ORIGINAL ARTICLE

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Emergent Spacetime and Entanglement. Quantum Mechanical Derivation of the Schwarzschild Radius

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Article History:

Submitted: October 04, 2025
Revised: December 21, 2025
Accepted: December 27, 2025
Published online: Dec 30, 2025

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ABSTRACT

We develop a relational extension of special relativity in which temporal structure is represented by a two-dimensional phase variable and the kinematics of free particles correspond to motion of fixed magnitude in this time phase plane. Spatial and temporal notions arise only through comparisons between physical systems, and the resulting "completed" relativistic framework reproduces standard Lorentz kinematics in inertial regimes while reducing to Newtonian dynamics in the non-relativistic limit. Gravitational phenomena are introduced through local phase shifts in the temporal and spatial components of particle trajectories, without invoking curvature of an underlying spacetime manifold. Within this formulation, entangled configurations correspond to superpositions occupying the same spacetime phase point from the observer's perspective, thereby preserving locality. Applying the phase evolution rules to particles propagating parallel to a gravitational field yields a critical radius identical to the Schwarzschild boundary, derived here from quantum and geometric resolvability conditions rather than from Einstein's equations. Information is confined to a finite shell outside the critical radius. No central singularity appears. Zero-frequency gravitons remain untapped by the horizon. An optical test is proposed using intersecting monochromatic laser beams. The model predicts enhanced photon–photon coupling when the temporal phases of the beams are aligned.

Keywords: Special relativity; Schwarzschild radius; Quantum theory; Gravitons; Photons

(MSC2020) Codes: 83A05, 83C57, 81Q05, 83C60, 81V10

1 | INTRODUCTION

Modern physics is based on two highly successful theories: general relativity and quantum mechanics. Both have been rigorously tested in their respective fields. However, they are built on incompatible conceptual foundations. General relativity describes gravity as a curvature of space-time, a geometric picture that is strongly supported by observations. Quantum mechanics, by

https://jirmcs.agasr.org/index.php/jirmcs, pISSN: 3005-4281, eISSN: 3005-429X
DOI: https://doi.org/10.62270/jirmcs.v4i2.57

contrast, describes matter and radiation in terms of discrete spectra, probability amplitudes, and nonlocal potential correlations. These features do not fit naturally into the classical description of space-time.

The conflict goes deeper than mere technical details. Relativity presupposes a space-time in which physical processes occur. Quantum theory, by contrast, suggests that fundamental concepts such as position and time take on meaning only through interactions between systems. Reconciling these two views has proven extremely difficult. This problem is particularly acute in systems where both gravity and quantum effects are fundamental, including regions near singularities, in the early universe, and at Planck energies. In such cases, neither theory alone provides a complete or internally consistent explanation.

From a quantum mechanical perspective, the Einstein-Podolsky-Rosen argument and Bell theory have shown that quantum correlations are inherently nonlocal and contextual [1], [2]. As a result, the classical separation principle has failed. From a gravitational perspective, black hole thermodynamics and Hawking radiation have revealed that space-time itself exhibits quantum properties, with both entropy and particle configuration playing key roles [3], [4]. These insights have spurred a wide range of approaches to the study of quantum gravity. Classical and loop programs attempt to directly quantify the geometry [5], while string theory and holographic dualism propose a more radical fundamental structure [6]. In contrast, thermodynamic and information-based approaches view gravity as arising from microscopic degrees of freedom [7], [8]. A common question in all these efforts is: is space-time fundamental, or does it arise from deeper relational structures?

That question naturally connects to Machian and relational approaches to dynamics [9] and to relational quantum mechanics [10]. In these perspectives, neither space nor time is taken as primitive. Physical quantities acquire meaning only through relations among systems. The spacetime continuum may therefore represent an effective large-scale description rather than a fundamental entity. Multi-time and two-time frameworks explore related extensions of kinematics [11], but they do not by themselves provide a concrete mechanism linking relational structure to the empirically verified content of relativity and quantum theory. The present work is motivated by this gap. We ask whether spacetime structure and gravitational dynamics can emerge from an intrinsic time phase geometry while remaining consistent with relativistic invariance, quantum discreteness, and observation.

In earlier work, we proposed such a geometry by extending time to a two-dimensional phase variable [12]. In this framework, each particle undergoes uniform motion in an intrinsic time phase plane. Observable time dilation appears as a projection effect, while massive and massless particles correspond to different phase orientations. Lorentz invariance is preserved but reinterpreted as a rotation in an internal temporal space. A subsequent analysis showed that standard quantum features follow from the same structure [13]. By requiring probability amplitudes to remain invariant under inertial transformations, uncertainty relations, exclusion behavior, and interference emerge as geometric constraints rather than independent postulates.

The framework was later extended into a covariant and dynamical form [14]. Discrete symmetry labels and a five-vector Lagrangian were introduced, allowing Lorentz transformations and particle attributes such as spin and charge to be treated within a single structure. Together, these results define what we call the completed theory of relativity, in which relativistic kinematics and quantum discreteness follow from a single postulate: uniform motion in a two-dimensional time space.

The present paper builds directly on this foundation by incorporating gravity into the same geometric language. Rather than introducing curvature of an external spacetime manifold, gravitational effects appear as local deformations of the temporal phase. In appropriate limits, this reproduces the classical predictions of general relativity. When quantum resolvability constraints are imposed, a critical radius emerges that coincides with the Schwarzschild boundary. This result follows from geometric and quantum considerations alone and does not require Einstein's field equations.

These ideas resonate with broader work linking gravity, information, and entanglement. Black hole complementarity and the stretched horizon picture treat the horizon as an information-bearing surface for external observers [15]. Holographic arguments further [13] suggest that information scales with boundary area rather than volume [23,24], while minimal length scenarios point to a fundamental resolution scale beyond which the continuum description breaks down [16], [17]. In entanglement-based approaches, spacetime geometry can in principle be reconstructed from entanglement structure [18], with entanglement entropy encoded in extremal surfaces [19], [20]. These ideas sharpen the black hole information problem and motivate proposals such as information scrambling and retrieval [21], as well as the firewall debate and refinements of complementarity [22].

This informational perspective is reinforced by experimental and phenomenological developments. Light-by-light scattering has been observed in heavy ion collisions [27], and strong field QED with ultra-intense lasers is now experimentally accessible [28]. Photon–photon interactions are therefore measurable effects. At the same time, generalized uncertainty principle models [29], emergent gravity proposals [30], and thermodynamic derivations of gravitational dynamics [8] support the view that classical spacetime geometry may arise from deeper microscopic or informational degrees of freedom.

Any reformulation of gravity must remain consistent with the high precision of existing tests of general relativity [31]. Nonetheless, many quantum gravity programs treat the spacetime continuum as effective rather than fundamental. Examples include loop quantum gravity [32, 33], causal set theory [34], and conformal super-space approaches [35]. Two-time physics [36] and entanglement-based reconstructions of spacetime [18] likewise motivate extended temporal or internal degrees of freedom underlying observed symmetries. Related constraints arise from quantum information limits and thermodynamic arguments [37, 38], as well as from work on quantum corrected entanglement entropy and entanglement renormalization [39, 40]. Together with classic results linking entropy and area [41–43], these studies emphasize the central role of information in shaping geometry and dynamics.

Against this background, the goal of the present work is specific. We extend the completed theory of relativity by incorporating gravitational phenomena into an intrinsic two-dimensional time phase geometry. Our focus is not only on formal consistency, but also on physical consequences. The framework naturally confines accessible information to a finite shell surrounding the horizon and avoids the formation of a central singularity. It also suggests an experimentally testable signature: an enhancement of photon–photon coupling when intersecting beams share a common time phase orientation.

