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A Novel Decision-Making Algorithm Enhancing Career Planning and Employment Guidance on Circular T-Spherical Fuzzy CRITIC WASPAS Method

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ABSTRACT

This study aims to improve decision-making in vocational training, career planning, and employment guidance by using the algorithm of the multi-attributes group decision-making (MAGDM) approach based on the circular T-spherical fuzzy (CTSF) framework. The criteria importance through the intercriteria correlation (CRITIC) method is used to assign importance to evaluation attributes based on their variation and relationship with each other, while the weighted aggregated sum product assessment (WASPAS) method is applied to rank different strategic alternatives and identify the most effective solution. The CTSF set (CTSFS) is an enhanced version of the traditional fuzzy extension that incorporates the radius and circular behavior of its grades. This enhancement enables CTSF to convey vagueness, interdependent relationships, and ambiguity more precisely than typical fuzzy sets (FS) and their extensions. We created CTSF weighted (CTSFWA) and CTSF weighted geometric (CTSFWG) to accomplish this, combining CTSF value data into a singleton set. A few fundamental properties are also examined, including idempotency, boundedness, and monotonicity. The model is experimented with the help of a hypothetical case study that is aimed at enhancing employment outcomes, and five alternatives are assessed in comparison to five important attributes. The findings indicate that the fifth option, i.e., "Digital Career Guidance Platforms," is the most ideal strategy of career planning and employment guidance. Sensitivity and comparative analyses are also conducted to confirm the reliability and stability of the proposed approach. The findings indicate that the model is practical, reliable, and useful for supporting strategic decisions in workforce development and employment planning.

Keywords: Circular T-spherical fuzzy set, CRITIC-WASPAS model, Career planning and employment guidance, multi-attributes group decision-making problem

(MSC2020) Codes: 03E72, 90B50, 91B65, 90B50

1 | INTRODUCTION

Many young people and job seekers today do not get enough good guidance when choosing career paths or planning their future work life. Schools and training centers often have limited career counseling services, so students are not well informed about what jobs are out there and what skills they need to succeed. This leaves many people confused and unable to make good career decisions that match their interests and abilities. Vocational training programs also experience challenges. Students often learn skills that are either obsolete or not relevant to actual work, and in most instances, training fails to match the needs of the job market. The employers sometimes lament that the graduates lack the hands-on skills demanded by employers and that they do not consult the industry greatly enough when making decisions on what to include in the training programs. In the absence of closer connections between training and industry demand, many graduates cannot secure decent employment despite completing courses, and therefore have problems with unemployment or underemployment.

FS theory is particularly helpful in situations involving trade-offs, as decision-making is rarely simple. The MAGDM approach provides useful frameworks for assessing these kinds of ambiguous and occasionally contradictory attributes. Researchers have made tremendous progress in this area in recent years, presenting fresh models and methods that enhance decision-making in the face of uncertainty. One that we could mention, [1] discussed an educational and career guidance model using fuzzy logic concepts. The fuzzy control algorithm was introduced by [2] in college students' personal career planning and entrepreneurship. A method for evaluating college students' career suitability based on a fuzzy clustering algorithm, primarily suggested by [3]. However, [4] elaborated on the development of a career planning assistance platform for college students combining fuzzy logic and deep learning.

1.1. Literature Study

The technique of selecting the optimal option from a range of options by assessing their possible outcomes is known as multi-attribute decision-making. By incorporating a group of decision-makers (Ɗ) who jointly assess options across several qualities, MAGDM expands upon MADM. It works well in situations involving a variety of parties and attributes and is helpful in complex evaluations, such as consumer value in purchasing behavior assessments. introduced artificial intelligence-based three-way decision-making introduced by [5], while [6] adopted hybrid models for decision-making based on rough Pythagorean fuzzy bipolar soft information, [7] developed Dombi aggregation operators based on complex q-rung orthopair fuzzy sets and their application to MADM, and [8] investigated the TOPSIS approach for MAGDM.

In this manuscript, we extend the CRITIC method and WASPAS method to the CTSF architecture. For thorough assessments, [9] suggested objective weights in multiple criteria problems. The weighted aggregated sum product assessment (WASPAS) method developed by [10], which combines the additive and the multiplicative models. It is a widely used approach to the MAGDM problem that takes into account both quantitative and qualitative aspects. Table 1 provides an overview of the WASPAS method.

Table 1. Previous work for the WASPAS method

References	Established	Worked on	Case Study
Keshavarz-Ghorabaee et al. [11]	(2020)	WASPAS	Green construction supplier evaluation
Ghoushchi et al. [12]	(2021)	SWARA-WASPAS	Landfill site selection for medical waste
Al-Barakati et al. [13]	(2022)	WASPAS	Renewable energy sources
Alrasheedi et al. [14]	(2023)	SWARA-WASPAS	Renewable energy sources selection
Anjum et al. [15]	(2024)	CRITIC-WASPAS	Cooperative intelligent transportation system scenarios
Introduced paper	(2025)	CRITIC-WASPAS	Career planning and employment guidance

However, a major drawback of typical WASPAS frameworks is that they are not made to handle data represented via CTSFS, as Table 1 illustrates. Given that CTSFS provides a more sophisticated method of representing uncertainty, this is a significant flaw.

1.2. Research Gap

1. Multi-attribute group decision-making (MAGDM) under uncertainty is gaining popularity, although there are still a number of important gaps in the research:
2. MAGDM issues have made extensive use of fuzzy, intuitionistic, image, and spherical fuzzy environments. There hasn't been much focus on the suggested CTSF framework, which provides improved capabilities in modeling hesitation and directional ambiguity.
3. In the area of CTSFS, few studies have combined several decision-making strategies. The breadth of precise and thorough decision assistance has been limited since no previous work has integrated the WASPAS approach for alternative ranking under CTSFS with the CRITIC method for objective weighing.
4. The majority of MAGDM models in fuzzy environments increase bias by significantly relying on judgment to provide weights to characteristics. Research using objective weighting techniques, such as CRITIC, in the CTSFS framework is lacking.
5. Despite the theoretical benefits of CTSFS, there is no empirical application of CTSFS-based decision-making models to real-world issues, including career planning and employment guidance.

1.3. Research Questions

The study aims to address the following important research concerns in light of the gaps found:

1. How can the MAGDM approach be enhanced in the CTSFS environment by combining the advantages of the proposed methods?

2. In CTSFS, can an objective, correlation-based weighting method (CRITIC) improve the dependability of decision-making results under high uncertainty?
3. In comparison to current fuzzy MAGDM models, how does the proposed model perform in an actual career planning decision-making scenario?
4. In terms of conveying hesitation and uncertainty in group decision-making, what are the advantages and disadvantages of employing CTSFS over conventional fuzzy environments?
5. This study suggests a unique extension of the CTSFS-based CRITIC-WASPAS model and describes an algorithm for using it in career planning and employment guidance decision-making situations in order to investigate these topics.

1.4. Fuzzy Environments Extension

There are several instances of uncertainty and indeterminacy in the real world. Crisp set theory and traditional binary logic are unable to model and make decisions effectively under these circumstances. The FS theory was introduced by [16] in 1965 to tackle this issue; it was the first theory that allowed for fundamental set membership to have values between 0 and 1. Because this theory allowed for the assessment of ambiguous data in degrees instead of a binary yes/no, the use of set theory in control systems and artificial intelligence has grown considerably. Even with its successes, classical FS theory continues to uncomfortably combine the ideas of membership and non-membership into one scalar value, along with the conflicting presence and absence of evidence. In 1986, [17] developed the Intuitionistic FS (IFS), as long as their degrees sum does not exceed one, to help address the issue. As a result, an IFS can offer a more thorough and complex depiction of uncertainty, especially in circumstances requiring ambiguity and decision-making with contradicting data.

Picture FS (PFS) is especially helpful in decision-making situations where stakeholders may express partial hesitation or indifference rather than complete agreement or disagreement when the abstinence component of IFS is added. A more comprehensive framework for simulating actual human opinions is provided by PFS. In 2013, [18] expanded the notion of IFS by adding the degree of abstinence. Every element in a PFS has three distinct membership degrees. Later on, [19] in 2019, developed the concept of spherical FS (SFS) and T-spherical FS (TSFS) by extending the domain of PFS.

