-(\text{ṅ}_{\mathfrak{z}})}{(\text{ṅ}_{\mathfrak{z}})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}},\\ \dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\left(\frac{1-(\text{ṅ}'_{\mathfrak{z}})}{(\text{ṅ}'_{\mathfrak{z}})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}}\end{array}\right)
\end{array}\right).
$$

$$
=\left(\left(\begin{array}{c}1-\dfrac{1}{1+\left(\frac{(\text{ṃ})}{1-(\text{ṃ})}\right)\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\right\}^{\frac{1}{\mathfrak{n}}}},\\ 1-\dfrac{1}{1+\left(\frac{(\text{ṃ}')}{1-(\text{ṃ}')}\right)\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\right\}^{\frac{1}{\mathfrak{n}}}}\end{array}\right),\left(\begin{array}{c}1-\dfrac{1}{1+\left(\frac{(\text{ạ})}{1-(\text{ạ})}\right)\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\right\}^{\frac{1}{\mathfrak{n}}}},\\ 1-\dfrac{1}{1+\left(\frac{(\text{ạ}')}{1-(\text{ạ}')}\right)\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\right\}^{\frac{1}{\mathfrak{n}}}}\end{array}\right),\left(\begin{array}{c}\dfrac{1}{1+\left(\frac{1-(\text{ṅ})}{(\text{ṅ})}\right)\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\right\}^{\frac{1}{\mathfrak{n}}}},\\ \dfrac{1}{1+\left(\frac{1-(\text{ṅ}')}{(\text{ṅ}')}\right)\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}}\vartheta_{\mathfrak{z}}\right\}^{\frac{1}{\mathfrak{n}}}}\end{array}\right)\right).
$$

$$
=\left(\left(1-\frac{1}{1+\left(\frac{(\text{ṃ})}{1-(\text{ṃ})}\right)},1-\frac{1}{1+\left(\frac{(\text{ṃ}')}{1-(\text{ṃ}')}\right)}\right),\left(1-\frac{1}{1+\left(\frac{(\text{ạ})}{1-(\text{ạ})}\right)},1-\frac{1}{1+\left(\frac{(\text{ạ}')}{1-(\text{ạ}')}\right)}\right),\left(\frac{1}{1+\left(\frac{1-(\text{ṅ})}{(\text{ṅ})}\right)},\frac{1}{1+\left(\frac{1-(\text{ṅ}')}{(\text{ṅ}')}\right)}\right)\right).
$$

$$\left((\text{ṃ},\text{ṃ}'),(\text{ạ},\text{ạ}'),(\text{ṅ},\text{ṅ}')\right)=\mathfrak{C}_i$$

Theorem 3: (**Boundedness property**) Take into consideration the collection of C-PFVs shown as

$$\mathfrak{C}_{\mathfrak{z}} = \left((\d{m}_{\mathfrak{z}}, \d{m}'_{\mathfrak{z}}), (\d{a}_{\mathfrak{z}}, \d{a}'_{\mathfrak{z}}), (\dot{n}_{\mathfrak{z}}, \dot{n}'_{\mathfrak{z}})\right), \text{ where } (\mathfrak{z} = 1,2, \dots, \mathfrak{y}).$$

$$\text{Let } \mathfrak{C}^- = min\{\mathfrak{C}_1, \mathfrak{C}_2, \dots, \mathfrak{C}_{\mathfrak{y}}\} \text{ and } \mathfrak{C}^+ = max\{\mathfrak{C}_1, \mathfrak{C}_2, \dots, \mathfrak{C}_{\mathfrak{y}}\}.$$

Then

$$\mathfrak{C}^- \leq C - PFDWA(\mathfrak{C}_1, \mathfrak{C}_2, \dots, \mathfrak{C}_{\mathfrak{y}}) \leq \mathfrak{C}^+.$$

Proof: Take the collection of C-PFVs as

$$\mathfrak{C}_{\mathfrak{z}} = \left((\d{m}_{\mathfrak{z}}, \d{m}'_{\mathfrak{z}}), (\d{a}_{\mathfrak{z}}, \d{a}'_{\mathfrak{z}}), (\dot{n}_{\mathfrak{z}}, \dot{n}'_{\mathfrak{z}})\right) \text{ for } (\mathfrak{z} = 1,2, \dots \mathfrak{y}).$$

$$\text{Define } \mathfrak{C}^- = min\{\mathfrak{C}_1, \mathfrak{C}_2, \dots, \mathfrak{C}_{\mathfrak{y}}\} = \left((\d{m}^-, \d{m}'^-), (\d{a}^-, \d{a}'^-)\right),$$

and

$$\mathfrak{C}^+ = max\{\mathfrak{C}_1, \mathfrak{C}_2, \dots, \mathfrak{C}_{\mathfrak{y}}\} = \left((\d{m}^+, \d{m}'^+), (\d{a}^+, \d{a}'^+)\right),$$

where

$(\d{m}^-, \d{m}'^-) = min_{\not{f}}\{\d{m}_{\not{f}}, \d{m}'_{\not{f}}\}, (\d{a}^-, \d{a}'^-) = max_{\not{f}}\{\d{a}_{\not{f}}, \d{a}'_{\not{f}}\}, (\d{m}^+, \d{m}'^+) = max_{\not{f}}\{\d{m}_{\not{f}}, \d{m}'_{\not{f}}\}$ and $(\d{a}^+, \d{a}'^+) = min_{\not{f}}\{\d{a}_{\not{f}}, \d{a}'_{\not{f}}\}, (\dot{n}^-, \dot{n}'^-) = min_{\not{f}}\{\d{m}_{\not{f}}, \d{m}'_{\not{f}}\}, (\dot{n}^+, \dot{n}'^+) = max_{\not{f}}\{\d{m}_{\not{f}}, \d{m}'_{\not{f}}\}$.