The remainder of the paper is organized as follows. Section 2 introduces the time phase coordinate system and emergent spacetime. Section 3 addresses entanglement and locality. Section 4 incorporates gravity as a phase deformation. Sections 5 and 6 analyze radial and orbital motion, respectively. Section 7 discusses gravitons and black holes, Section 8 outlines an experimental test, and Section 9 summarizes the results.

2 | NEW COORDINATE SYSTEM AND EMERGENT SPACE TIME

A central challenge for any background-independent formulation of physics is to construct a coordinate system that does not assume a pre-existing spacetime manifold but instead derives temporal and spatial structure from relations among physical systems. In previous work [12–14], we introduced such a construction by assigning each particle an intrinsic two-dimensional temporal degree of freedom and defining spatial quantities solely through observable separations. This section presents a concise and rigorous summary of that framework and establishes how familiar spacetime concepts emerge from it.

2.1. Fundamental Postulates of the Relational Framework.

The relational coordinate system introduced in [12–14] is built on the following postulates:

I. Two-dimensional intrinsic time.

Each particle carries an internal time variable represented by a complex number ($T = |T|e^{i\phi}$). Physical evolution corresponds to motion in this two-dimensional time phase plane. No external time parameter is assumed.

II. Discrete temporal orientation ρ.

The direction of motion in the time phase plane is encoded by a binary quantum number $\rho = \pm 1$, which is defined only relationally. For two particles, $\rho = -1$ indicates mutual approach, and $\rho = +1$ indicates mutual recession in the internal time direction.

III. Relational spatial coordinates.

Space is not presupposed as an independent manifold. The spatial relation between particles a and b in the $i - th$ direction is defined by the magnitude of their separation. $|x_{ab}^{i}|$, which is treated as an observable but has no meaning without at least two particles. Spatial orientation is determined only up to sign.

IV. Discrete spatial symmetry labels β_i.

For each spatial dimension $i \in \{2,3,4\}$, the label $\beta_i = \pm 1$ encodes the two possible orientations along that axis. These labels allow relationally defined coordinate systems to distinguish left/right or forward/backward relative orientations without invoking an absolute embedding space. These postulates define the minimal structure necessary for particles to compare relative motion, assign consistent spatial separations, and define temporal flow.

2.2. Relational Description of Two Non-Accelerating Particles

Consider two massive particles moving with constant relative velocity along a chosen axis X^2. From the perspective of an observer O defined relationally with respect to the pair, the following quantities characterize the system:

- The geometric separation $|x_{rg}|$,
- The relative speed $|v_{rg}|$,
- The opposite spatial orientations $\beta_2^g = -1, \beta_2^r = +1$,
- The temporal orientation $\rho^g = \rho^r = +1$, (mutual recession).

These quantities form a complete relational specification of the configuration. No external coordinate origin or background metric is invoked. All observables are defined purely through comparisons among the particles.

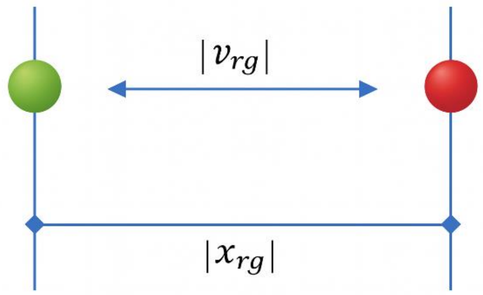

Figure 1. Relational description of two massive particles with separation $|x_{rg}|$, and relative speed $|v_{rg}|$. The particles possess opposite spatial orientation labels. $\beta_2^g = -1, \beta_2^r = +1$ along the x-axis and share the same temporal direction parameter $\rho^g = \rho^r = 1$, corresponding to mutual recession in the intrinsic time phase plane.

This setup also illustrates a general feature of the framework: a "reference observer" is not an independent entity but simply another particle whose relational state with respect to the others is taken as defining a coordinate chart (see fig 1).

2.3. A Limiting Case: A Universe Containing a Single Particle

To illustrate the relational assumptions sharply, consider a hypothetical universe containing a single particle. In such a universe, the following quantities are undefined:

- The relative speed $|v_r|$, because there is no system to compare temporal evolution against.
- The relational distance $|x_r|$, because distance requires at least two distinguishable locations.
- The spatial orientation β_2^r, because symmetry distinctions require multiplicity.
- The temporal direction ρ^r, cannot be assigned, because notions such as approach and recession are only defined relative to another particle.

As a result, space and time, as meaningful physical quantities, do not exist in the case of a single particle. This assertion is not philosophical but a direct consequence of the practical definitions used here: observable quantities require relations, and relations require at least two entities. This claim stands in sharp contrast to classical Cartesian coordinates, in which position and velocity can be determined relative to an arbitrarily chosen origin. In the framework of relations, such an origin does not exist. Each particle defines its own reference frame. The evolution of time becomes meaningful only when comparisons with another reference frame are possible. Therefore, relative time is inherently finite and cannot be arbitrarily extended in the absence of interaction.

2.4. Emergence of Spatial, Temporal, and Gravitational Structure

Consider again the limiting construction introduced above, and suppose that a second particle is added to the system. The new particle moves with constant velocity relative to the first. At the instant $t = 0$, only a single particle exists, and no relational observables are defined. For $t > 0$, the situation changes discontinuously: the existence of a second system makes comparison possible. As soon as two particles are present, a minimal set of relational quantities becomes well defined. These include

- a relative speed $|v_{rg}|$, defined through comparison of temporal evolution,
- A spatial separation $|x_{rg}|$, defined as the magnitude of their mutual displacement,
- discrete spatial orientation labels β_2^g , β_2^r,
- temporal orientation parameters ρ^g, ρ^r.

These quantities exhaust the information required to specify the configuration. No reference to an external coordinate system or background geometry is introduced. All observables arise from relations between the particles themselves. Once such relations exist, space and time acquire operational meaning. In this framework, spacetime is not an independently existing arena; it is an effective description of the relational data encoded in particle separations, relative motion, and intrinsic time phase orientations, and nothing beyond these relations is assumed. An important consequence follows immediately: When at least two particles exist, and an observer can be defined relationally, gravitational effects also become meaningful. In the theory developed here, gravity is not identified with the curvature of a background spacetime. Instead, it appears through local variations in the intrinsic time phase orientations associated with different particles. These variations have no physical content in the single particle limit. They become observable only once relational comparisons are possible. Therefore, the introduction of a second particle has a dual role. It allows the definition of spatial and temporal variables and provides the relative context necessary for the emergence of gravitational behavior. In this sense, gravity is not a primary interaction in the current framework, but a secondary structure, arising from deeper relative and phase degrees of freedom, in accordance with broader approaches to emergent gravity.

3 | ENTANGLEMENT AND LOCALITY

Entanglement is one of the most sensitive tools for studying the structure of space-time. Therefore, it serves as a crucial test for any proposed modification or reinterpretation of relativistic geometry. In classical quantum mechanics, the correlations between entangled particles remain constant even over very large spatial distances. This behavior challenges classical notions of locality and is the basis for the fundamental debates initiated by Einstein, Podolsky, and Rosen. The tension arises because spatial separation appears to have no limiting influence on observable correlations. Within the relational time phase framework developed here, these features admit a coherent interpretation.

Spatial distinctions are not imposed by an external background. Directional labels and separations instead arise relationally, through comparisons between physical systems. Correlations, therefore, remain well defined without invoking background-dependent degrees of freedom. In this section, we develop a consistent description of bipartite entanglement within the time phase approach. We show that the framework reproduces standard quantum predictions. At the same time, operational locality is preserved. Nonlocal correlations emerge without requiring superluminal signaling or violations of causal structure.