In order to overcome this constraint, Atanassov [20] presented circular IFS (CIFS) in 2020 as a novel extension of IFS. However, CIFS has a relatively narrow domain. To overcome this limitation, [21] created circular Pythagorean FS (CPyFS), which expands the domain of CIFS under the condition that the radius falls within a unit interval and the sum of squares of membership and non-membership degrees is less than or equal to 1. Alternative ideas in FS theory investigated by [22], such as circular fermatean FS (CFFS). While [23] created the idea of circular q-rung orthopair FS (Cq-ROFS), which broadens the scope of CFFS, while requiring that the radius must belong to the unit interval and the sum of their powers of degrees is less than or equal to 1. The three membership degrees are still present in the CPFS instance studied by [24] in 2025, but they are regarded as a circular area with their degrees instead of as fixed values. This circular depiction allows for the modeling of variability, hesitation, or uncertainty radius around each assessment point. The concept of CSFS was established by [25] in 2025 by expanding the CPFS domain. From a conceptual standpoint, it indicates that the actual membership denoted by ($\mathcal{M}$)abstinence denoted by ($\mathcal{B}$), non-membership denoted by ($\mathcal{N}$) degrees, and radius denoted by ($\mathcal{R}$) may fluctuate within a range that is delineated by the circle. The main distinctions and characteristics of CIFS, CPyFS, Cq-ROFS, CPFS, and CSFS are compiled in Table 2:

Table 2. FS theory generalization

Environments	Functions	Conditions	Aspect Benefits
FS	$\mathcal{M}$	$0 \leq \mathcal{M} \leq 1$	Permits partial membership
IFS	$\mathcal{M}, \mathcal{N}$	$0 \leq \mathcal{M} + \mathcal{N} \leq 1$	Includes an explicit statement of non-membership
PFS	$\mathcal{M}, \mathcal{B}, \mathcal{N}$	$0 \leq \mathcal{M} + \mathcal{B} + \mathcal{N} \leq 1$	Established abstinence
CIFS	$\mathcal{M}, \mathcal{N}$, with radius $\mathcal{R}$	$0 \leq \mathcal{M} + \mathcal{N} \leq 1$ (center)	Circular region around $(\mathcal{M}, \mathcal{N})$
CPFS	$\mathcal{M}, \mathcal{B}, \mathcal{N}$, with radius $\mathcal{R}$	$0 \leq \mathcal{M} + \mathcal{B} + \mathcal{N} \leq 1$ (center)	Circular region around $(\mathcal{M}, \mathcal{B}, \mathcal{N})$
CSFS	$\mathcal{M}, \mathcal{B}, \mathcal{N}$, with radius $\mathcal{R}$	$0 \leq \mathcal{M}^2 + \mathcal{B}^2 + \mathcal{N}^2 \leq 1$ (center)	Circular region around $(\mathcal{M}, \mathcal{B}, \mathcal{N})$
CTSFS	$\mathcal{M}, \mathcal{B}, \mathcal{N}$, with radius $\mathcal{R}$	$0 \leq \mathcal{M}^t + \mathcal{B}^t + \mathcal{N}^3 \leq 1$ (center)	Circular region around $(\mathcal{M}, \mathcal{B}, \mathcal{N})$

1.5. Novelty and Contribution

In the context of consumer value evaluation, this study significantly advances the subject of making MAGDM under uncertainty in a number of ways:

1. The study creatively integrates the WASPAS and CRITIC approaches into the CTSF setting. Membership, abstinence, non-membership, and radius-based hesitation are all included in this paradigm for the first time, enabling a more complex depiction of uncertainty.
2. By enabling a dual-perspective review, CTSFS enhances the precision and adaptability of decision outcomes in MAGDM issues.
3. Businesses looking to align their strategy can benefit from the model's application to the case study on career planning.

4. There has not been much research done on the integration of CTSFS in MAGDM. This study closes a significant vacuum in the literature and creates opportunities for additional applications in challenging decision-making situations in a variety of fields, including marketing, strategic planning, and healthcare.

1.6. Paper Structure

The article is organized as follows: the introduction is given in Section 1, and the preliminary information is given in Section 2. The definition of CTSFS, fundamental characteristics, and operators needed for this manuscript are covered in Section 3. We expand the CRITIC-WASPAS model within the CTSFS framework in Section 4. Section 5 applies the suggested model to the case study for the demonstration, and Section 6 uses sensitivity analysis to examine the model's performance. In order to demonstrate the benefits of the initiated work, Section 7 contrasts our methodology with previous research. Section 8 concludes with recommendations for further research, limitations, and conclusions.

2 | PRELIMINARIES

Definition 1 [25]: A circular spherical fuzzy set A on the universe set Ṳ is of the form

$$A = \{(\mathcal{M}(\chi), \mathcal{B}(\chi), \mathcal{N}(\chi);\ \mathcal{R}(\chi)) | \chi \in Ṳ\}$$

with the condition: $0 \leq (\mathcal{M}(\chi))^2 + (\mathcal{B}(\chi))^2 + (\mathcal{N}(\chi))^2 \leq 1$ of $\chi \in$ Ṳ, where $\mathcal{M}(\chi) \in [0,1]$ is the membership degree, $\mathcal{B}(\chi) \in [0,1]$ is the abstinence degree, $\mathcal{N}(\chi) \in [0,1]$ is the non-membership degree, and $\mathcal{R}(\chi) \in [0,1]$ is the radius of the circle. Additionally, $\mathcal{F}(\chi) = \sqrt{1 - (\mathcal{M}(\chi))^2 - (\mathcal{B}(\chi))^2 - (\mathcal{N}(\chi))^2}$ is said to be the refusal degree, and the CSF values (CSFVs) are denoted by $A_J = (\mathcal{M}_J, \mathcal{B}_J, \mathcal{N}_J, \mathcal{R}_J)$; $(J = 1,2, \ldots, ŋ)$.

Definition 2 [25]: Any two CSFVs $A_1 = (\mathcal{M}_1, \mathcal{B}_1, \mathcal{N}_1, \mathcal{R}_1)$ and $A_2 = (\mathcal{M}_2, \mathcal{B}_2, \mathcal{N}_2, \mathcal{R}_2)$ will satisfied the following results:

$$A_1 \oplus A_2 = \left(\sqrt{\mathcal{M}^2{}_1 + \mathcal{M}^2{}_2 - \mathcal{M}^2{}_1 \mathcal{M}^2{}_2}, \mathcal{B}_1\mathcal{B}_2, \mathcal{N}_1\mathcal{N}_2; \mathcal{R}_1\mathcal{R}_2\right)$$

$$A_1 \otimes A_2 = \left(\mathcal{M}_1\mathcal{M}_2, \mathcal{B}_1\mathcal{B}_2, \sqrt{\mathcal{N}^2{}_1 + \mathcal{N}^2{}_2 - \mathcal{N}^2{}_1 \mathcal{N}^2{}_2}; \mathcal{R}_1\mathcal{R}_2\right)$$

$$ƛA_1 = \left(\sqrt{1 - (1 - \mathcal{M}^2{}_1)^{ƛ}}, \mathcal{B}_1{}^{ƛ}, \mathcal{N}_1{}^{ƛ}; \mathcal{R}_1{}^{ƛ}\right)$$

$$A_1{}^{ƛ} = \left(\mathcal{M}_1{}^{ƛ}, \mathcal{B}_1{}^{ƛ}, \sqrt{1 - (1 - \mathcal{N}^2{}_1)^{ƛ}}; \mathcal{R}_1{}^{ƛ}\right)$$

$$(A_1)^c = (\mathcal{N}_1, \mathcal{B}_1, \mathcal{M}_1, \mathcal{R}_1)$$

Remark 1: Since each SFS A is of the form of CSFS, when we consider $\mathcal{R}(\chi) = 0$

$$A = \{(\mathcal{M}(\chi), \mathcal{B}(\chi), \mathcal{N}(\chi); 0) | \chi \in Ṳ\}$$

This means that any SFS can be considered as a CSFS. Therefore, the notion of CSFS is a generalization of the notion of SFS. But CSFS has some limitations, e.g., if we consider $\mathcal{M}(\chi) = 0.8$, $\mathcal{B}(\chi) = 0.4$, $\mathcal{N}(\chi) = 0.6$. Then, according to the definition, $(0.8)^2 + (0.4)^2 + (0.6)^2 = 1.16 > 1$, it is not a CSFS.

3 | CIRCULAR T-SPHERICAL FUZZY FRAMEWORK

This section discusses the need for CTSFS and how to generalize fuzzy extension. A circular association between the three degrees that is membership, abstinence, and non-membership is more suitable in some circumstances, though. According to remark 1, we observed that CSFS, as we recall from Definition 1, has limitations and boundedness. This consequence introduces and presents the CTSFS to account for this. In this section, we also explain their accuracy function, scoring function, and basic algebraic laws as required in this manuscript.