$$1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\d{m}^-)}{1-(\d{m}^-)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \leq 1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\d{m}_{\mathfrak{z}})}{1-(\d{m}_{\mathfrak{z}})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \leq 1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{i}\left(\frac{(\d{m}^+)}{1-(\d{m}^+)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}},$$

$$1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\d{m}'^-)}{1-(\d{m}'^-)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \leq 1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\d{m}'_{\mathfrak{z}})}{1-(\d{m}'_{\mathfrak{z}})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \leq 1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\d{m}'^+)}{1-(\d{m}'^+)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}},$$

$$1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\d{a}^-)}{1-(\d{a}^-)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \leq 1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\d{a}_{\mathfrak{z}})}{1-(\d{a}_{\mathfrak{z}})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \leq 1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\d{a}^+)}{1-(\d{a}^+)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}},$$

$$1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\d{a}'^-)}{1-(\d{a}'^-)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \leq 1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\d{a}'_{\mathfrak{z}})}{1-(\d{a}'_{\mathfrak{z}})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \leq 1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\d{a}'^+)}{1-(\d{a}'^+)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}},$$

$$1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\dot{n}^+)}{1-(\dot{n}^+)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \leq 1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\dot{n}_{\mathfrak{z}})}{1-(\dot{n}_{\mathfrak{z}})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \leq 1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\dot{n}^-)}{1-(\dot{n}^-)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}},$$

$$1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\dot{n}'^+)}{1-(\dot{n}'^+)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \leq 1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\dot{n}'_{\mathfrak{z}})}{1-(\dot{n}'_{\mathfrak{z}})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \leq 1 - \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\frac{(\dot{n}'^-)}{1-(\dot{n}'^-)}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}},$$

$\mathfrak{C}^- \leq C - PFDWA(\mathfrak{C}_1, \mathfrak{C}_2, \dots, \mathfrak{C}_{\mathfrak{y}}) \leq \mathfrak{C}^+$.

Theorem 4: (**Monotonicity property**) Let us assume that for all $\mathfrak{z}$ the sets of C-PFVs represented as $\mathfrak{C}_{i_{\mathfrak{z}}}$ and $\mathfrak{C}^*_{i_{\mathfrak{z}}} (\mathfrak{z} = 1,2, \dots, \mathfrak{y})$ such that $\mathfrak{C}_{i_{\mathfrak{z}}} \leq \mathfrak{C}^*_{i_{\mathfrak{z}}}$. Then

$$C - PFDWA(\mathfrak{C}_1, \mathfrak{C}_2, \dots, \mathfrak{C}_{\mathfrak{y}}) \leq C - PFDWA\left(\mathfrak{C}^*_1, \mathfrak{C}^*_2, \dots, \mathfrak{C}^*_{\mathfrak{y}}\right).$$

Proof: Straightforward.

4.2 | Complex-Picture Fuzzy DOMBI Weighted Geometric (C-PFDWG) Operator

The objective is to present the C-PFDWG operator. To ensure more accurate and dependable results in decision-making processes, the C-PFDWG operator efficiently manages and aggregates complicated and imprecise data.

Definition 8: Take into consideration the collection of C-PFVs shown as

$$\mathfrak{C}_{\mathfrak{z}} = \left((\mathfrak{m}_{\mathfrak{z}}, \mathfrak{m}'_{\mathfrak{z}}), (\mathfrak{a}_{\mathfrak{z}}, \mathfrak{a}'_{\mathfrak{z}}), (\dot{\mathfrak{n}}_{\mathfrak{z}}, \dot{\mathfrak{n}}'_{\mathfrak{z}})\right) \text{ for } (\mathfrak{z} = 1,2, \ldots, \mathfrak{y}).$$

Then, a function from $\mathfrak{C}_{\mathfrak{i}}^{\mathfrak{y}} \longrightarrow \mathfrak{C}_{\mathfrak{i}}$ describe the C-PFDWG operator for $\mathfrak{y}$, where

$$C - PFDWG(\mathfrak{C}_1, \mathfrak{C}_2, \ldots, \mathfrak{C}_{\mathfrak{y}}) = \otimes_{\mathfrak{z}}^{\mathfrak{y}} \left(\mathfrak{C}_{(\mathfrak{z})}\right)^{\vartheta_{\mathfrak{z}}}$$

$$= \left(\begin{pmatrix} \dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}} \left(\frac{1-(\mathfrak{m}_{(\mathfrak{z})})}{(\mathfrak{m}_{(\mathfrak{z})})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}}, \\ \dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}} \left(\frac{1-(\mathfrak{m}'_{(\mathfrak{z})})}{(\mathfrak{m}'_{(\mathfrak{z})})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \end{pmatrix}, \begin{pmatrix} \dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}} \left(\frac{1-(\mathfrak{a}_{(\mathfrak{z})})}{(\mathfrak{a}_{(\mathfrak{z})})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}}, \\ \dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}} \left(\frac{1-(\mathfrak{a}'_{(\mathfrak{z})})}{(\mathfrak{a}'_{(\mathfrak{z})})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \end{pmatrix}, \begin{pmatrix} 1-\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}} \left(\frac{(\dot{\mathfrak{n}}_{(\mathfrak{z})})}{1-(\dot{\mathfrak{n}}_{(\mathfrak{z})})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}}, \\ 1-\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}} \left(\frac{(\dot{\mathfrak{n}}'_{(\mathfrak{z})})}{1-(\dot{\mathfrak{n}}'_{(\mathfrak{z})})}\right)^{\mathfrak{n}}\right\}^{\frac{1}{\mathfrak{n}}}} \end{pmatrix} \right). \quad (6)$$

Theorem 5: (**Idempotency property**) Take into consideration the collection of C-PFVs shown as $\mathfrak{C}_{\mathfrak{z}} = \left((\mathfrak{m}_{\mathfrak{z}}, \mathfrak{m}'_{\mathfrak{z}}), (\mathfrak{a}_{\mathfrak{z}}, \mathfrak{a}'_{\mathfrak{z}}), (\dot{\mathfrak{n}}_{\mathfrak{z}}, \dot{\mathfrak{n}}'_{\mathfrak{z}})\right)$ for $(\mathfrak{z} = 1,2, \ldots, \mathfrak{y})$, all of them are the same, i.e., $\mathfrak{C}_{\mathfrak{z}} = \mathfrak{C}$ for all $\mathfrak{z}$, where, $\mathfrak{C} = \left((\mathfrak{m}, \mathfrak{m}'), (\mathfrak{a}, \mathfrak{a}')\right)$. Then $C - PFDWG\left(\mathfrak{C}_1, \mathfrak{C}_2, \ldots, \mathfrak{C}_{\mathfrak{y}}\right) = \mathfrak{C}$.