3.1. Relational Setup for Two-Particle Entanglement

Consider two identical massive particles created in a localized interaction. Immediately after creation, the particles recoil in opposite spatial directions along the relational x-axis. In this framework, the internal spatial phase label β_2 distinguishes the two possible orientations available to each particle. Conservation laws and exclusion constraints imply that if one particle acquires $\beta_2 = +1$, the other must acquire $\beta_2 = -1$ [14]. Since no preferred spatial direction exists, both assignments occur with equal probability.

The relational configuration separation $|x_{rg}|$, relative speed $|v_{rg}|$, and opposite spatial phase orientations are illustrated in Fig. 2.

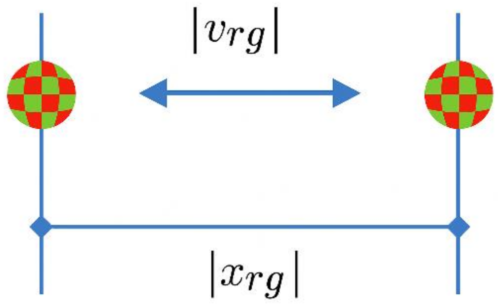

Figure 2. Schematic illustration of two entangled particles separating along the x-axis with opposite spatial phase orientations and equal relative speed.

3.2. Representation of the Entangled State

Let β_2^r and β_2^g denote the spatial phase orientations of the red and green particles, respectively. Immediately after the interaction, the entangled state in the β_2 -basis is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\beta_2^r = 1, \beta_2^g = -1\rangle - |\beta_2^r = -1, \beta_2^g = +1\rangle\right), \quad \rho^r = \rho^g = +1,$$

where ρ denotes the temporal orientation associated with separation in the two-dimensional time plane. The minus sign reflects anti-symmetry. This anti-symmetry is required by particle identity and exclusion considerations. The underlying relational geometry enforces a further constraint. The two spatial phase labels always appear with opposite sign. This remains true regardless of detector placement.

3.3. Measurement and Observer-Dependent Assignments

As the particles spread out, the distance between them increases. A detector located on the right side of the coordinate axis records each particle that reaches this spatial phase position. The model does not require the particle to move through classical space. Instead, each particle occupies one of two relational branches, determined by its spatial phase direction. The detector chooses between these two branches. No further dynamical assumptions are made. If an observer records a particle on the right detector, there are two relational possibilities. Symmetry determines their weights.

- Probability $\frac{1}{2}$ that the detected particle is the red particle ($\beta_2^r = -1$),
- Probability $\frac{1}{2}$ that the detected particle is the green particle ($\beta_2^g = +1$).

These probabilities follow directly from symmetry considerations. They reproduce the standard quantum predictions. No superluminal influence is required. The measurement merely identifies which relational branch intersects the detector.

3.4. Observer Displacement and Transformation of Labels

To examine locality more carefully, consider an observer positioned to the left of both particles. From this vantage point, each particle lies to the observer's right. The spatial phase labels are therefore identical, with $\beta_2^r = \beta_2^g = +1$. The temporal orientation, however, differs between the two particles. One particle is receding relative to the observer, while the other is approaching. The observer, therefore, assigns the relational state.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\,\rho^r = +1, \rho^g = -1\rangle - |\,\rho^r = -1, \rho^g = +1\rangle),\ \ \beta_2^r = \ \beta_2^g = +1.$$

At first sight, this expression appears different from the representation obtained in the β_2 basis. The difference is only apparent. Both expressions describe the same physical state. The change in functional form reflects the observer's displacement, not a change in the underlying relational physics. This behavior is directly analogous to standard quantum mechanics. Spinor components transform under spatial rotations, while the physical state itself remains invariant.

3.5. Observer Invariant Relational Wave Function

Across all inertial observers, a simple invariance property holds. A change in observational perspective may reverse the temporal orientation ρ. When this occurs, the associated spatial phase orientation β_2 reverses simultaneously. The product $\rho\beta_2$, however, remains unchanged.

This constraint fixes the observer invariant form of the entangled state. It may be written as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\left|\, \rho^r \beta_2^r = +1, \rho^g \beta_2^g = -1\right\rangle - \left|\, \rho^r \beta_2^r = -1, \rho^g \beta_2^g = +1\right\rangle\right).$$

This representation is invariant under changes of inertial observer. All observers, therefore, assign the same physical state, even though the individual labels ρ and β_2 may differ. Consistency with relativistic locality is preserved. No contradictions arise from observer-dependent descriptions, because only the invariant relational combinations enter the physical state.

3.6. Relational Resolution of the Locality Problem

Within the time phase framework, locality is addressed at the level of ontology. Before measurement, the two particles share a common spacetime phase origin. Separate classical positions are not assigned. The theory, therefore, does not describe particles as following distinct spatial trajectories before an observation is made. Measurement plays a limited role. It does not induce physical changes at distant locations. Instead, it selects a single branch of an already existing relational structure. No dynamical influence propagates between spatially separated points at the moment of detection. From this perspective, correlations between entangled particles pose no threat to locality. The underlying description does not contain spatially separated classical entities to which superluminal signals could be applied. Entanglement instead reflects pre-existing relational constraints encoded in the joint time phase geometry. This resolution closely parallels the treatment of nonlocality in quantum information theory. There, entanglement generates correlations without enabling signaling. Relativistic causality remains intact. The same logic applies here, expressed in explicitly relational terms.

3.7. Classical Analogy for Operational Locality

Consider a mechanism that routes two distinguishable balls (red and green) into two sealed containers, controlled by a binary switch triggered by a radioactive decay:

- A decay event sets the switch left, sending red to the left container and green to the right.
- No decay sets the switch right, reversing the assignment.
- Until an observer opens one container, the system is operationally indistinguishable from a superposition.
- Observing one ball immediately reveals the color of the other, regardless of distance.
- No information is transmitted faster than light; the correlations were embedded at the origin.

Entanglement in the time phase approach functions analogously: relational labels encode correlations established at creation, not mediated through space at the time of measurement.

We can summarize the discussion as

- The spatial phase label β_2 and temporal orientation ρ provides a relational encoding of two particle correlations.
- Entangled states expressed in these variables maintain invariance under all observer translations, preserving locality.
- The framework reproduces the empirical structure of quantum entanglement while offering a geometric explanation grounded in emergent spacetime.
- Apparent nonlocality arises only when relational variables are misinterpreted as classical spatial degrees of freedom.

4 | GRAVITY AND ITS IMPACT ON THE PHASES OF SPACE AND TIME

In our previous developments of the Completed Theory of Relativity (CTR) [12–14], we obtained a Lagrangian describing the motion of non-accelerating particles in flat spacetime. The corresponding action takes the form.

$$Action = S = m\int_a^b \rho T dt + \frac{1}{2}m\int_a^b (\beta_3\, r^2\dot{\alpha})dt$$

which can be written compactly as

$$\mathcal{L} = m\left(\rho T + \frac{1}{2}\beta_3 r^2\dot{\alpha}\right)$$

or, equivalently,

$$\mathcal{L} = m\left(e^{i\dot{\theta}} + \frac{1}{2}\beta_3 r^2\dot{\alpha}\right)$$

where $T = e^{i\dot{\theta}}$ represents the intrinsic two-dimensional time phase, α is the spatial phase associated with directional motion, and β_3 encodes orientation symmetry in the spatial manifold. In the presence of a weak gravitational field, this Lagrangian must remain valid over an infinitesimal interval $dx \to 0, dt \to 0$, where spacetime may still be approximated as locally flat. The question addressed in this section is how gravitational effects modify the intrinsic spatial and temporal phases (α, θ) that characterize the CTR description of particle dynamics. Following the framework outlined in [14], we adopt two postulates governing how gravity influences the phase variables of a particle:

- Gravity perpendicular to the direction of motion of any particle changes its phase in space. (α). The change in angular momentum of the particle is a function of its velocity and the perpendicular component of gravity.
- Gravity parallel to the direction of motion of any particle changes its phase in time. (θ). The change in the energy of the particle is a function of its velocity and the parallel component of gravity.