Definition 3: A circular T-spherical fuzzy set A on the universe set $\ddot{\mathrm{U}}$ is of the form

$$A = \{(\mathcal{M}(\chi), \mathcal{B}(\chi), \mathcal{N}(\chi);\ \mathcal{R}(\chi)) | \chi \in \ddot{\mathrm{U}}\}$$

with the condition: $0 \le (\mathcal{M}(\chi))^t + (\mathcal{B}(\chi))^t + (\mathcal{N}(\chi))^t \le 1$ for some $t \in \mathbb{Z}^+$ for all $\chi \in \ddot{\mathrm{U}}$, where $\mathcal{M}(\chi) \in [0,1]$ is the membership degree, $\mathcal{B}(\chi) \in [0,1]$ is the abstinence degree, $\mathcal{N}(\chi) \in [0,1]$ is the non-membership degree. The radius of the circle around each degree is represented by $\mathcal{R}(\chi.) \in [0,1]$. Additionally, $\mathcal{F}(\chi) = \sqrt[t]{1 - (\mathcal{M}(\chi))^t - (\mathcal{B}(\chi))^t - (\mathcal{N}(\chi))^t}$ is called the refusal degree, and the CTSF values (CTSFVs) are denoted by $A_\mathrm{J} = (\mathcal{M}_\mathrm{J}, \mathcal{B}_\mathrm{J}, \mathcal{N}_\mathrm{J}, \mathcal{R}_\mathrm{J})$; $(\mathrm{J} = 1,2, \dots, \mathfrak{y})$.

Example 1: Assume that the universe set has three elements, that is, $\ddot{\mathrm{U}} = \{\chi_1, \chi_2, \chi_3\}$. Then, the given set $A = \{(\chi_1, 0.9, 0.5, 0.3; 0.7), (\chi_2, 0.7, 0.5, 0.6; 0.8), (\chi_3, 0.8, 0.7, 0.4; 0.6)\}$ is CTSFS, for $t = 3$.

Definition 4: Any two CTSFVs $A_1 = (\mathcal{M}_1, \mathcal{B}_1, \mathcal{N}_1, \mathcal{R}_1)$ and $A_2 = (\mathcal{M}_2, \mathcal{B}_2, \mathcal{N}_2, \mathcal{R}_2)$ will satisfy the following results:

$$A_1 \oplus A_2 = \left(\sqrt[t]{\mathcal{M}^t{}_1 + \mathcal{M}^t{}_2 - \mathcal{M}^t{}_1\mathcal{M}^t{}_2}, \mathcal{B}_1\mathcal{B}_2, \mathcal{N}_1\mathcal{N}_2; \mathcal{R}_1\mathcal{R}_2\right) \quad (1)$$

$$A_1 \otimes A_2 = \left(\mathcal{M}_1\mathcal{M}_2, \mathcal{B}_1\mathcal{B}_2, \sqrt[t]{\mathcal{N}^t{}_1 + \mathcal{N}^t{}_2 - \mathcal{N}^t{}_1\mathcal{N}^t{}_2}; \mathcal{R}_1\mathcal{R}_2\right) \quad (2)$$

$$\lambda A_1 = \left(\sqrt[t]{1 - (1 - \mathcal{M}^t{}_1)^{\lambda}}, \mathcal{B}_1{}^{\lambda}, \mathcal{N}_1{}^{\lambda}; \mathcal{R}_1{}^{\lambda}\right) \quad (3)$$

$$A_1{}^{\lambda} = \left(\mathcal{M}_1{}^{\lambda}, \mathcal{B}_1{}^{\lambda}, \sqrt[t]{1 - (1 - \mathcal{N}^t{}_1)^{\lambda}}; \mathcal{R}_1{}^{\lambda}\right) \quad (4)$$

$$(A_1)^c = (\mathcal{N}_1, \mathcal{B}_1, \mathcal{M}_1, \mathcal{R}_1) \quad (5)$$

Definition 5: Let $A = (\mathcal{M}, \mathcal{B}, \mathcal{N}, \mathcal{R})$ be the CTSFV. The score function $\d{S}$ presented in Eq. (6) converts a fuzzy evaluation into a single numerical value to represent its overall preference level. A higher score indicates a more desirable alternative. The accuracy function $\d{A}$ presented in Eq. (7) measures the reliability of a fuzzy evaluation and is used to distinguish alternatives with equal or similar score values:

$$\d{S}(A)=\frac{1}{4}\left(\mathcal{M}-\mathcal{B}-\mathcal{N}+\sqrt{2\mathcal{R}}(2\beta-1);\alpha\in[0,1]\right)\in[-1,1] \tag{6}$$

$$\d{A}(A)=(\mathcal{M}+\mathcal{B}+\mathcal{N})\in[0,1] \tag{7}$$

Accordingly, decision-makers adjust the influence of the radius on the range of the score function through the parameter β. When β is 0, the radius is completely excluded from the decision-making process. As β approaches 1, the radius increasingly affects the score output.

Definition 6: The relation between any two CTSFS is as follows:

- $\d{S}(A_1)>\d{S}(A_2)\Rightarrow A_1>A_2$ and $\d{S}(A_1)=\d{S}(A_2)\Rightarrow A_1=A_2$
- $\d{A}(A_1)>\d{A}(A_2)\Rightarrow A_1>A_2$ and $\d{A}(A_1)=\d{A}(A_2)\Rightarrow A_1=A_2$

3.1. A Novel Linguistic Term Structure in the CTSFS Environment for Case Study

In this study, the linguistic assessments provided by experts, such as outstanding, exceptional, substantial, significant, considerable, reasonable, acceptable, minimal, and negligible, are represented using CTSFS. Each linguistic term is modeled through a combination of each degree along with an associated radius phase. This representation allows us to capture not only the strength of the experts' opinions but also the uncertainty and periodic fluctuations inherent in real-world decision-making. For the alternative evaluation, linguistic terms are suggested in CTSFS, stored in Table 3.

Table 3. Linguistic keywords inside the CTSFS framework

Linguistic term	$\mathcal{M}$	$\mathcal{B}$	$\mathcal{N}$	$\mathcal{R}$
Outstanding (OST)	1.00	0.00	0.00	0.33
Exceptional (EXT)	0.90	0.10	0.20	0.40
Substantial (SBT)	0.80	0.20	0.30	0.43
Significant (SGT)	0.70	0.30	0.40	0.47
Considerable (CON)	0.60	0.40	0.50	0.50
Reasonable (RSB)	0.50	0.50	0.40	0.47
Acceptable (APT)	0.30	0.35	0.60	0.42
Minimal (MIN)	0.10	0.20	0.80	0.37
Negligible (NEG)	0.00	0.00	1.00	0.33

3.2. Circular T-Spherical Fuzzy Weighted Averaging Operator

The CTSFS framework introduces the weighted averaging operator to aggregate several CTSFS variables according to their relevance weights. The subsection provides the CTSF weighted averaging (CTSFWA) operator with numerical examples to illustrate its use. Furthermore, this operator preserves important mathematical properties, including idempotency, boundedness, and monotonicity.

Definition 7: The CTSFWA for CTSFVs $A_{\mathrm{J}}(\mathrm{J}=1,2,3,\ldots,\eta)$ is of the form:

$$CTSFWA(A_1,A_2,\ldots,A_{\mathrm{J}})=\sum_{\mathrm{J}=1}^{\eta}\omega_{\mathrm{J}}A_{\mathrm{J}} \tag{8}$$

where the weights satisfied the constraint $\omega_{\mathrm{J}}\geq 0$ and $\sum\omega_{\mathrm{J}}=1$.

Theorem 1: Aggregating CTSFVs A_{J} using Definition 7 and its functional characteristics yields a CTSFV, expressed as:

$$CTSFWA(A_1,A_2,\ldots,A_{\mathrm{J}})=\left(\sqrt[t]{1-\prod_{\mathrm{J}=1}^{\eta}\left(1-\mathcal{M}^{t}{}_{\mathrm{J}}\right)^{\omega_{\mathrm{J}}}},\prod_{\mathrm{J}=1}^{\eta}\mathcal{B}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}},\prod_{\mathrm{J}=1}^{\eta}\mathcal{N}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}},\prod_{\mathrm{J}=1}^{\eta}\mathcal{R}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}}\right) \tag{9}$$

Proof: We must use the mathematical induction approach to verify the outcome, which is displayed as:

For $\eta=2$

$$\omega_1A_1=\left(\sqrt[t]{1-(1-\mathcal{M}^{t}{}_{1})^{\omega_1}},\mathcal{B}_1{}^{\omega_1},\mathcal{N}_1{}^{\omega_1},\mathcal{R}_1{}^{\omega_1}\right),\ \omega_2A_2=\left(\sqrt[t]{1-(1-\mathcal{M}^{t}{}_{2})^{\omega_2}},\mathcal{B}_2{}^{\omega_2},\mathcal{N}_2{}^{\omega_2},\mathcal{R}_2{}^{\omega_2}\right)$$

$$\omega_1A_1\oplus\omega_2A_2=\left(\sqrt[t]{1-(1-\mathcal{M}^{t}{}_{1})^{\omega_1}},\mathcal{B}_1{}^{\omega_1},\mathcal{N}_1{}^{\omega_1},\mathcal{R}_1{}^{\omega_1}\right)\oplus\left(\sqrt[t]{1-(1-\mathcal{M}^{t}{}_{2})^{\omega_2}},\mathcal{B}_2{}^{\omega_2},\mathcal{N}_2{}^{\omega_2},\mathcal{R}_2{}^{\omega_2}\right)$$

$$=\left(\begin{array}{c}\sqrt[t]{\left(\sqrt[t]{1-(1-\mathcal{M}^{t}{}_{1})^{\omega_1}}\right)^{t}+\left(\sqrt[t]{1-(1-\mathcal{M}^{t}{}_{2})^{\omega_2}}\right)^{t}-\left(\sqrt[t]{1-(1-\mathcal{M}^{t}{}_{1})^{\omega_1}}\right)^{t}\left(\sqrt[t]{1-(1-\mathcal{M}^{t}{}_{2})^{\omega_2}}\right)^{t}},\\ \mathcal{B}_1{}^{\omega_1}\mathcal{B}_2{}^{\omega_2},\mathcal{N}_1{}^{\omega_1}\mathcal{N}_2{}^{\omega_2},\mathcal{R}_1{}^{\omega_1}\mathcal{R}_2{}^{\omega_2}\end{array}\right)$$

$$=\left(\sqrt[t]{1-(1-\mathcal{M}^{t}{}_{1})^{\omega_1}(1-\mathcal{M}^{t}{}_{2})^{\omega_2}},\mathcal{B}_1{}^{\omega_1}\mathcal{B}_2{}^{\omega_2},\mathcal{N}_1{}^{\omega_1}\mathcal{N}_2{}^{\omega_2},\mathcal{R}_1{}^{\omega_1}\mathcal{R}_2{}^{\omega_2}\right)$$

$$=\left(\sqrt[t]{1-\prod_{\mathrm{J}=1}^{2}\left(1-\mathcal{M}^{t}{}_{\mathrm{J}}\right)^{\omega_{\mathrm{J}}}},\prod_{\mathrm{J}=1}^{2}\mathcal{B}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}},\prod_{\mathrm{J}=1}^{2}\mathcal{N}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}},\prod_{\mathrm{J}=1}^{2}\mathcal{R}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}}\right)$$

For $\eta=l$

$$CTSFWA(A_1,A_2,A_3,\ldots,A_{\mathrm{J}})=\left(\sqrt[t]{1-\prod_{\mathrm{J}=1}^{l}\left(1-\mathcal{M}^{t}{}_{\mathrm{J}}\right)^{\omega_{\mathrm{J}}}},\prod_{\mathrm{J}=1}^{l}\mathcal{B}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}},\prod_{\mathrm{J}=1}^{l}\mathcal{N}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}},\prod_{\mathrm{J}=1}^{l}\mathcal{R}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}}\right)$$

For $\eta=l+1$

$$CTSFWA(A_1,A_2,A_3,\ldots,A_{\mathrm{J}},A_{\mathrm{J}+1})=\sum_{\mathrm{J}=1}^{l+1}\omega_{\mathrm{J}}A_{\mathrm{J}}=\sum_{\mathrm{J}=1}^{l}\omega_{\mathrm{J}}A_{\mathrm{J}}\oplus\omega_{\mathrm{J}+1}A_{\mathrm{J}+1}$$

$$=\left(\begin{array}{c}\left(\sqrt[t]{1-\prod_{\mathrm{J}=1}^{l}\left(1-\mathcal{M}^{t}{}_{\mathrm{J}}\right)^{\omega_{\mathrm{J}}}},\prod_{\mathrm{J}=1}^{l}\mathcal{B}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}},\prod_{\mathrm{J}=1}^{l}\mathcal{N}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}},\prod_{\mathrm{J}=1}^{l}\mathcal{R}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}}\right)\oplus\\ \left(\sqrt[t]{1-(1-\mathcal{M}^{t}{}_{l+1})^{\omega_{l+1}}},\mathcal{B}_{l+1}{}^{\omega_{l+1}},\mathcal{N}_{l+1}{}^{\omega_{l+1}},\mathcal{R}_{l+1}{}^{\omega_{l+1}}\right)\end{array}\right)$$

For $\eta=2$

$$= \left(\sqrt[t]{1 - \prod_{\mathrm{J}=1}^{l+1} \left(1 - \mathcal{M}^{t}{}_{\mathrm{J}}\right)^{\omega_{\mathrm{J}}}}, \prod_{\mathrm{J}=1}^{l+1} \mathcal{B}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}}, \prod_{\mathrm{J}=1}^{l+1} \mathcal{N}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}}, \prod_{\mathrm{J}=1}^{l+1} \mathcal{R}_{\mathrm{J}}{}^{\omega_{\mathrm{J}}} \right)$$

Hence, the result holds for $\eta = l + 1$.

Example 2: Assume that the universe set has two elements, that is, $\ddot{\mathrm{U}} = \{\chi_1, \chi_2\}$. Then, demonstrate the CTSFWA operator, where CTSFS is characterized as $A = \{(0.80,0.40,0.60; 0.50), (0.75,0.45,0.55; 0.65)\}$ with the corresponding weighted vector $\omega = (0.45,0.55)$ for $t = 3$.

Solution: $CTSFWA(\chi_1, \chi_2) = \begin{pmatrix} \sqrt[3]{1 - \left(\left(1 - 0.80^3\right)^{0.45}\right)\left(\left(1 - 0.75^3\right)^{0.55}\right)}, \\ (0.40)^{0.45}(0.45)^{0.55}, \\ (0.60)^{0.45}(0.55)^{0.55}, \\ (0.50)^{0.45}(0.65)^{0.55} \end{pmatrix}$

$$= (0.78,0.43,0.57,0.58)$$

Therefore, it is clear that our suggested operator, as stated in Eq. (9), produces a CTSFV.

Remarks 2: The CTSFWA operator properties in the framework of CTSFS are described as follows:

Idempotency: Assume the collection of CTSFVs $\chi_{\mathrm{J}} = \left(\mathcal{M}_{\chi\mathrm{J}}, \mathcal{B}_{\chi\mathrm{J}}, \mathcal{N}_{\chi\mathrm{J}}, \mathcal{R}_{\chi\mathrm{J}}\right)$ such that for all $\mathrm{J} = 1,2, \dots, \eta$. Then, the idempotency of the CTSFWA operator is:

$$\mathrm{CTSFWA}\left(\chi_1, \chi_2, \dots, \chi_\eta\right) = \chi$$

Boundedness: Assume the collection of CTSFVs $\chi_{\mathrm{J}} = \left(\mathcal{M}_{\chi\mathrm{J}}, \mathcal{B}_{\chi\mathrm{J}}, \mathcal{N}_{\chi\mathrm{J}}, \mathcal{R}_{\chi\mathrm{J}}\right)$ such that for all $\mathrm{J} = 1,2, \dots, \eta$. If $\chi^- = \min_{\mathrm{J}}\left(\chi_{\mathrm{J}}\right)$ and $\chi^+ = \max_{\mathrm{J}}\left(\chi_{\mathrm{J}}\right)$ such that for all $\mathrm{J} = 1,2, \dots, \eta$. Then, the boundedness of the CTSFWA operator is:

$$\chi^- \leq \mathrm{CTSFWA}\left(\chi_1, \chi_2, \dots, \chi_\eta\right) \leq \chi^+$$

Monotonicity: Assume the two collections of CTSFVs $\chi_{\mathrm{J}} = \left(\mathcal{M}_{\chi\mathrm{J}}, \mathcal{B}_{\chi\mathrm{J}}, \mathcal{N}_{\chi\mathrm{J}}, \mathcal{R}_{\chi\mathrm{J}}\right)$ and $\chi'_{\mathrm{J}} = \left(\mathcal{M}'_{\chi\mathrm{J}}, \mathcal{B}'_{\chi\mathrm{J}}, \mathcal{N}'_{\chi\mathrm{J}}, \mathcal{R}'_{\chi\mathrm{J}}\right)$ such that for all $\mathrm{J} = 1,2, \dots, \eta$. If $\chi_{\mathrm{J}} \leq \chi'_{\mathrm{J}}$ then, the CTSFWA operator exhibits monotonicity:

$$\mathrm{CTSFWA}\left(\chi_1, \chi_2, \dots, \chi_\eta\right) \leq \mathrm{CTSFWA}\left(\chi_1', \chi_2', \dots, \chi_\eta'\right)$$

3.3. Circular T-Spherical Fuzzy Weighted Geometric Operator

The CTSFS framework introduces the weighted geometric operator to aggregate several CTSFS variables according to their relevance weights. The subsection provides the CTSF weighted geometric (CTSFWG) operator with numerical examples to illustrate its use. Furthermore, this operator preserves important mathematical properties, including idempotency, boundedness, and monotonicity.