Proof: Straightforward.

Theorem 6: (**Boundedness property**) Take into consideration the collection of C-PFVs shown as

$$\mathfrak{C}_{\mathfrak{z}} = \left((\mathfrak{m}_{\mathfrak{z}}, \mathfrak{m}'_{\mathfrak{z}}), (\mathfrak{a}_{\mathfrak{z}}, \mathfrak{a}'_{\mathfrak{z}}), (\dot{\mathfrak{n}}_{\mathfrak{z}}, \dot{\mathfrak{n}}'_{\mathfrak{z}})\right), \text{ for } (\mathfrak{z} = 1,2, \ldots, \mathfrak{y}).$$

Assume that $\mathfrak{C}^- = \min\{\mathfrak{C}_1, \mathfrak{C}_2, \ldots, \mathfrak{C}_{\mathfrak{y}}\}$ and $\mathfrak{C}^+ = \max\{\mathfrak{C}_1, \mathfrak{C}_2, \ldots, \mathfrak{C}_{\mathfrak{y}}\}$.

Following that: $\mathfrak{C}^- \leq C - PFDWG(\mathfrak{C}_1, \mathfrak{C}_2, \ldots, \mathfrak{C}_{\mathfrak{y}}) \leq \mathfrak{C}^+$.

Proof: Similarly, as in **Theorem 3**.

Theorem 7: (**Monotonicity property**) Let us assume that for all $\mathfrak{z}$ the sets of C-PFVs represented as $\mathfrak{C}_{\mathfrak{i}_{\mathfrak{z}}}$

and $\mathfrak{C}^*_{\mathfrak{i}_{\mathfrak{z}}}$ $(\mathfrak{z} = 1,2, \ldots, \mathfrak{y})$ such that $\mathfrak{C}_{\mathfrak{i}_{\mathfrak{z}}} \leq \mathfrak{C}^*_{\mathfrak{i}_{\mathfrak{z}}}$. Then

$$C - PFDWG(\mathfrak{C}_1, \mathfrak{C}_2, \ldots, \mathfrak{C}_{\mathfrak{y}}) \leq C - PFDWG\ (\mathfrak{C}^*_1, \mathfrak{C}^*_2, \ldots, \mathfrak{C}^*_{\mathfrak{y}}).$$

5 | APPROACH FOR SOLVING MARTIAL ARTS TEACHING SKILLS.

In the case when attribute values are represented as C-PFVs and attribute weights are real numbers, researchers present a MADM approach that employs the C-PFDWA and C-PFDWG operators. This strategy utilizes the MADM framework and C-PF data to establish an innovative method of assessing and advancing innovation in evaluation and implementation. Let $\dot{\mathrm{P}} =$

$\{\dot{\mathrm{P}}_1, \dot{\mathrm{P}}_2, \dots, \dot{\mathrm{P}}_{\mathfrak{y}}\}$ indicate the sequence of alternatives to be assessed, and $\text{Ẓ} = \{\text{Ẓ}_1, \text{Ẓ}_2, \dots, \text{Ẓ}_{n}\}$ indicate the finite collection of attributes that are chosen for a MADM problem. $\vartheta = \{\vartheta_1, \vartheta_2, \dots \dots, \vartheta_{\mathfrak{y}}\}$ is supposed to represent the weight vector of the attributes, where $\vartheta_{\hbar}(\hbar = 1,2,3, \dots \dots, \mathfrak{y})$ satisfies $\vartheta_{\hbar} > 0$ and $\sum_{\hbar=1}^{\mathfrak{y}} \vartheta_{\hbar} = 1$. Moreover, let Ẓ be the C-PFVs decision matrix, represented as

$$\text{Ẓ} = (\mathcal{L}_{ij})_{\text{Ẓ}\times\mathfrak{y}} = \left((\text{ṃ}_{ij}, \text{ṃ}'_{ij}), (\text{ạ}_{ij}, \text{ạ}'_{ij}), (\dot{n}_{ij}, \dot{n}'_{ij})\right),$$

where $\mathcal{L}_{ij}$ denotes the feasible value at which the attribute Ẓ_i meets the alternative $\dot{\mathrm{P}}_i$.

5.1 | Algorithm

In the subsequent technique, we use the C-PFDWA and C-PFDWG operators to solve a MADM problem with the C-PF dataset.

Step 1: Demonstrate a C-PF decision matrix represented by the following:

$$\text{Ẓ} = (\mathcal{L}_{ij})_{\text{Ẓ}\times\mathfrak{y}} = \left((\text{ṃ}_{ij}, \text{ṃ}'_{ij}), (\text{ạ}_{ij}, \text{ạ}'_{ij}), (\dot{n}_{ij}, \dot{n}'_{ij})\right).$$

Step 2: Apply $\eth_{\hbar}$ combined information to the alternative Ẓ_i by utilizing the equations as follows:

$$\eth_{\hbar} = C-PFDWA(\mathfrak{C}_1, \mathfrak{C}_2, \dots, \mathfrak{C}_{\mathfrak{y}}) = \begin{pmatrix} \left(1 - \dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\dfrac{(\text{ṃ}_{\hbar\mathfrak{z}})}{1-(\text{ṃ}_{\hbar\mathfrak{z}})}\right)^{n}\right\}^{\frac{1}{n}}}, 1 - \dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\dfrac{(\text{ṃ}'_{\hbar\mathfrak{z}})}{1-(\text{ṃ}'_{\hbar\mathfrak{z}})}\right)^{n}\right\}^{\frac{1}{n}}}\right), \\ \left(1 - \dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\dfrac{(\text{ạ}_{\hbar\mathfrak{z}})}{1-(\text{ạ}_{\hbar\mathfrak{z}})}\right)^{n}\right\}^{\frac{1}{n}}}, 1 - \dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\dfrac{(\text{ạ}'_{\hbar\mathfrak{z}})}{1-(\text{ạ}'_{\hbar\mathfrak{z}})}\right)^{n}\right\}^{\frac{1}{n}}}\right), \\ \left(\dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\dfrac{1-(\dot{n}_{\hbar\mathfrak{z}})}{(\dot{n}_{\hbar\mathfrak{z}})}\right)^{n}\right\}^{\frac{1}{n}}}, \dfrac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}}\left(\dfrac{1-(\dot{n}'_{\hbar\mathfrak{z}})}{(\dot{n}'_{\hbar\mathfrak{z}})}\right)^{n}\right\}^{\frac{1}{n}}}\right) \end{pmatrix} \quad (7)$$