We can express this as follows:

$$\dot{\alpha} = g\left(G_{\mathrm{i}}^{\perp}, V_{\mathrm{i}}\right)$$

$$\dot{\theta} = \tan^{-1}\left(f\left(G_{\mathrm{i}}^{||}, V_{\mathrm{i}}\right)\right)$$

where:

- $G_{\mathrm{i}}^{\perp}$ is the component of the gravitational field orthogonal to the particle's velocity,
- $G_{\mathrm{i}}^{||}$ is the component parallel to the particle's velocity,
- v_i is the particle's coordinate speed,
- g and f are functions encoding how gravity couples to intrinsic temporal and spatial phase evolution.

These relations specify how the local gravitational environment deforms the intrinsic phases of spacetime, thereby determining particle trajectories within CTR.

5 | PARTICLE MOVING TOWARDS A MASSIVE OBJECT

In this section, we analyze the motion of a particle, first a photon and then a general relativistic particle, falling radially toward a massive, spherically symmetric object within the CTR. The aim is to derive the temporal phase evolution. $\dot{\theta}$ under a purely parallel gravitational field, and to identify the minimum radius r_{min} at which the theory remains well defined. Imposing quantum resolvability constraints on the temporal phase then yields the Schwarzschild boundary. $r_s = 2GM$ directly from CTR, without appealing to spacetime curvature

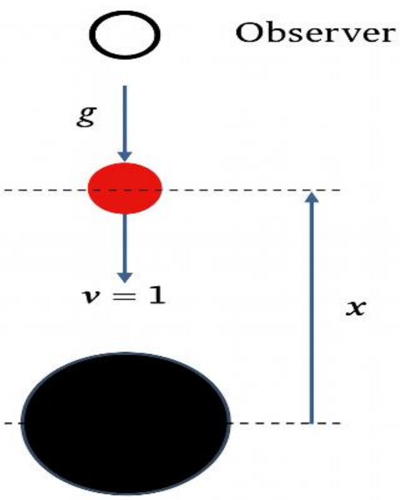

Figure 3. Radial infall of a photon toward a massive object as viewed by a stationary observer. The photon experiences only the parallel component of the gravitational field, leading to a change in temporal phase. $\dot{\theta}$ while the spatial phase remains constant $\dot{\alpha} = 0$.

Figure 3 depicts a photon propagating radially inward toward a massive object along a direction parallel to the gravitational field. Because the gravitational field has no perpendicular component along such a radial trajectory, the spatial phase does not evolve. $\dot{\alpha} = 0,$ consistent with the postulate that only transverse gravity alters spatial phase. The temporal phase, however, changes through the relation.

$$\dot{\theta} = \tan^{-1}\left(f\left(G_{\mathrm{i}}^{||}, \lambda_{\mathrm{i}}\right)\right)$$

where $\lambda_{\mathrm{i}}$is the local wavelength and $G_{\mathrm{i}}^{||}$is the parallel component of the gravitational field. For a photon, the CTR kinematics require

$$v = 1, \ \ \dot{\alpha} = 0, \ \dot{\theta} \neq constant, \ \phi = 0.$$

Since no transverse gravitational component exists, the photon trajectory is strictly radial. The temporal phase rate $\dot{\theta}$ increases monotonically as the photon approaches the massive body, reflecting the growing influence of the parallel gravitational field. This reproduces the classical general relativistic result that radially infalling photons follow straight spatial geodesics.

The CTR framework is governed by two principles

1. The velocity in time of all particles is governed by the equation $T = \rho e^{i\dot{\theta}}$ [12].
2. A gravitational field parallel to the direction of motion of any particle changes its velocity/frequency [14].

Consider now a particle falling radially toward a black hole. Since the gravitational field is parallel to the motion, its influence appears exclusively in $\dot{\theta}$.

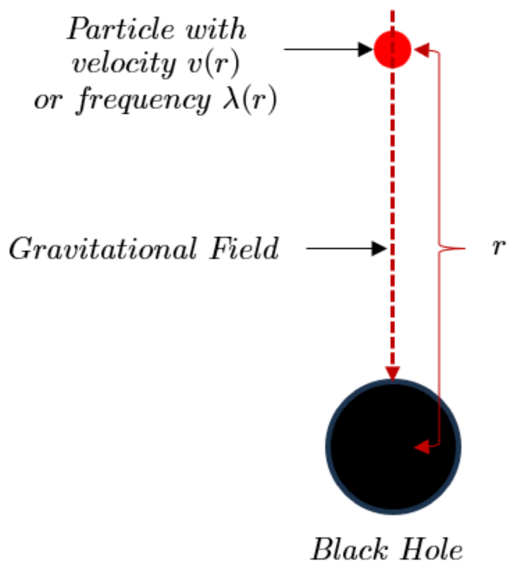

Figure 4. Radial infall of a particle toward a black hole. The gravitational field acts along the direction of motion, modifying the particle's temporal phase and its locally measured velocity or frequency. No perpendicular gravitational component is present along the radial path.

In Special Relativity, one has $\dot{\theta} = f(v) = constant$. In the gravitational field, this becomes

$$\dot{\theta}(v_r) = \tan^{-1}(v_r) \; and \; v_r = f(r, v_0, r_0, G, M)$$

for massive particles, and analogously

$$\dot{\theta}(\lambda_r) = \tan^{-1}(-\lambda_r) \; and \; \lambda_r = f(r, \lambda_0, r_0, G, M)$$

for photons. The function $f(v)$ appearing in v_r or λ_r encodes the effect of gravity on temporal phase evolution. Also

$$v_0, \lambda_0 = \; velocity/frequency \; of \; the \; particle \; at \; some \; initial \; radius \; r_0 \text{ (observer)}$$

$$v_r = \; velocity \; of \; the \; particle \; at \; any \; radius \; r$$

$$G, M = \; gravitational \; constant \; and \; the \; mass \; of \; the \; black \; hole$$

Therefore, for a massive particle:

$$T = \rho e^{i\dot{\theta}} = \rho e^{if(v_r)} = \rho e^{i\tan^{-1}(f(r,v_0,r_0,G,M))}$$

$$T = \rho e^{i\dot{\theta}} = \rho e^{if(-\lambda_r)} = \rho e^{i\tan^{-1}(-f(r,\lambda_0,r_0,G,M))}$$

Since CTR treats velocities as non-directional scalars, the allowed temporal phase range for massive particles obeys

$$(1-\rho)\frac{\pi}{2} \le \dot{\theta} < \frac{\pi}{4} + (1-\rho)\frac{\pi}{2}$$

In the same way, the limit to the wavelength of a photon is 1, and therefore, we also have limits to its phase in time [12]:

$$-\frac{\pi}{4} + (1-\rho)\frac{\pi}{2} < \dot{\theta} < (1-\rho)\frac{\pi}{2}$$

To simplify, let's assume that the particle is always moving towards the massive object and therefore from the relative perspective of the massive object and the moving particle, $\rho = -1$ (this also means that $r_0 \ge r$). From the perspective of an observer at the initial radius of the trajectory, the trajectory of the particle in the θ dimension in time is then:

$$\theta_m(r) = \int_a^b \dot{\theta}\, dt = \int_a^b \tan^{-1}\big(f(r, v_0, r_0, G, M)\big)\, dt$$

$$\theta_\gamma(r) = \int_a^b \dot{\theta}\, dt = \int_a^b \tan^{-1}\big(-f(r, \lambda_0, r_0, G, M)\big)\, dt$$