Definition 8: The CTSFWG operator for CTSFVs $A_{\mathrm{J}} (\mathrm{J} = 1,2,3, \dots, \eta)$ is of the form

$$CTSFWG\left(A_1, A_2, \dots, A_\mathrm{J}\right) = \prod_{\mathrm{J}=1}^{\mathfrak{y}} A_\mathrm{J}^{\omega_\mathrm{J}} \tag{10}$$

where the weights satisfied the constraint $\omega_\mathrm{J} \geq 0$ and $\sum \omega_\mathrm{J} = 1$.

Theorem 2: Aggregating CTSFVs A_J using the Definition 8 and their functional characteristics yields a CTSFV, expressed as:

$$CTSFWG\left(A_1, A_2, \dots, A_\mathrm{J}\right) = \left(\prod_{\mathrm{J}=1}^{\mathfrak{y}} \mathcal{M}_\mathrm{J}^{\omega_\mathrm{J}}, \prod_{\mathrm{J}=1}^{\mathfrak{y}} \mathcal{B}_\mathrm{J}^{\omega_\mathrm{J}}, \sqrt[t]{1 - \prod_{\mathrm{J}=1}^{\mathfrak{y}} \left(1 - \mathcal{N}^t{}_\mathrm{J}\right)^{\omega_\mathrm{J}}}, \prod_{\mathrm{J}=1}^{\mathfrak{y}} \mathcal{R}_\mathrm{J}^{\omega_\mathrm{J}}\right) \tag{11}$$

Proof: We must use the mathematical induction approach to verify the outcome, which is displayed as:

For $\mathfrak{y} = 2$,

$$\omega_1 A_1 = \left(\boldsymbol{\mathcal{M}}_1^{\omega_1}, \mathcal{B}_1^{\omega_1}, \sqrt[t]{1 - (1 - \mathcal{N}^t{}_1)^{\omega_1}}, \boldsymbol{\mathcal{R}}_1^{\omega_1}\right), \omega_2 A_2 =$$
$$\left(\boldsymbol{\mathcal{M}}_2^{\omega_2}, \mathcal{B}_2^{\omega_2}, \sqrt[t]{1 - (1 - \mathcal{N}^t{}_2)^{\omega_2}}, \boldsymbol{\mathcal{R}}_2^{\omega_2}\right)$$

$$\omega_1 A_1 \otimes \omega_2 A_2 = \left(\boldsymbol{\mathcal{M}}_1^{\omega_1}, \mathcal{B}_1^{\omega_1}, \sqrt[t]{1 - (1 - \mathcal{N}^t{}_1)^{\omega_1}}, \boldsymbol{\mathcal{R}}_1^{\omega_1}\right) \otimes \left(\boldsymbol{\mathcal{M}}_2^{\omega_2}, \mathcal{B}_2^{\omega_2}, \sqrt[t]{1 - (1 - \mathcal{N}^t{}_2)^{\omega_2}}, \boldsymbol{\mathcal{R}}_2^{\omega_2}\right)$$

$$= \left(\begin{array}{c} \boldsymbol{\mathcal{M}}_1^{\omega_1} \boldsymbol{\mathcal{M}}_2^{\omega_2}, \mathcal{B}_1^{\omega_1} \mathcal{B}_2^{\omega_2}, \\ \sqrt[t]{\left(\sqrt[t]{1 - (1 - \mathcal{N}^t{}_1)^{\omega_1}}\right)^t + \left(\sqrt[t]{1 - (1 - \mathcal{N}^t{}_2)^{\omega_2}}\right)^t - \left(\sqrt[t]{1 - (1 - \mathcal{N}^t{}_1)^{\omega_1}}\right)^t \left(\sqrt[t]{1 - (1 - \mathcal{N}^t{}_2)^{\omega_2}}\right)^t}, \\ \boldsymbol{\mathcal{R}}_1^{\omega_1} \boldsymbol{\mathcal{R}}_2^{\omega_2} \end{array}\right)$$

$$= \left(\boldsymbol{\mathcal{M}}_1^{\omega_1} \boldsymbol{\mathcal{M}}_2^{\omega_2}, \mathcal{B}_1^{\omega_1} \mathcal{B}_2^{\omega_2}, \sqrt[t]{1 - (1 - \mathcal{N}^t{}_1)^{\omega_1} (1 - \mathcal{N}^t{}_2)^{\omega_2}}, \boldsymbol{\mathcal{R}}_1^{\omega_1} \boldsymbol{\mathcal{R}}_2^{\omega_2}\right)$$

$$= \left(\prod_{\mathrm{J}=1}^{2} \boldsymbol{\mathcal{M}}_\mathrm{J}^{\omega_\mathrm{J}}, \prod_{\mathrm{J}=1}^{2} \mathcal{B}_\mathrm{J}^{\omega_\mathrm{J}}, \sqrt[t]{1 - \prod_{\mathrm{J}=1}^{2} \left(1 - \mathcal{N}^t{}_\mathrm{J}\right)^{\omega_\mathrm{J}}}, \prod_{\mathrm{J}=1}^{2} \boldsymbol{\mathcal{R}}_\mathrm{J}^{\omega_\mathrm{J}}\right)$$

For $\mathfrak{y} = l$

$$CTSFWG\left(A_1, A_2, A_3, \dots, A_\mathrm{J}\right) = \left(\prod_{\mathrm{J}=1}^{l} \boldsymbol{\mathcal{M}}_\mathrm{J}^{\omega_\mathrm{J}}, \prod_{\mathrm{J}=1}^{l} \mathcal{B}_\mathrm{J}^{\omega_\mathrm{J}}, \sqrt[t]{1 - \prod_{\mathrm{J}=1}^{l} \left(1 - \mathcal{N}^t{}_\mathrm{J}\right)^{\omega_\mathrm{J}}}, \prod_{\mathrm{J}=1}^{l} \boldsymbol{\mathcal{R}}_\mathrm{J}^{\omega_\mathrm{J}}\right)$$

For $\mathfrak{y} = l + 1$

For $\mathfrak{y} = l + 1$

$$CTSFWG\left(A_1, A_2, A_3, \dots, A_\mathrm{J}, A_{\mathrm{J}+1}\right) = \prod_{\mathrm{J}=1}^{l+1} A_\mathrm{J}^{\omega_\mathrm{J}} = \prod_{\mathrm{J}=1}^{l} A_\mathrm{J}^{\omega_\mathrm{J}} \otimes A_{\mathrm{J}+1}^{\omega_{\mathrm{J}+1}}$$

$$= \left(\left(\prod_{\mathrm{J}=1}^{l} \boldsymbol{\mathcal{M}}_\mathrm{J}^{\omega_\mathrm{J}}, \prod_{\mathrm{J}=1}^{l} \mathcal{B}_\mathrm{J}^{\omega_\mathrm{J}}, \sqrt[t]{1 - \prod_{\mathrm{J}=1}^{l} \left(1 - \mathcal{N}^t{}_\mathrm{J}\right)^{\omega_\mathrm{J}}}, \prod_{\mathrm{J}=1}^{l} \boldsymbol{\mathcal{R}}_\mathrm{J}^{\omega_\mathrm{J}}\right) \otimes \left(\boldsymbol{\mathcal{M}}_{l+1}^{\omega_{l+1}}, \mathcal{B}_{l+1}^{\omega_{l+1}}, \sqrt[t]{1 - (1 - \mathcal{N}^t{}_{l+1})^{\omega_{l+1}}}, \boldsymbol{\mathcal{R}}_{l+1}^{\omega_{l+1}}\right)\right)$$

For $\mathfrak{y} = 2$

$$= \left(\prod_{\mathrm{J}=1}^{l+1} \boldsymbol{\mathcal{M}}_\mathrm{J}^{\omega_\mathrm{J}}, \prod_{\mathrm{J}=1}^{l+1} \mathcal{B}_\mathrm{J}^{\omega_\mathrm{J}}, \sqrt[t]{1 - \prod_{\mathrm{J}=1}^{l+1} \left(1 - \mathcal{N}^t{}_\mathrm{J}\right)^{\omega_\mathrm{J}}}, \prod_{\mathrm{J}=1}^{l+1} \boldsymbol{\mathcal{R}}_\mathrm{J}^{\omega_\mathrm{J}}\right)$$

Hence, the result holds for $\eta = l + 1$.