$$\eth_{\mathfrak{f}} = C-PFDWG(\mathfrak{C}_1, \mathfrak{C}_2, \ldots, \mathfrak{C}_{\mathfrak{y}}) = \left(\begin{array}{c} \left(\frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}} \left(\frac{1-(\mathrm{m}_{\mathfrak{f}\mathfrak{z}})}{(\mathrm{m}_{\mathfrak{f}\mathfrak{z}})}\right)^{\mathrm{n}}\right\}^{\frac{1}{\mathrm{n}}}}, \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}} \left(\frac{1-(\mathrm{m}'_{\mathfrak{f}\mathfrak{z}})}{(\mathrm{m}'_{\mathfrak{f}\mathfrak{z}})}\right)^{\mathrm{n}}\right\}^{\frac{1}{\mathrm{n}}}} \right), \\ \left(\frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}} \left(\frac{1-(\mathrm{a}_{\mathfrak{f}\mathfrak{z}})}{(\mathrm{a}_{\mathfrak{f}\mathfrak{z}})}\right)^{\mathrm{n}}\right\}^{\frac{1}{\mathrm{n}}}}, \frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}} \left(\frac{1-(\mathrm{a}'_{\mathfrak{f}\mathfrak{z}})}{(\mathrm{a}'_{\mathfrak{f}\mathfrak{z}})}\right)^{\mathrm{n}}\right\}^{\frac{1}{\mathrm{n}}}} \right), \\ \left(1-\frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}} \left(\frac{(\dot{\mathrm{n}}_{\mathfrak{f}\mathfrak{z}})}{1-(\dot{\mathrm{n}}_{\mathfrak{f}\mathfrak{z}})}\right)^{\mathrm{n}}\right\}^{\frac{1}{\mathrm{n}}}}, 1-\frac{1}{1+\left\{\sum_{\mathfrak{z}=1}^{\mathfrak{y}} \vartheta_{\mathfrak{z}} \left(\frac{(\dot{\mathrm{n}}'_{\mathfrak{f}\mathfrak{z}})}{1-(\dot{\mathrm{n}}'_{\mathfrak{f}\mathfrak{z}})}\right)^{\mathrm{n}}\right\}^{\frac{1}{\mathrm{n}}}} \right) \end{array} \right). \quad (8)$$

Step 3: For each alternative $\mathfrak{Z}_i$, find the score values $\mathfrak{s}(\eth_i)$.

Step 4: Rank the alternatives.

6 | ASSESSMENT OF MATRIAL ARTS TEACHING SKILLS

Martial Arts Teaching Skills are the characteristics, performance, and approaches an instructor needs to embody to educate their students on martial arts concepts. Such skills allow students to learn the actual physical tactics of the combat sport and instill discipline, respect, and mental strength. Typically defined as effective martial arts teaching, the primary attributes are technique mastery, student engagement, physical fitness, discipline, and professionalism.

After developing an algorithm based on the proposed approach, we applied it to actual evaluations of martial arts instructors, emphasizing the crucial instructing proficiencies. In particular, the proposed approach assessed the collection of instructors (alternatives) based on a set of important skills (criteria) such as the level of student's involvement in activities, physical training, and work manners using C-PFS and the Dombi AO. This setup allows us to specify the level of proficiency in a class and helps us control the variations involved in instruction quantity and quality.

Since many of these skills involve assessments with little objective standard, fuzzy logic is useful for quantifying these evaluations smartly and correctly. The explicit consideration of uncertainty and ambiguity characterizes the use of judgment by C-PFS. In the following subsections, we will utilize the C-PFDWA and C-PFDWG AOs for illustration purposes with an example of an assessment of martial arts instructors. These operators are used to sum up evaluations of the several attributes to a single score for every instructor, considering the priorities of the attribute. Our challenge is to assess the effectiveness of four martial arts instructors in teaching skills performance. These instructors are:

- Instructor $\dot{P}_1$.
- Instructor $\dot{P}_2$.
- Instructor $\dot{P}_3$.
- Instructor $\dot{P}_4$.

Evaluation of attributes:

In the development of our proposed work for teaching skills of martial arts, we give importance to a few truly important attributes such as technique, reaction, fitness level, and discipline/ professionalism of the students. The attributes involved in the set mathematical model are liable for the formation of the set of quantitative and qualitative indicators, as well as an overall evaluation of the effectiveness of an instructor. It concerns learning achievement a great deal, as technique does when participation correlates with student performance and interest. In addition, these attributes are in line with the principal philosophy of martial arts, where one is supposed to develop technique and personality at the same time. Finally, all the attributes are well-defined and can be quantified using easily comprehensible measurements, which can conveniently be incorporated into the proposed C-PF for refining the evaluation approach. To quantify these attributes, we employ certain coefficients that indicate the level of efficiency of one or the other of them. The application of these metrics by the C-PFS in the proposed algorithm makes the processing of these metrics present a holistic and more transparent performance assessment of the teaching of martial arts skills.

- **Technique mastery Z_1:** This attribute measures how well an instructor can perform or instruct beginner's martial arts moves correctly. Understanding the technique required is very important, especially in a bid to enhance the development of skills by the students, in a safe manner.
- **Student engagement Z_2:** This criterion assesses an instructor's capacity to engage students in lessons and ensure their participation, motivation, and interest in the chosen kind of art.
- **Physical fitness Z_3:** Martial arts involve a lot of physical strength, flexibility, and sometimes a good measure of endurance. Physical fitness is compulsory for present-day instructors to become models to the learners and to demonstrate different skills.
- **Discipline and professionalism Z_4:** This attribute embraced ethical practices and professionalism that ought to go hand with martial arts training. It provides a positive model to students and encourages their respect, commitment, and businesslike attitude toward practice.