To avoid repeating equations, we will continue assuming the particle is a photon. We want to find the function f such that $\lambda(r) = f(r, \lambda_0, r_0, G, M)$ That explains the movement of the particle in the gravitational field. At an infinite radius, the gravitational field is null, and the formula must collapse to the Minkowski metric of Completed Special Relativity [12]:

$$\dot{\theta} = \theta_\gamma(r \to \infty) \to \tan^{-1}\big(-\lambda(r \to \infty)\big) \to \tan^{-1}(-\lambda_0)$$

$$f_{(r \to \infty)} = \lambda_0$$

A black hole has a surface where the change of the phase in time approaches a limit; there is a minimum radius for which the function is defined:

$$\dot{\theta} \to \frac{3\pi}{4} \; then \; \lambda(r \to r_{min}) \to 1$$

$$f_{(r \to r_{min})} = 1$$

From the perspective of an observer in the inertial reference frame of the black hole, the wavelength of the photon increases as the photon approaches the black hole (Ref 3):

$$f_{(r_a)} < f_{(r_b)}$$

In the limit $\lambda_0 \to 1$, gravity can't have any more impact on its wavelength (in the same way that a particle traveling at the speed of light would not be affected by gravity parallel to its propagation):

$$\lambda_0 \to 1 \; then \; f(r) \to 1 \qquad (1)$$

From the observer's perspective, when the initial radius is larger, the wavelength at any subsequent smaller radius is larger:

$$r_0 > r_1 \to f(r, r_0) > f(r, r_1)$$

when

$$r_0 = r \to f(r) = \lambda_0$$

We know Newton's Universal Law of Gravitation:

$$F = \frac{Gm_1m_2}{r^2}$$

Therefore, for a particle in the gravitational field of a large mass:

$$a = \frac{GM}{r^2}$$

and the corresponding velocity is:

$$v = -\frac{GM}{r} + constant = -\frac{GM}{r} + v_0$$

Newton's equations need to approximate the equations of Completed Relativity at low velocities, and since we know that. $\dot{\theta}$ is a function of the velocity/frequency of the particle, we can assume that f is also a function of $\frac{GM}{r}$. We know from [14] that the relation between frequency and velocity in Completed Special Relativity is the reason for the appearance of space contraction:

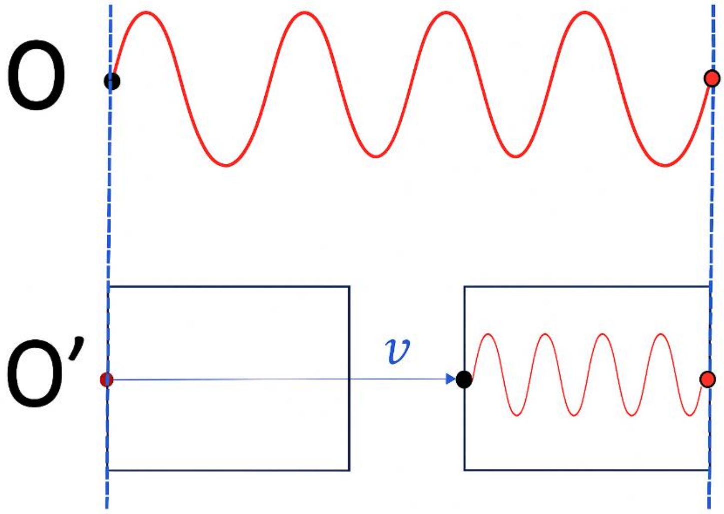

Figure 5. Photon propagation from an observer O' inside a moving train car, emitting a wave toward the front of the car. Observer O, stationary outside the car, perceives a different wavelength and velocity configuration due to the motion of the car, yet both observers agree on the total number of wave cycles.

$$\lambda' = \sqrt{1-v^2}\,\lambda$$

The following equation satisfies all the above constraints and is dimensionally correct (remember we are using $c = 1$):

$$f(r) = \left(1 - \sqrt{1-\left(\frac{GM}{r}\right)^2} \Big/ \sqrt{1-\left(\frac{GM}{r_0}\right)^2}\right)\frac{GM}{r} + \left(\sqrt{1-\left(\frac{GM}{r}\right)^2} \Big/ \sqrt{1-\left(\frac{GM}{r_0}\right)^2}\right)\lambda_0 \qquad (2)$$

For very low velocities with respect to the speed of light (in other words, for small $\frac{GM}{r}$):

$$\sqrt{1-\left(\frac{GM}{r}\right)^2} \approx 1$$

therefore

$$f(r) \approx \lambda_0$$

This is exactly what we expect, negligible changes in frequency at low velocities. We now proceed to calculate the minimum allowable radius in the case of a black hole (the Schwarzschild Radius) using Equation (2). We know that the wavelength is a quantized number [12] and that we have a limit to the value of the wavelength. We also know that Special Relativity requires that the wavelength be less than one in the same way a massive particle may never reach the speed of light ($\lambda < 1$):

$$\lambda(r) \in \left\{\frac{1}{2}, \frac{1}{3}, \dots\right\}$$

Therefore, the largest allowable value for the wavelength is ½, which at the same time is the limit for r because Equation (2) says that as the radius decreases, the wavelength increases:

$$\lambda(r_{min}) = \frac{1}{2}$$

We have been making up to now an error in our assumption in Equation (1); the particle never reaches the centre of a black hole because our equations are not defined at a radius smaller than GM, and therefore r is actually measured from the surface of the black hole, not its centre.

In Equation (2) $r < GM$ is undefined, therefore, we need to adjust the equation and measure r from the surface of the black hole $r_{surface} = r_{centre} - GM$ and set its limit to ½.

$$f(r) = \frac{1}{2}\left(1 - \sqrt{1-\left(\frac{GM}{r-GM}\right)^2} \Big/ \sqrt{1-\left(\frac{GM}{r_0}\right)^2}\right)\frac{GM}{r-GM} + \left(\sqrt{1-\left(\frac{GM}{r-GM}\right)^2} \Big/ \sqrt{1-\left(\frac{GM}{r_0}\right)^2}\right)\lambda_0 \tag{3}$$

This adjustment is trivial when the radius of the massive object is much greater than GM, like most of our experiences; it becomes crucial when the massive objects are black holes. What is the meaning of quantized frequency? It means that an observer in a stationary reference frame can only obtain information about a particle in motion to a resolution between integer multiples of the frequency. In this case, we know that at a distance of $r = GM$ from the centre of the black hole, the theoretical frequency would have a value of 1, so what we need to find out is the distance in the time trajectory between a frequency of 1 and a frequency of ½ in the vicinity of the black hole. We picture this in the following diagram:

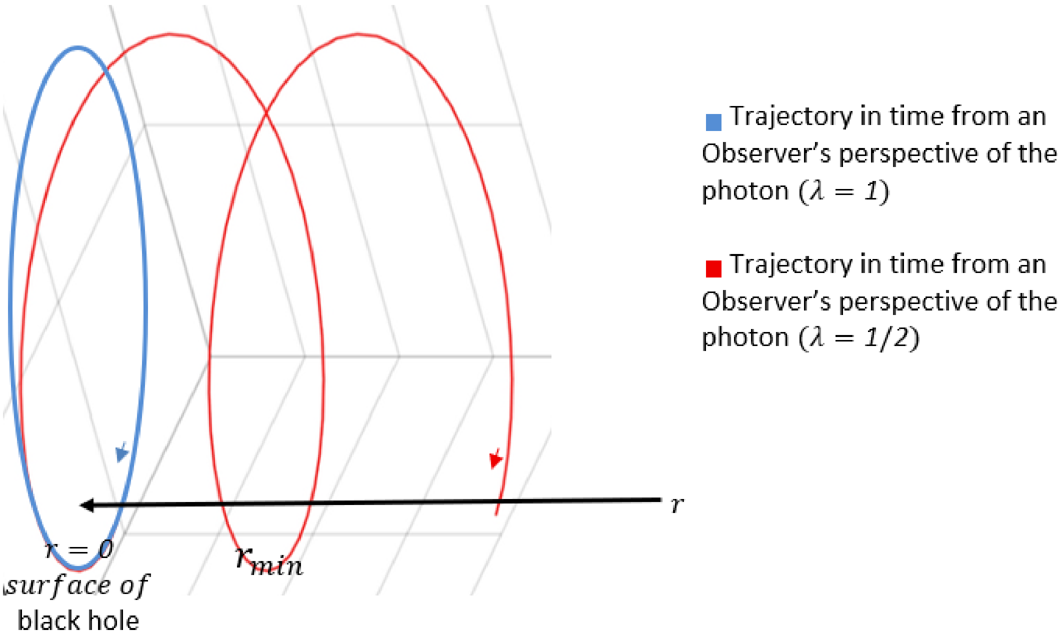

Figure 6. Comparison of the temporal phase evolution of a photon relative to its position with respect to the surface of a black hole. The red curve ($\lambda = \frac{1}{2}$) exhibits still a component in the linear time dimension, while there is no linear component for the blue curve ($\lambda = 1$). Information about the photon is no longer available beyond the radius r_{min}, which corresponds to the observer-dependent Schwarzschild boundary.

To explain what is happening in the physical world, we want to know at what position in space the uncertainty starts to influence the particle, and we can no longer get information about it (in other words, we can no longer see it) [13]. Another way to think about this is at what position is it still possible to place the particle so that it is aligned with the gravitational field of the black hole. From Equation (3), we know that.

$$r_{min} = GM$$

But this is measured from the surface of the black hole. If we measure from the centre of the black hole, we have the Schwarzschild radius:

$$r_s = r_{min} + GM = 2GM$$

We obtained the Schwarzschild radius directly from quantum principles. All this says is that once we reach a radius of 2GM from the black hole, we can't obtain any information from the particle due to the uncertainty principle (which is equivalent to the quantization of frequency, which means we can't have a photon with a wavelength higher than ½), and we perceive this as the event horizon of a black hole. When a particle moves closer than $r = GM$ to the surface of the black hole, an observer can't access any information about the direction that the particle is moving in, and we can't emit a photon in a parallel direction to the gravitational field of the black hole, as this would violate the uncertainty principle. At the surface of the black hole, particles can, in theory, only move perpendicularly to the centre of the black hole, so that the equation for frequency is still valid and we do not obtain an inflection point. This results in two major implications for the physics of black holes:

- There is no singularity at the centre of a black hole.
- All particles (information) are contained in a shell around the black hole of size $[2GM, GM)$.
- The movement of a particle past the event horizon is unknown for any observer.

6 | ORBITAL MOTION

We now examine orbital motion within the CTR, focusing on a test particle of negligible mass moving in a vacuum around a perfectly spherical massive object. Because the orbital trajectory is everywhere perpendicular to the local gravitational field, this regime isolates the influence of the transverse gravitational component on the spatial phase α, while leaving the temporal phase θ unaffected to leading order.

6.1. Description of a Particle Orbiting a Large Object

Figure 7 illustrates the configuration. A test particle (red) moves along a circular orbit of radius r around a central mass (black). The instantaneous velocity of the particle is tangent to the orbit and denoted v, while the spatial phase angle αcharacterizes its evolution within the intrinsic spatial phase plane of CTR. The gravitational field points radially inward, and its magnitude is determined solely by the radial coordinate.

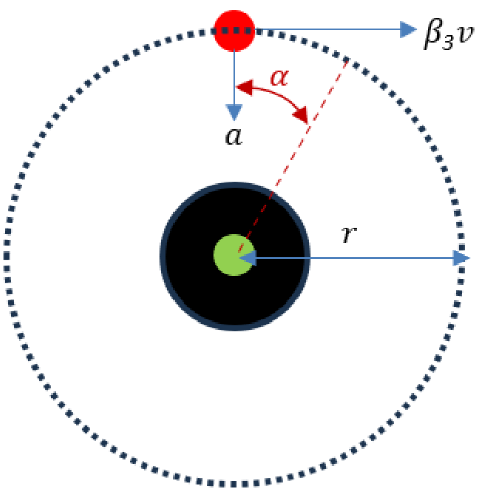

Figure 7. Circular orbit of a test particle around a spherical mass. The particle's velocity is tangential while the gravitational field is radial, causing only the spatial phase α to evolve, with the temporal phase remaining constant along the orbit.

In this special case, from the perspective of an observer in the same IRF as the massive object:

$$v = r = a = \dot{\alpha} = \tan^{-1}(v) \ \ is\ constant\ .$$

For orbital motion, the direction of the velocity is at a right angle to the gravitational field. Using the center of the massive object as the origin of a polar coordinate system, we can define our coordinate system as:

$$R^{\mu} = (t, \theta, r, \alpha, \varphi)$$

where, from the perspective of the observer:

$v = constant\ (since\ the\ gravitational\ field\ is\ constant\ and\ perpendicular\ to\ v)$

$r = \ constant$

$\dot{\alpha} = \dfrac{2\pi\beta_3 v}{r}\ where\ \beta_3 = -1\ when\ clockwise\ otherwise\ \beta_3 = 1$

$\dot{\theta} = \tan^{-1}(v) = constant$

$\varphi\ = 0$

Inserting into the Lagrangian:

$$\mathcal{L} = \ M\left(e^{i\dot{\theta}} + \beta_3 r^2 \dot{\alpha}\right)$$

$$\mathcal{L} = \ M\left(e^{i\dot{\theta}} + 2\pi r v\right)$$

β_3 defines the direction of rotation of the particle around the massive object. The velocity in our coordinate system [12] is just a scalar without direction.

The solution to the Euler-Lagrange Equation for this Lagrangian is the same as we showed in our previous work [14], because the time component is the same, and the rate of change of the phase in space is also multiplied by a constant, therefore:

$$\ddot{\alpha} = 0\ and\ \ddot{\theta} = 0$$

Thus, in the absence of radial motion, the spatial and temporal phases evolve independently in the expected manner: the temporal phase remains fixed, reflecting the lack of parallel gravitational influence, while the spatial phase evolves uniformly due to the constant transverse component of the gravitational field. Orbital dynamics, therefore, emerge in CTR exactly as expected from general relativity in the weak field regime.

6.2. Orbital Motion Close to the Surface of the Massive Object

As the orbiting particle approaches the surface of the central mass, transverse gravitational effects become increasingly important. This regime is reached when the orbital radius nearly coincides with the physical radius of the object. At that point, local variations dominate the dynamics. To analyze this behavior, we consider an observer located at the surface of the massive object. For definiteness, this observer is identified with the green particle. The analysis then focuses on the local relational dynamics as the orbiting particle, taken to be the red particle, passes through its point of closest approach. The discussion is framed as a thought experiment. Within this setting, the relevant geometric and temporal relations can be isolated and examined explicitly. In this thought experiment, we let

$$r_v \to r$$

If the observer is placed at the centre of the massive object, ρ is always 1. Now, from the perspective of the observer at the surface of the object, $\rho = -1$ in the upper part of the trajectory and $\rho = 1$ in the lower half of the trajectory.