Example 3: Assume that the universe set has two elements, that is, $\mathrm{U} = \{\chi_1, \chi_2\}$. Then, demonstrate the CTSFWA operator, where CTSFS is characterized as $A = \{(0.80, 0.40, 0.60; 0.50), (0.75, 0.45, 0.55; 0.65)\}$ with the corresponding weighted vector $\omega = (0.45, 0.55)$ for $t = 3$.

Solution: $$CTSFWG(\chi_1, \chi_2) = \begin{pmatrix} (0.80)^{0.45}(0.75)^{0.55}, \\ (0.40)^{0.45}(0.45)^{0.55}, \\ \sqrt[3]{1 - \left(\left(1 - 0.60^3\right)^{0.45}\right)\left(\left(1 - 0.55^3\right)^{0.55}\right)}, \\ (0.50)^{0.45}(0.65)^{0.55} \end{pmatrix}$$
$$= (0.77, 0.43, 0.57, 0.78)$$

Therefore, it is clear that our suggested operator, as stated in Eq. (11), produces a CTSFV.

Remarks 3: The CTSFWG operator properties in the framework of CTSFS are described as follows:

Idempotency: Assume the collection of CTSFVs $\chi_{\mathrm{J}} = \left(\boldsymbol{\mathcal{M}}_{\chi\mathrm{J}}, \mathcal{B}_{\chi\mathrm{J}}, \mathcal{N}_{\chi\mathrm{J}}, \boldsymbol{\mathcal{R}}_{\chi\mathrm{J}}\right)$ such that for all $\mathrm{J} = 1, 2, \ldots, \eta$. Then, the idempotency of the CTSFWG operator is:

$$\mathrm{CTSFWG}\left(\chi_1, \chi_2, \ldots, \chi_\eta\right) = \chi$$

Boundedness: Assume the collection of CTSFVs $\chi_{\mathrm{J}} = \left(\boldsymbol{\mathcal{M}}_{\chi\mathrm{J}}, \mathcal{B}_{\chi\mathrm{J}}, \mathcal{N}_{\chi\mathrm{J}}, \boldsymbol{\mathcal{R}}_{\chi\mathrm{J}}\right)$ such that for all $\mathrm{J} = 1, 2, \ldots, \eta$. If $\chi^- = \min_{\mathrm{J}}\left(\chi_{\mathrm{J}}\right)$ and $\chi^+ = \max_{\mathrm{J}}\left(\chi_{\mathrm{J}}\right)$ such that for all $\mathrm{J} = 1, 2, \ldots, \eta$. Then, the boundedness of the CTSFWG operator is:

$$\chi^- \leq \mathrm{CTSFWG}\left(\chi_1, \chi_2, \ldots, \chi_\eta\right) \leq \chi^+$$

Monotonicity: Assume the two collections of CTSFVs $\chi_{\mathrm{J}} = \left(\boldsymbol{\mathcal{M}}_{\chi\mathrm{J}}, \mathcal{B}_{\chi\mathrm{J}}, \mathcal{N}_{\chi\mathrm{J}}, \boldsymbol{\mathcal{R}}_{\chi\mathrm{J}}\right)$ and $\chi'_{\mathrm{J}} = \left(\boldsymbol{\mathcal{M}}'_{\chi\mathrm{J}}, \mathcal{B}'_{\chi\mathrm{J}}, \mathcal{N}'_{\chi\mathrm{J}}, \boldsymbol{\mathcal{R}}'_{\chi\mathrm{J}}\right)$ such that for all $\mathrm{J} = 1, 2, \ldots, \eta$. If $\chi_{\mathrm{J}} \leq \chi'_{\mathrm{J}}$ then, the CTSFWG operator exhibits monotonicity:

$$\mathrm{CTSFWG}\left(\chi_1, \chi_2, \ldots, \chi_\eta\right) \leq \mathrm{CTSFWG}\left(\chi_1', \chi_2', \ldots, \chi_\eta'\right)$$

4 | ALGORITHM OF MAGDM IN THE ENVIRONMENT OF CIRCULAR T-SPHERICAL FUZZY SET

In this paper, we propose a hybrid decision-making method that incorporates both the CRITIC methodology of calculating the objective weights and the WASPAS methodology of assessing the entire alternative. It gives a good foundation for considering alternative opinions and arriving at prudent decisions during difficult circumstances. To be more definite, Figure 1 illustrates that the hybrid CRITIC-WASPAS model can easily resolve MAGDM issues within the CTSF framework. Below is a description of the sequential procedure of the suggested model:

Step 1: To characterize the decision matrix (DM) for a decision-making problem, identify a collection of alternatives and a set of evaluation attributes. In this investigation, there are kth Ɗ to utilize the linguistic terms stored in Table 3.

$$M^{k}{}_{\text{ɱ}\times\text{ŋ}} = \begin{array}{c} \\ Ꞥ_1 \\ Ꞥ_2 \\ \vdots \\ Ꞥ_{\text{ɱ}} \end{array}\begin{bmatrix} \text{Ꞔ}_1 & \text{Ꞔ}_2 & \cdots & \text{Ꞔ}_{\text{ŋ}} \\ \chi^{k}{}_{11} & \chi^{k}{}_{12} & \cdots & \chi^{k}{}_{1\text{ŋ}} \\ \chi^{k}{}_{21} & \chi^{k}{}_{22} & \cdots & \chi^{k}{}_{2\text{ŋ}} \\ \vdots & \vdots & \ddots & \vdots \\ \chi^{k}{}_{\text{ɱ}1} & \chi^{k}{}_{\text{ɱ}2} & \cdots & \chi^{k}{}_{\text{ɱ}\text{ŋ}} \end{bmatrix} \tag{12}$$

Step 2: To determine the weight of kth Ɗ, where $(k = 1,2,3,\dots,r)$. For this, we utilized the score function shown in Eq. (6) and then used Eq. (13), where the summation of kth Ɗ weight must be equal to 1.

$$\frac{\sum_{k} \text{Ş}\left(\chi^{k}{}_{\text{ЇJ}}\right)}{\sum_{k}^{\text{r}}\left(\sum_{k} \text{Ş}\left(\chi^{k}{}_{\text{ЇJ}}\right)\right)} \tag{13}$$

Step 3: Used the weight of kth Ɗ and the DM shown in Eq. (12), aggregate based on Eq. (11). The aggregated DM is depicted in Eq. (14), where a set of alternatives $Ꞥ_{\text{Ї}}(\text{Ї} = 1,2,\dots,\text{ɱ})$ regards ɱ under multiple attributes Ꞔ_{J} $(\text{J} = 1,2,\dots,\text{ŋ})$ regrades ŋ attributes represent the CTSFVs.

$$M_{\text{ɱ}\times\text{ŋ}} = \begin{array}{c} \\ Ꞥ_1 \\ Ꞥ_2 \\ \vdots \\ Ꞥ_{\text{ɱ}} \end{array}\begin{bmatrix} \text{Ꞔ}_1 & \text{Ꞔ}_2 & \cdots & \text{Ꞔ}_{\text{ŋ}} \\ \chi_{11} & \chi_{12} & \cdots & \chi_{1\text{ŋ}} \\ \chi_{21} & \chi_{22} & \cdots & \chi_{2\text{ŋ}} \\ \vdots & \vdots & \ddots & \vdots \\ \chi_{\text{ɱ}1} & \chi_{\text{ɱ}2} & \cdots & \chi_{\text{ɱ}\text{ŋ}} \end{bmatrix} \tag{14}$$

Step 4.1: Employing the provided Eq. (15), calculate the total score for each value of the aggregated DM as indicated in Eq. (14).

$$\text{Ş}\left(\chi_{\text{ЇJ}}\right) = \frac{1}{4}\left(\mathcal{M}_{\text{ЇJ}} - \mathcal{B}_{\text{ЇJ}} - \mathcal{N}_{\text{ЇJ}} + \sqrt{2\mathcal{R}_{\text{ЇJ}}}(2\alpha - 1); \alpha \in [0,1]\right) \in [-1,1] \tag{15}$$

Step 4.2: Apply the conversion formula given in Eq. (16) to convert the attribute score values into a standard matrix, where $\text{Ş}\left(\chi_{\text{ЇJ}}\right)^{+} = \max_{\text{Ї}}\left(\text{Ş}\left(\chi_{\text{ЇJ}}\right)\right)$ and $\text{Ş}\left(\chi_{\text{ЇJ}}\right)^{-} = \min_{\text{Ї}}\left(\text{Ş}\left(\chi_{\text{ЇJ}}\right)\right)$.