Following Figure 1 shows the detailed flowchart of the methodology.

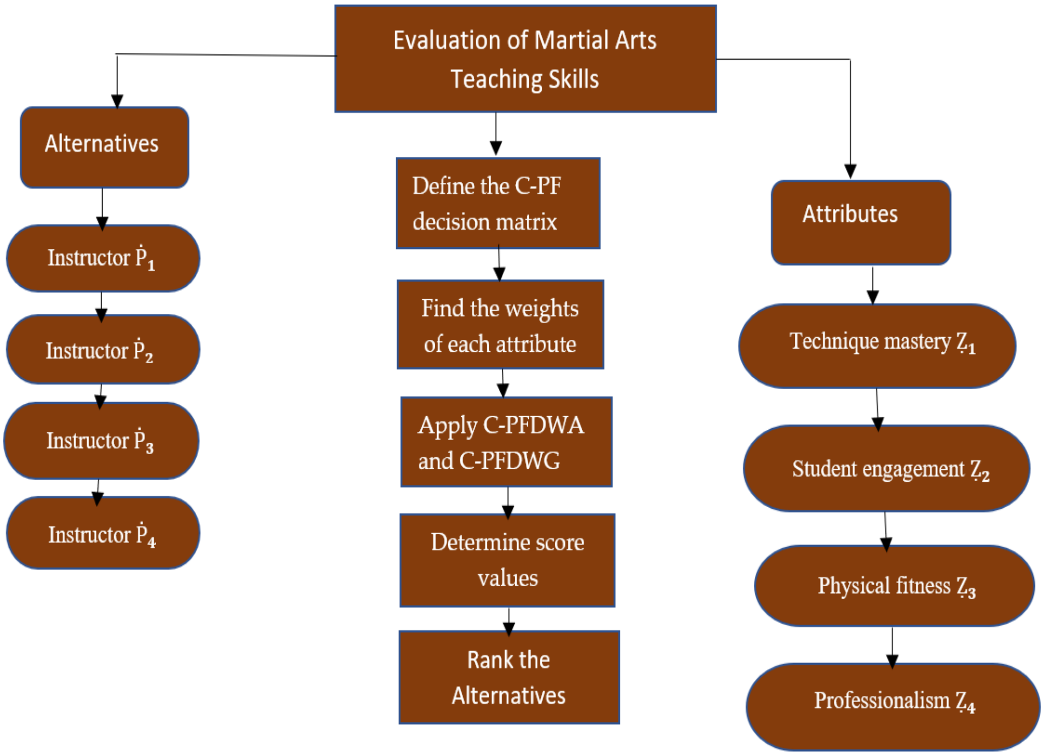

Figure 1. Flowchart of the methodology.

Step 1: Table 1 presents the C-PF decision matrix for these evaluations, showing each of the four criteria. Each instructor is evaluated according to these criteria.

Table 1. Complex- Picture Fuzzy Decision Matrix.

	$\underset{\cdot}{Z}_1$	$\underset{\cdot}{Z}_2$	$\underset{\cdot}{Z}_3$	$\underset{\cdot}{Z}_4$
$\dot{P}_1$	$\begin{pmatrix}(0.4,0.5),\\(0.4,0.1),\\(0.2,0.4)\end{pmatrix}$	$\begin{pmatrix}(0.3,0.2),\\(0.2,0.4),\\(0.2,0.4)\end{pmatrix}$	$\begin{pmatrix}(0.7,0.3),\\(0.1,0.3),\\(0.2,0.4)\end{pmatrix}$	$\begin{pmatrix}(0.3,0.1),\\(0.3,0.5),\\(0.4,0.1)\end{pmatrix}$
$\dot{P}_2$	$\begin{pmatrix}(0.3,0.3),\\(0.1,0.1),\\(0.5,0.5)\end{pmatrix}$	$\begin{pmatrix}(0.4,0.1),\\(0.5,0.1),\\(0.1,0.5)\end{pmatrix}$	$\begin{pmatrix}(0.6,0.2),\\(0.2,0.1),\\(0.2,0.5)\end{pmatrix}$	$\begin{pmatrix}(0.1,0.2),\\(0.5,0.6),\\(0.4,0.2)\end{pmatrix}$
$\dot{P}_3$	$\begin{pmatrix}(0.2,0.7),\\(0.3,0.2),\\(0.4,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.5,0.6),\\(0.1,0.2),\\(0.3,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.5,0.1),\\(0.1,0.2),\\(0.3,0.6)\end{pmatrix}$	$\begin{pmatrix}(0.1,0.1),\\(0.6,0.7),\\(0.3,0.2)\end{pmatrix}$
$\dot{P}_4$	$\begin{pmatrix}(0.1,0.2),\\(0.4,0.1),\\(0.3,0.5)\end{pmatrix}$	$\begin{pmatrix}(0.6,0.5),\\(0.1,0.4),\\(0.3,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.4,0.1),\\(0.3,0.1),\\(0.3,0.7)\end{pmatrix}$	$\begin{pmatrix}(0.2,0.1),\\(0.6,0.8),\\(0.2,0.1)\end{pmatrix}$

Step 2: This step involves assigning each criterion a weight and set $ɳ = 3$, which is then applied to the C-PFDWA and C-PFDWG aggregating operators. The results are shown in Table 2.

Table 2. Applying C-PFDWA and C-PFDWG operators.

	C-PFDWA	C-PFDWG
$\dot{P}_1$	$\begin{pmatrix}(0.598,0.394),(0.316,0.412),\\(0.215,0.149)\end{pmatrix}$	$\begin{pmatrix}(0.341,0.146),(0.146,0.149),\\(0.306,0.377)\end{pmatrix}$
$\dot{P}_2$	$\begin{pmatrix}(0.493,0.247),(0.425,0.486),\\(0.166,0.281)\end{pmatrix}$	$\begin{pmatrix}(0.149,0.160),(0.134,0.109),\\(0.430,0.476)\end{pmatrix}$
$\dot{P}_3$	$\begin{pmatrix}(0.458,0.604),(0.469,0.577),\\(0.311,0.122)\end{pmatrix}$	$\begin{pmatrix}(0.156,0.123),(0.116,0.2121),\\(0.332,0.501)\end{pmatrix}$
$\dot{P}_4$	$\begin{pmatrix}(0.506,0.403),(0.438,0.651),\\(0.277,0.131)\end{pmatrix}$	$\begin{pmatrix}(0.130,0.138),(0.142,0.116),\\(0.294,0.589)\end{pmatrix}$

Step 3: Determine the score function values for each possibility, comprised from $\dot{P}_1$ to $\dot{P}_4$, in Table 3.