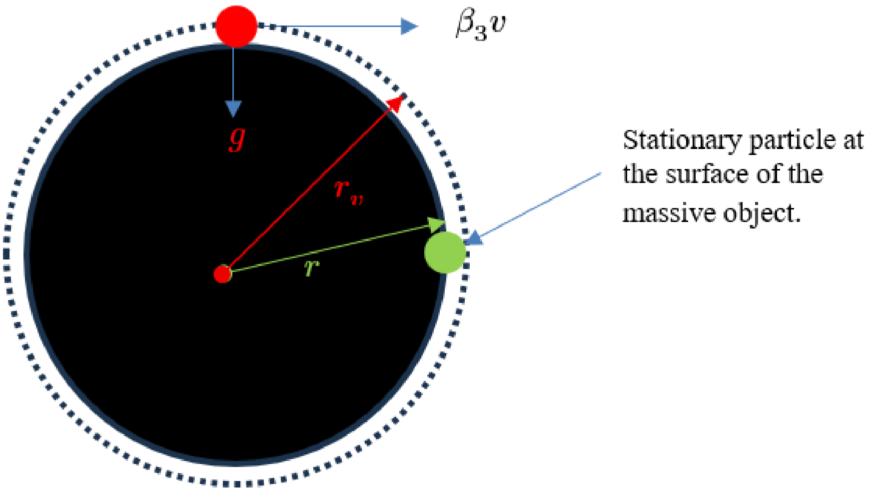

Figure 8. Orbital motion near the surface of a massive object. A test particle (red) moves tangentially with velocity. $\beta_3 v$ while experiencing the local gravitational field g. An observer particle (green) is located on the surface, defining the relational separation r and the instantaneous distance v_r to the orbiting particle. The configuration illustrates how the proximity to the surface alters the spatial phase evolution and leads to an inflection point in the orbital trajectory.

When looking at an infinitesimal segment $d\alpha$ at the time that the two particles reach the closest distance to one another (when ρ changes sign instantaneously), we face an inflection point that is equivalent to the one described in our previous work [14]. At that point $\dot{\alpha} \to \infty$. Notice that the massive object is composed of particles that fill its volume; therefore, this situation occurs in every surface point, not only where we have arbitrarily placed the green particle. Let's magnify and redefine α as the change in the angle with respect to the red and the green particle (no longer the centre of the massive object):

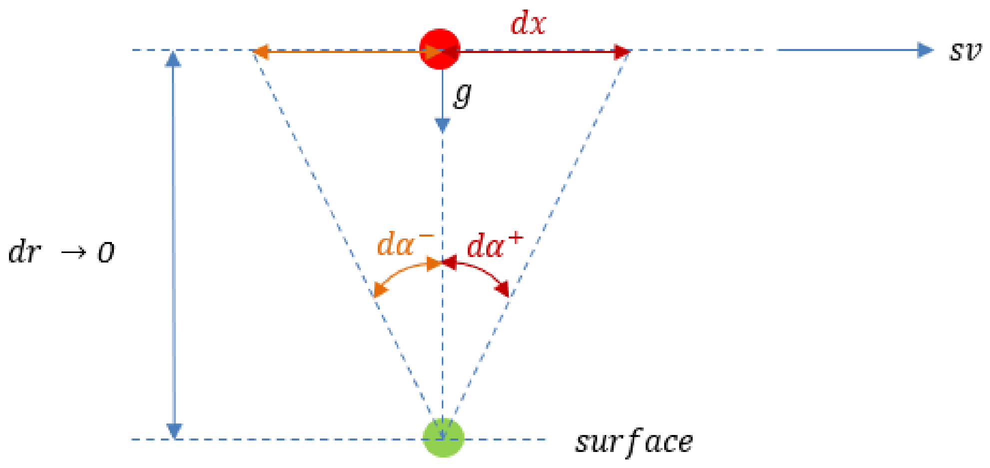

Figure 9. Magnified geometry showing the angular change $d\alpha$ between the orbiting particle and the surface observer as the particle passes its point of closest approach, where the relational parameter ρ changes sign, and the angular rate momentarily diverges.

Because the green particle is not at the centre of the massive object, the red particle is moving closer to it before arriving at its closest point ($\rho = -1$) and moving away from it after crossing the closest point ($\rho = 1$). We obtain the same result as for special relativity [14]:

$$\alpha = \tan^{-1}\left(\frac{dx}{dr}\right)$$

The result of this equation is well defined provided that $r \gg 0$, but we face an inflection point when both:

$$dx \to 0 \text{ and } dr \to 0$$

$$\alpha = \tan^{-1}\left(\frac{dx}{dr}\right) \to \tan^{-1}\left(\frac{0}{0}\right), \textit{ which is undefined}$$

This occurs when the orbit comes close enough to the surface for the uncertainty principle to act [13], and as expected, we observe a collision. All experiments to date result in a collision if a particle has an orbit that is too close to a massive object. We have shown that the result for orbital motion using the Completed Theory of Relativity describes nature as expected. Einstein's theory is a perfect approximation except when the orbit of the particle is very close to the surface of the object (because it does not predict a collision).

7 I THE GRAVITON AND BLACK HOLES

In general relativity, gravity is identified with curvature of spacetime. At the event horizon of a black hole, this curvature becomes sufficiently strong that no causal trajectory allows escape once the horizon is crossed. The horizon, therefore, acts as a one-way boundary for matter and radiation. In quantum field theory, however, gravitational interactions are usually described in terms of a force carrier, the graviton, which is assumed to propagate along null trajectories. When these two descriptions are combined, a conceptual tension arises. If gravitons mediate the gravitational interaction in the same sense as photons mediate electromagnetism, it is natural to ask how the gravitational field of a fully formed black hole persists outside the horizon. If gravitons were trapped in the same manner as photons or massive particles, the exterior gravitational field would rapidly decay after horizon formation. This behavior would be inconsistent with both classical predictions and observation.

This tension reflects a deeper incompatibility between the standard formulations of general relativity and quantum field theory. However, this incompatibility is not apparent in full relativity. In this framework, particles, including the graviton, are described not only by their orbits in space-time, but each particle is assigned an orbit in an internal two-dimensional phase-time space. This internal time structure, rather than the curvature of space-time alone, determines how particles respond to gravitational influence. A key feature of the graviton in this description is the vanishing of its intrinsic frequency. Its phase direction is opposite to that of a stationary particle. As a result, gravitational effects that trap matter and radiation by changing their phase do not hinder the motion of the graviton. Thus, the graviton remains free to propagate outward from the event horizon, maintaining the external gravitational field even after the formation of a black hole.

In the complete theory, the gravitational component affects the time-space of a particle. This effect depends on the internal structure of the particle. For particles with mass, the speed of time changes directly. For photons, however, the same interaction manifests itself as a change in frequency. This distinction is the basis for the confinement of matter and radiation in the event horizon, while the graviton remains unchanged. Gravity remains effective outside the black hole without the need for information to pass from the event horizon. Figure 10 illustrates the regions of motion allowed for different types of particles in the plane of the internal time-space. Matter and photon states occupy separate kinetic sectors, while other regions are excluded due to the limitations of the complete theory. This representation explains why different particles respond differently to the influence of gravity and why the graviton occupies a special role in this framework.