$$\widetilde{\text{Ş}\left(\chi_{\text{ЇJ}}\right)} = \begin{cases} \dfrac{\text{Ş}\left(\chi_{\text{ЇJ}}\right) - \text{Ş}\left(\chi_{\text{ЇJ}}\right)^{-}}{\text{Ş}\left(\chi_{\text{ЇJ}}\right)^{+} - \text{Ş}\left(\chi_{\text{ЇJ}}\right)^{-}}; & \text{for benefit attribute} \\ \dfrac{\text{Ş}\left(\chi_{\text{ЇJ}}\right)^{+} - \text{Ş}\left(\chi_{\text{ЇJ}}\right)}{\text{Ş}\left(\chi_{\text{ЇJ}}\right)^{+} - \text{Ş}\left(\chi_{\text{ЇJ}}\right)^{-}}; & \text{for cost attribute} \end{cases} \tag{16}$$

Step 4.3: used the given Eq. (17) to calculate the standard deviations of each attribute, where $\overline{\text{Ş}\left(\chi_{\text{ЇJ}}\right)} = \sum_{\text{Ї}=1}^{\text{ɱ}} \frac{\widetilde{\text{Ş}\left(\chi_{\text{ЇJ}}\right)}}{\text{ɱ}}$.

$$\mathfrak{D}_{\mathrm{J}} = \sqrt{\frac{\sum_{\ddot{\mathrm{I}}=1}^{\mathfrak{m}} \left(\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right) - \overline{\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right)} \right)^2}{\mathfrak{m}}} \tag{17}$$

Step 4.4: Use Eq. (18) to determine the attribute's correlation coefficient.

$$\mathcal{C}_{\mathrm{Jt}} = \frac{\sum_{\ddot{\mathrm{I}}=1}^{\mathfrak{m}} \left(\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right) - \overline{\mathfrak{S}\left(\chi_{\mathrm{J}}\right)} \right) \left(\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right) - \overline{\mathfrak{S}(\chi_{t})} \right)}{\sqrt{\sum_{\ddot{\mathrm{I}}=1}^{\mathfrak{m}} \left(\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right) - \overline{\mathfrak{S}\left(\chi_{\mathrm{J}}\right)} \right)^2 \sum_{\ddot{\mathrm{I}}=1}^{\mathfrak{m}} \left(\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right) - \overline{\mathfrak{S}(\chi_{t})} \right)^2}} \tag{18}$$

Step 4.5: used Eq. (19) to determine the information for each property.

$$\mathrm{G}_{\mathrm{J}} = \mathfrak{D}_{\mathrm{J}} \sum_{\mathrm{J}=1}^{\mathfrak{n}} \left(1 - \mathcal{C}_{\mathrm{Jt}}\right) \tag{19}$$

Step 4.6: Eq. (20) is used to determine each attribute's weight prioritizing.

$$\omega_{\mathrm{J}} = \frac{\mathrm{G}_{\mathrm{J}}}{\sum_{\mathrm{J}=1}^{\mathfrak{n}} \mathrm{G}_{\mathrm{J}}} \tag{20}$$

Step 5.1: Utilize Eq. (21) for the normalization of attributes.

$$\mathcal{W}\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right) = \begin{cases} \dfrac{\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right)}{\max_{\ddot{\mathrm{I}}}\left(\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right)\right)}; & \text{for benefit attribute} \\[2ex] \dfrac{\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right)}{\min_{\ddot{\mathrm{I}}}\left(\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right)\right)}; & \text{for cost attribute} \end{cases} \tag{21}$$

Step 5.2: WASPAS method-based, the weighted sum model (WSM) findings are represented by $Q_{\ddot{\mathrm{I}}}^{1}$, which is supplied using Eq. (22).

$$Q_{\ddot{\mathrm{I}}}^{1} = \sum_{\mathrm{J}=1}^{\mathfrak{n}} \mathcal{W}\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right) \omega_{\mathrm{J}} \tag{22}$$

Step 5.3: WASPAS method-based, the weighted product model (WPM) findings are represented by $Q_{\ddot{\mathrm{I}}}^{2}$, which is supplied using Eq. (23).

$$Q_{\ddot{\mathrm{I}}}^{2} = \prod_{\mathrm{J}=1}^{\mathfrak{n}} \mathcal{W}\mathfrak{S}\left(\chi_{\ddot{\mathrm{I}}\mathrm{J}}\right)^{\omega_{\mathrm{J}}} \tag{23}$$

Step 5.4: Utilized the convex formula given in Eq. (24), the WASPAS approach, represented by $Q_{\ddot{\mathrm{I}}}$, combines the advantages of the additive and multiplicative models to improve ranking accuracy as $\lambda \in [0,1]$:

$$Q_{\ddot{\mathrm{I}}} = \lambda Q_{\ddot{\mathrm{I}}}^{1} + (1-\lambda) Q_{\ddot{\mathrm{I}}}^{2} \tag{24}$$

Step 5.5: Choose the alternative with the highest rating, based on its score values or accuracy.

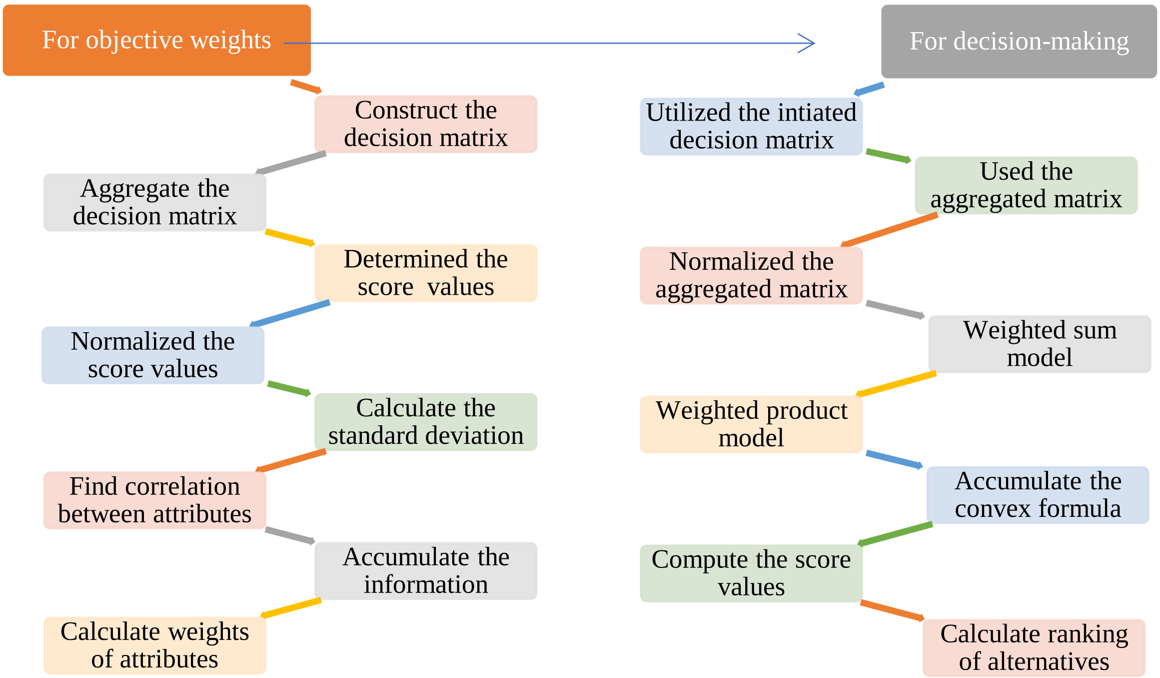

Figure 1. Procedure of prioritized weights and decision-making

5 | CASE STUDY

This case study focuses on a vocational training center that aims to improve career planning and employment guidance for young job seekers. Many trainees complete their courses but still struggle to choose the right career path or secure suitable jobs. The center offers technical skills training, but limited career counseling, weak industry connections, and outdated course content reduce the effectiveness of the program.

The study looks at the difficulties companies, trainers, and students have in matching vocational training to the demands of the labor market today. It also examines the effects of inadequate career counseling on long-term employment outcomes, job preparedness, and student motivation. The case study identifies gaps between career planning, training, and employment assistance services by examining these problems.

5.1. Five Best Alternatives

Various alternative techniques can be taken into consideration to address the issues in career planning, employment advising, and vocational training. These options concentrate on raising employment prospects, enhancing skill relevance, and offering improved career assistance.

- **Ȃ_1: Industry-Based Training Programs**
- Work closely with industries to design training courses based on real job requirements. This helps learners gain practical and in-demand skills that employers actually need.
- **Ȃ_2: Career Counseling and Mentorship Services**
  Provide regular career guidance sessions, one-on-one counseling, and mentorship from experienced professionals to help students make better career decisions.