Table 3. Score values.

	C-PFDWA	C-PFDWG
$\dot{P}_1$	-0.050	-0.246
$\dot{P}_2$	-0.309	-0.420
$\dot{P}_3$	-0.209	-0.441
$\dot{P}_4$	-0.294	-0.437

Step 4: Rank the alternatives as shown in Table 4.

Table 4. Ranking of alternatives.

	Ranking
C-PFDWA	$\dot{P}_1 > \dot{P}_3 > \dot{P}_4 > \dot{P}_2$
C-PFDWG	$\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$

Figure 2, shows the graphical representation of score values based on C-PFDWA and C-PFDWG AOs.

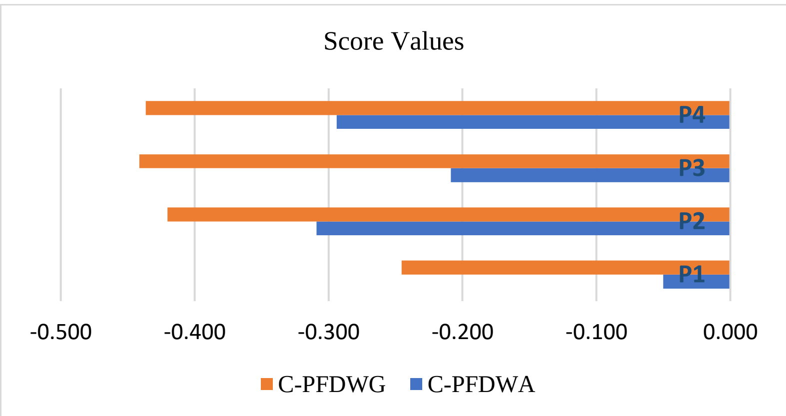

Figure 2. Graphical representation of score values.

6.1 | Result Discussion

Overall, instructor $\dot{P}_1$ has been rated the highest throughout both the aggregate methods in all four teaching criteria. The difference in the rank of instructor $\dot{P}_2$, instructor $\dot{P}_3$, and instructor $\dot{P}_4$ Shows how the C-PFDWG and C-PFDWA operator supports the input data. The C-PFDWG method seems to exaggerate lower performance more significantly, and that shows the instructor. $\dot{P}_3$ is rated lower by the C-PFDWG method than the other method C-PFDWA. In summary, these results indicate that the intelligent algorithm can be a reliable and complex means of assessing the teaching performance of martial arts by reflecting the differences between candidates to a higher degree than a simple rating system.

7 | COMPARATIVE STUDY

In the following Table 5, we contrast the new proposed C-PFDWA and C-PFDWG AOs with other literature operators. The problem of these traditional operators including Intuitionistic Fuzzy Weighted Averaging (IFWA) [38], Intuitionistic Fuzzy Weighted Geometric (IFWG) [39], Complex Intuitionistic Fuzzy Weighted Averaging (C-IFWA) [40], Complex Intuitionistic Fuzzy Weighted Geometric (C-IFWG) [41], Pythagorean Fuzzy Weighted Averaging (PyFWA) [42], Pythagorean Fuzzy Weighted Geometric (PyFWG) [42], Complex Pythagorean Fuzzy Weighted Averaging (C-PyFWA) [43], Complex Pythagorean Fuzzy Weighted Geometric (C-PyFWG) [43], q-Rung Orthopair Fuzzy Weighted Averaging (QROFWA) [44], q-Rung Orthopair Fuzzy Weighted Geometric (QROFWG) [44], Complex q-Rung Orthopair Fuzzy Weighted Averaging (C-QROFWA) [45], and Complex q-Rung Orthopair Fuzzy Weighted Geometric (C-QROFWG) [45], is that they are not capable of managing the abstinence degree, which is most important in the picture fuzzy environment. Moreover, some operators failed to work in Picture Fuzzy Weighted Averaging (PFWA) [46] and Picture Fuzzy Weighted Geometric (PFWG) [46] because their operators do not work in a complex fuzzy environment. However, the present study introduced two new operators, namely C-PFDWA and C-PFDWG, and the developed operators overcame these issues by integrating both the abstinence degree and the complex fuzzy logic. This enabled them to create precise scores, such as where $\dot{P}_1 > \dot{P}_3 > \dot{P}_4 > \dot{P}_2$ and $\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$ in the C-PFVs setting to exhibit efficiency in solving a problem where other approaches fail. This comparison shows that our proposed methods are more appropriate when handling the challenges associated with the C-PF structure.

Table 5. Comparison analysis based on different operators.

Operators	Environment	Score values
IFWA	IFVs	Failed
IFWG	IFVs	Failed
C-IFWA	C-IFVs	Failed

C-IFWG	C-IFVs	Failed
PyFWA	PyFVs	Failed
PyFWG	PyFVs	Failed
C-PyFWA	C-PyFVs	Failed
C-PyFWG	C-PyFVs	Failed
QROFWA	QROFVs	Failed
QROFWG	QROFVs	Failed
C-QROFWA	C-QROFVs	Failed
C-QROFWG	C-QROFVs	Failed
PFWA	PFVs	Failed
PFWG	PFVs	Failed
PFDWA	PFVs	Failed
PFDWG	PFVs	Failed
C-PFWA	C-PFVs	Failed
C-PFWG	C-PFVs	Failed
C-PFDWA (Proposed work)	C-PFVs	$\dot{P}_1 > \dot{P}_3 > \dot{P}_4 > \dot{P}_2$
C-PFDWG (Proposed work)	C-PFVs	$\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$