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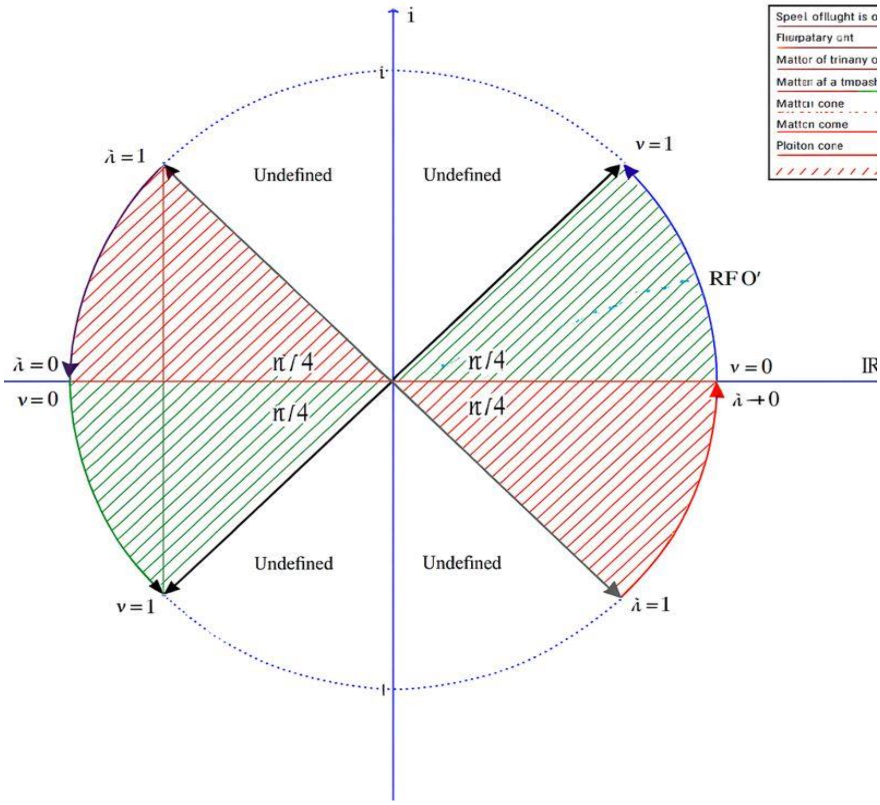

Figure 10. Allowed regions of particle motion in the two-dimensional time phase plane. Matter and photon states occupy distinct. $\frac{\pi}{4}$ cones, while the remaining sectors are forbidden by CTR kinematic constraints. The boundary circle marks the limiting velocity $c = 1$. The graviton occupies the pi position in the time phase plane (infinite velocity / no frequency

A more precise statement follows from [14]. The velocity associated with the graviton is directed opposite to that of an observer at rest in the intrinsic time phase plane. This orientation reflects the vanishing intrinsic frequency of the graviton and plays a central role in its behavior under gravitational influence. Within the present framework, the parallel component of gravity acts on a particle by modifying its temporal phase. For massive particles, this produces a direct change in temporal velocity. For photons, the same interaction manifests as a shift in frequency. This distinction is essential. It explains why matter and radiation are trapped at the event horizon, while the graviton is not. As the graviton approaches the horizon, its propagation speed decreases, but it does not vanish. Instead, the graviton continues to move outward along null directions in the time phase plane. Consequently, the gravitational field generated by the black hole is not extinguished after horizon formation. It persists through the continuous outward propagation of gravitons.

This behavior resolves the apparent conflict between general relativity and quantum field theory. The external gravitational field does not require information to escape from within the horizon. Rather, it is sustained by gravitons whose intrinsic time phase structure prevents them from being trapped. Because gravity modifies temporal phase rather than acting as a conventional force, the graviton remains unaffected by the same mechanism that confines matter and radiation. In this way, the completed theory provides a consistent account of how a black hole can retain a stable external gravitational field while respecting both relativistic causality and quantum principles.

8 | ON HOW TO EXPERIMENTALLY PROVE THE COMPLETED THEORY

We propose that photons of identical frequency cannot occupy the same position in spacetime; when forced into coincidence, they are expected to interact analogously to matter particles [13]. The following experiment provides a direct means of testing this prediction. Two high-intensity lasers are arranged such that their beams intersect within a sealed enclosure. The enclosure must be constructed to prevent any external light from entering, and its internal surfaces must detect photons that scatter from the interaction region. The lasers are first adjusted to emit monochromatic light at two different wavelengths, for example, red and violet. Both beams are projected into the enclosure for a fixed duration, and the photon scattering rate is recorded. The lasers are then tuned to emit light at closely matched frequencies, such as two nearly identical red wavelengths. The procedure is repeated under the same conditions, and the resulting scattering rate is measured. According to the theory, photon scattering should be significantly enhanced when the beams possess similar frequencies compared to when their frequencies differ.

9 | CONCLUSION AND DISCUSSION

This work develops the completed theory of relativity as a unified framework. Relativistic kinematics, quantum discreteness, and gravity are treated within a single structure. The starting point is deliberately minimal. Physical motion is described as uniform motion in an intrinsic two-dimensional time phase space. Space and time are not assumed from the outset. They emerge only through relational comparisons between physical systems.

Within this framework, Lorentz transformations take a simple form. They correspond to rotations in the time phase plane. Quantum features follow from the same structure. Uncertainty, exclusion, and interference arise from the requirement that probability amplitudes remain invariant under inertial transformations. No additional quantum postulates are introduced. Gravity is incorporated without invoking curvature of an external spacetime manifold. Instead, gravitational effects appear as local variations in temporal phase. In regimes where general relativity has been extensively tested, this description reproduces its classical predictions.

A central result concerns black hole. When limits on temporal and frequency resolution are imposed, a Schwarzschild boundary appears naturally. It follows from geometric and operational constraints within the time phase framework. Beyond this boundary, temporal distinctions collapse. No observer can extract further relational information. As a result, accessible information is confined to a finite shell surrounding the horizon. This conclusion does not rely on Einstein's field equations.

The framework also clarifies the role of the graviton. In the completed theory, the graviton occupies a special position in the time phase plane. Its intrinsic frequency vanishes. Because the trapping mechanism at a black hole horizon act through modifications of temporal phase or frequency, the graviton is not confined in the same way as matter or radiation. It remains able to propagate outward. The external gravitational field, therefore, persists after horizon formation. This resolves a long-standing tension between general relativity and quantum field theory.

Further consequences follow from the relational structure of time itself. Temporal resolution is finite and depends on interaction. Two photons with identical frequency cannot occupy the same spacetime phase point. This restriction leads to an effective interaction between photons that depends on their relative phase orientation. When beams are phase aligned, the interaction is enhanced. This provides a concrete experimental signature of the framework and a direct probe of the underlying time phase geometry.

Taken together, these results point to a common origin of spacetime structure, gravity, and quantum behavior. In the present approach, they arise from a single relational and geometric principle. Known physical results are recovered in appropriate limits. At the same time, the framework avoids several conceptual tensions present in standard formulations. It also makes falsifiable predictions, which is essential for further progress.

Several directions remain open. Gravitational field equations may be formulated directly in the time phase language. The coupling of quantum fields to this structure remains to be explored. Many body systems and entangled states in curved time phase backgrounds also deserve further study. These extensions may shed additional light on the role of information, horizons, and relational structure in fundamental physics.

Acknowledgment: We are thankful to the editors and anonymous reviewers for their constructive suggestions.

Author Contributions: Conceptualization, R.S.; methodology, R.S.; software, R.S.; validation, S.A.M.; writing original draft preparation, R.S.; writing, review, and editing, S.A.M. All authors agreed to the manuscript.

Declaration of Conflicting Interest: The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding: The author(s) received no financial support for the research, authorship, and/or publication of this article.

Data Availability Statement: Data that supports the findings of this study are available on request from the corresponding author.

Plagiarism Statement: This article was scanned by the plagiarism program. No plagiarism was detected.

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