Table 6 compares different FS and Dombi-based aggregation methods, measured in membership, abstinence function, non-membership function, dependent relationship, flexibility, parameter control, sensitivity, and reversibility. IFS, PyFS, and QROFS, for example, while providing for computing membership and non-membership, do not support abstinence, flexibility, sensitivity, or reversibility. Integrated by the degree of abstinence named, PFS and C-PFS become more flexible and perhaps more easily reversible but still retain their necessary drawbacks. IFDA, PyFDA, and QROFDA, the Dombi-based aggregation methods, also exhibit better parameter control and sensitivity because the first derivative of the Dombi operator serves as the transition function. However, flexibility and reversibility still have severe limitations in these approaches. The C-IFDA, C-PyFDA, and C-QROFDA present further optimization by increasing flexibility and regulation of parameters and displaying greater sensitivity and reversibility. The proposed approach is the only method that is specific in achieving all the properties- membership, non-membership, and abstinence computation and ability, as well as relationship dependency, high flexibility parameter, strong parameters control, high sensitivity, and reversibility. This makes the proposed approach much more effective and flexible than the other methods listed in the following table.

Table 6. Comparison of fuzzy set and Dombi-based aggregation approaches based on properties.

Approach	MD	AD	NMD	Flexibility	Parameter Control	Sensitivity	Reversibility
IFS	✓	×	✓	×	×	×	×
PyFS	✓	×	✓	×	×	×	×
QROFS	✓	×	✓	×	✓	×	×
PFS	✓	✓	✓	×	×	×	×
C-PFS	✓	✓	✓	×	×	×	×
IFDA	✓	×	✓	✓	×	✓	×
PyFDA	✓	×	✓	✓	×	✓	×
QROFDA	✓	×	✓	✓	✓	✓	×
PFDA	✓	✓	✓	✓	×	✓	×
C-IFDA	✓	×	✓	✓	×	✓	×
C-PyFDA	✓	×	✓	✓	×	✓	×
C-QROFDA	✓	×	✓	✓	✓	✓	×
Proposed Approach	✓	✓	✓	✓	✓	✓	✓

The comparison in Table 7 identifies ways of assessing the skills of teachers of martial arts, though concentrating on the measurement of accuracy, flexibility, and managing measurement of risk and uncertainty. C-PDWA and C-PFDWG both show better performance in handling uncertainty because of the features of the Dombi AO used in this model, which could minimize the imprecision in aggregating PF information. C-PDWA is highly accurate and flexible thus it can be used in many different situations and C-PFDWG is again highly accurate with the geometric operator that allows for variations in the input data. Compared with other methods such as 2TLPFN-ExpTODIM-GRA and SERVQUAL, the novel methods introduced in this study possess a higher performance for coping with sophisticated linguistic vagueness which is involved frequently in the assessment of martial arts disciplines. There are some other methods like the Aikido-based framework and fuzzy logic-based approach other methods that have more efficiency in specific areas rather than having balanced flexibility and precision in comparison with the above-discussed models. This case comparison shows that not only are C-PDWA and C-PFDWG successful in delivering precise fundamental line fitting, but the implementations are highly adaptable in various instances, perfect for multi-faced evaluations of martial arts education.

Table 7. Comparison of proposed methods with existing techniques.

Method	Accuracy	Flexibility	Ability	Optimization Algorithms	Scenarios Evaluated
C-PFDWA (Proposed)	High	High	Excellent	Dombi averaging	Multiple
C-PFDWG (Proposed)	High	High	Excellent	Dombi geometric	Multiple
TODIM-GRA [47]	Moderate	Moderate	Good	TODIM-GRA	Limited
TOPSIS [48]	Moderate	High	Moderate	TOPSIS methodology	Limited
Martial Arts [49]	Low	Low	Low	Traditional evaluation	Limited
Aikido-based analysis [16]	High	Moderate	Good	Aikido-based analysis	Limited
Fuzzy Clustering [17]	High	Moderate	High	Fuzzy clustering	Multiple

7.1 | Sensitivity Analysis

In Table 8, a sensitivity analysis is conducted by varying the parameter values (ɳ) of the Dombi AO to examine its effect on the final rankings under two different approaches: C-PFDWA and C-PFDWG. An examination of the ranking of alternatives among each other is done as ɳ rises from **3** to **100**. The obtained outcome reveals that regardless of the type of the used methodology the $\dot{P}_1$ Option is always ranked as the most preferable; however, the position of other options, $\dot{P}_2$, $\dot{P}_3$, and $\dot{P}_4$, differ when applying different methods of aggregation. The sensitivity analysis shows how delicate the rankings are to shifts in the Dombi parameter, which underlines the stability of the model intact. When comparing the different rankings of the preferences of the decision-making model, it became clear that small changes to the value of the Dombi parameter resulted in significant shifts in the rankings of the alternatives which verifies the relative sensitivity of the Dombi parameter to changes in input parameters. This shows the applicability of the model and provides information on how the AO parameters affect the rankings. Also, Figure 3. shows the graphical representation of sensitivity analysis.

Table 8. Sensitive analysis.

ɳ	Ranking based on C-PFDWA	Ranking based on C-PFDWG
ɳ = 3	$\dot{P}_1 > \dot{P}_3 > \dot{P}_4 > \dot{P}_2$	$\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$
ɳ = 4	$\dot{P}_1 > \dot{P}_3 > \dot{P}_2 > \dot{P}_4$	$\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$
ɳ = 5	$\dot{P}_1 > \dot{P}_3 > \dot{P}_2 > \dot{P}_4$	$\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$
ɳ = 10	$\dot{P}_1 > \dot{P}_3 > \dot{P}_2 > \dot{P}_4$	$\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$
ɳ = 15	$\dot{P}_1 > \dot{P}_3 > \dot{P}_2 > \dot{P}_4$	$\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$

ɳ = 20	$\dot{P}_1 > \dot{P}_3 > \dot{P}_2 > \dot{P}_4$	$\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$
ɳ = 30	$\dot{P}_1 > \dot{P}_3 > \dot{P}_2 > \dot{P}_4$	$\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$
ɳ = 40	$\dot{P}_1 > \dot{P}_3 > \dot{P}_2 > \dot{P}_4$	$\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$
ɳ = 50	$\dot{P}_1 > \dot{P}_3 > \dot{P}_2 > \dot{P}_4$	$\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$
ɳ = 100	$\dot{P}_1 > \dot{P}_3 > \dot{P}_2 > \dot{P}_4$	$\dot{P}_1 > \dot{P}_2 > \dot{P}_4 > \dot{P}_3$

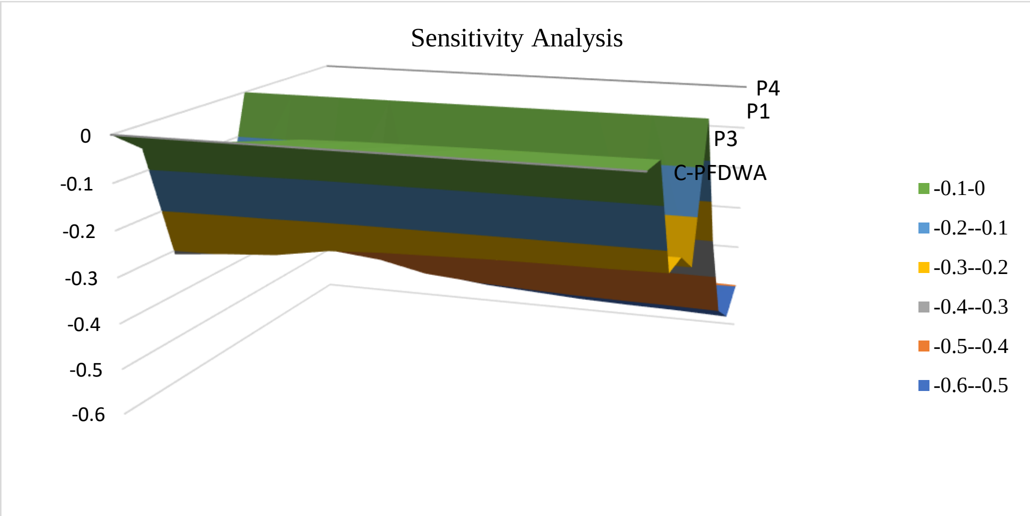

Figure 3: Sensitivity analysis.

7.2 | Advantages

The advantages of this research are summed up as follows:

- The C-PFDWA and C-PFDWG operators handle scenarios involving uncertainty and imprecision well because they are useful in the subjective region, such as martial arts teaching skills assessment.
- It helps particularly when several criteria can be assessed simultaneously, like technique, student involvement, physical shape, and professional demeanor, giving a complete picture of an instructor's performance.
- These operators allow for greater aggregation of multiple sources of expert opinion, even where some membership, abstinence, and non-membership degrees may be present, which produces better results.
- This increases improved decision-making because the number of elements allows better distinctions ranking and evaluations of options to prevent subjectivity in an environment that is usually subjective.
- The advantages of this approach are that it is quite adaptable for use in natural contexts; for example, when it comes to assessing the quality of martial arts teachers, using the

reported traditional tests may fail or be insufficient to describe the intricacies of the teaching and performing abilities.

- Due to the processing of ambiguous data and the ability to consider various attributes more accurately and predict their interaction, this method provides more valuable outcomes for the decisions made in such a sphere, which requires many subjective estimations.

8 | COMPARATIVE STUDY

This study offers a new and workable approach to measuring martial arts teaching skills through the influence of the complex picture fuzzy Dombi AO. Applying ideas of the C-PFDWA and C-PFDWG operators makes it possible to address vagueness, humans' inability to make decisions, and uncertainty typical of subjective assessments. Such operators provide a choice of how and to what extent many criteria can be combined and integrated into the assessments conditioned by multifaceted interconnections between the factors that determine teaching proficiency in martial arts. This is particularly important as it makes it easier and more realistic to capture the dynamic and multi-faceted nature of skill evaluation, giving a fair, valid, and holistic evaluation of the subject. This fuzzy-based process shows the possibilities of its use in other educational and training environments that require decision-making based on vague information. The results emphasize using the Dombi AOs to improve the decision-making process in complicated and ambiguous situations, showing a great positive influence on the intelligent evaluation process.

8.1 | Limitations

Therefore, there are several theoretical and practical limitations inherent in this research and can be discussed here.

- First, the operators of both the C-PFDWA and C-PFDWG in the C-PFS add a level of complexity through the use of PF and complex numbers, which makes it more difficult for individuals without high levels of mathematical understanding and analysis to implement in real-world environments unless additional software or knowledge is used.
- Second, many indicatives may be used as operators in the formula and the efficiency of such operators depends on the quality and accuracy of entering data.
- Lack of consistency and completeness weakens the impact of the results and makes an assessment unreliable, which is critical in a constantly changing environment.
- However, these works do not adequately address the scalability of the proposed algorithm. Whether or not the evaluation system can be applied to a large number of martial arts instructors, and how the system can be implemented across different subject areas, thus using the results of the present study in other educational and training contexts, remains unknown.

- Lastly, a higher level of empirical evidence needs to be provided for this line of thought using larger sets of large datasets, and its usability in environments defined by a large number of low-quantifiability attributes.

8.2 | Future Research Directions

Future studies on both C-PFDWA and C-PFDWG operators should pay more attention to the issue of computational complexity to enhance their application, as well as developing techniques for using data that might be noisy or contain missing points for greater accuracy and robustness, as well as the ability to scale to larger datasets. The rating system used for the assessment of teaching skills for martial arts can be reused in practically all other fields, making our work versatile. Like in sports coaching, it is possible to induce a coach's effectiveness by evaluating certain parameters according to the algorithm, which can encompass technique, athletes' interests, and decision-making. For the performing arts, it can be used to measure creative choreography and performance, theatre, and musical ability and interactivity. Also, the approach may be used in learning institutions to measure the efficiency of teaching and students' interest. These versatile aspects indicate that our framework could be useful for performing skill evaluation in various domains, which also increases the scale of its applicability and applications. Furthermore, extending the use studies of these models, such as in the domains of neutrosophic information [50], transportation sharing practices [51], medical diagnosis [52], and robot selection [53], would expand the practical usage of these models in decision-making under uncertainty. These enhancements will extend the applicability of these operators in various realistic applications.

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