- **$Ꜳ_3$: Work-Based Learning and Apprenticeships**
  Combine classroom learning with on-the-job training through internships and apprenticeships so learners gain real work experience.
- **$Ꜳ_4$: Strong Job Placement and Employer Partnerships**
  Build strong relationships with employers to support job placements, campus interviews, and continuous feedback on training quality.
- **$Ꜳ_5$: Digital Career Guidance Platforms**
  Use online tools and platforms to offer career assessments, job matching, training recommendations, and labor market information.

5.2. Five Best Attributes

Key characteristics should be taken into account while selecting the best option for enhancing career planning, employment counseling, and vocational training. These characteristics aid in assessing each option's efficacy, viability, and sustainability in preparing individuals for the labor market.

- **$Ꞡ_1$: Relevance to Job Market Needs**
  The solution should focus on skills and knowledge that employers currently need and value.
- **$Ꞡ_2$: Accessibility and Inclusiveness**
  The program should be easy to access and support learners from different backgrounds, including disadvantaged groups.
- **$Ꞡ_3$: Quality of Career Guidance**
  It should provide clear, accurate, and personalized career advice to help individuals make informed decisions.
- **$Ꞡ_4$: Practical Skill Development**
  The approach should offer hands-on training and real-world experience, not just theoretical knowledge.
- **$Ꞡ_5$: Employment Outcomes and Sustainability**
  The solution should lead to real job opportunities and be able to continue effectively over time.

5.3. Analyzing the Results

Step 1: To characterize the decision-making problem, identify a collection of alternatives and a set of evaluation attributes. The Ɗ investigates the DM of CTSFVs depicted in Table 4. In this investigation, there are kth Ɗ to utilizing the linguistic terms stored in Table 3 (where $t = 3$ and $\lambda = 0.5$).

Table 4. Three Ɗ investigate the data for the evaluation of alternatives

Ɗ	Ꜳ/Ꞡ	$Ꞡ_1$	$Ꞡ_2$	$Ꞡ_3$	$Ꞡ_4$	$Ꞡ_5$
$Ɗ_1$	$Ꜳ_1$	EXT	CON	EXT	SBT	SGT
	$Ꜳ_2$	SGT	EXT	CON	EXT	CON
	$Ꜳ_3$	SBT	SGT	SBT	OST	SBT

	A_4	SGT	OST	SGT	CON	SGT
	A_5	EXT	SGT	OST	SGT	OST
D_2	A_1	MIN	SBT	RSB	SGT	SGT
	A_2	SBT	EXT	SBT	EXT	SBT
	A_3	EXT	SGT	EXT	APT	EXT
	A_4	SBT	OST	CON	EXT	CON
	A_5	EXT	SBT	SBT	EXT	OST
D_3	A_1	SGT	SBT	OST	SGT	SGT
	A_2	OST	OST	CON	EXT	CON
	A_3	SBT	SGT	EXT	OST	EXT
	A_4	RSB	EXT	SGT	SBT	SGT
	A_5	SBT	CON	RSB	SGT	OST

Step 2: Use the linguistic concepts mentioned in Table 3 and Eq. (13) to determine the weights of all D. The weights listed in Table 5 must add up to 1.

Table 5. Weights of decision-makers

kth D	Role	Responsibility	Decision	Weights
D_1	SBT	CON	EXT	0.30
D_2	EXT	SGT	OST	0.37
D_3	OST	CON	SBT	0.33

Step 3: Table 6 displays the results of aggregating the DM listed in Table 4 using Eq. (11).

Table 6. Aggregated DM

A/C	C_1	C_2	C_3	C_4	C_5
A_1	$\begin{pmatrix}0.37,0.19,0.63,\\ 0.41\end{pmatrix}$	$\begin{pmatrix}0.73,0.25,0.39,\\ 0.45\end{pmatrix}$	$\begin{pmatrix}0.75,0.00,0.30,\\ 040.\end{pmatrix}$	$\begin{pmatrix}0.73,0.27,0.38,\\ 0.46\end{pmatrix}$	$\begin{pmatrix}0.70,0.30,0.40,\\ 0.47\end{pmatrix}$
A_2	$\begin{pmatrix}0.83,0.00,0.31,\\ 0.41\end{pmatrix}$	$\begin{pmatrix}0.93,0.00,0.18,\\ 0.38\end{pmatrix}$	$\begin{pmatrix}0.67,0.31,0.45,\\ 0.47\end{pmatrix}$	$\begin{pmatrix}0.90,0.10,0.20,\\ 0.40\end{pmatrix}$	$\begin{pmatrix}0.67,0.31,0.45,\\ 0.47\end{pmatrix}$
A_3	$\begin{pmatrix}0.84,0.15,0.27,\\ 0.42\end{pmatrix}$	$\begin{pmatrix}0.70,0.30,0.40,\\ 0.47\end{pmatrix}$	$\begin{pmatrix}0.87,0.12,0.24,\\ 0.41\end{pmatrix}$	$\begin{pmatrix}0.64,0.00,0.44,\\ 0.36\end{pmatrix}$	$\begin{pmatrix}0.87,0.12,0.24,\\ 0.41\end{pmatrix}$
A_4	$\begin{pmatrix}0.66,0.31,0.37,\\ 0.45\end{pmatrix}$	$\begin{pmatrix}0.97,0.00,0.14,\\ 0.35\end{pmatrix}$	$\begin{pmatrix}0.66,0.33,0.44,\\ 0.48\end{pmatrix}$	$\begin{pmatrix}0.77,0.19,0.37,\\ 0.44\end{pmatrix}$	$\begin{pmatrix}0.66,0.33,0.44,\\ 0.48\end{pmatrix}$
A_5	$\begin{pmatrix}0.87,0.13,0.24,\\ 0.41\end{pmatrix}$	$\begin{pmatrix}0.70,0.28,0.41,\\ 0.46\end{pmatrix}$	$\begin{pmatrix}0.73,0.00,0.32,\\ 0.41\end{pmatrix}$	$\begin{pmatrix}0.77,0.20,0.35,\\ 0.44\end{pmatrix}$	$\begin{pmatrix}1.00,0.00,0.00,\\ 0.33\end{pmatrix}$

Step 4.1: Table 7 displays the score values of the aggregate DM (shown in Table 6), which are derived using Eq. (15) whenever $\alpha = 1$.

Table 7. Combined DM score values

A/C	C_1	C_2	C_3	C_4	C_5
A_1	0.11	0.26	0.34	0.26	0.24
A_2	0.35	0.41	0.22	0.37	0.22
A_3	0.33	0.24	0.35	0.26	0.35
A_4	0.23	0.42	0.22	0.29	0.22
A_5	0.35	0.24	0.33	0.29	0.45

Step 4.2: Every attribute is aware of the advantages. Eq. (16) is used to generate the normalized DM, which is shown in Table 8.

Table 8. Normalized DM

A/G	G_1	G_2	G_3	G_4	G_5
A_1	0.00	0.13	0.88	0.00	0.11
A_2	1.00	0.94	0.04	1.00	0.02
A_3	0.90	0.00	1.00	0.02	0.58
A_4	0.50	1.00	0.00	0.22	0.00
A_5	0.98	0.00	0.84	0.25	1.00

Step 4.3: We calculated each attribute's standard deviation using Eq. (17) and displayed the results in Table 9.

Table 9. Standard deviation for each attribute

Standard deviations	G_1	G_2	G_3	G_4	G_5
$\mathfrak{D}_J$	0.38	0.46	0.44	0.37	0.39

Step 4.4: Eq. (18) can be used to determine the correlation coefficient between any two qualities; the results are displayed in Table 10.

Table 10. Correlation coefficient of each attribute

$\mathcal{C}_{Jt}$	G_1	G_2	G_3	G_4	G_5
G_1	1.00	0.05	-0.12	0.52	0.49
G_2	0.05	1.00	-0.99	0.65	-0.75
G_3	-0.12	-0.99	1.00	-0.69	0.67
G_4	0.52	0.65	-0.69	1.00	-0.32
G_5	0.49	-0.75	0.67	-0.32	1.00

Step 4.5: Each attribute's information is determined by applying Eq. (19) and is listed in Table 11.

Table 11. Recalculate attributes

Attributes	G_1	G_2	G_3	G_4	G_5
G_J	1.18	2.30	2.25	1.41	1.53

Step 4.6: Eq. (20) is used to get the weight for each attribute, which is shown in Table 12.

Table 12. Finalized weights of the attribute

Weights	G_1	G_2	G_3	G_4	G_5
ω_J	0.14	0.25	0.25	0.16	0.18

Step 5.1: Normalize the score values DM provided in Table 7 using Eq. (21); the resulting normalized DM is presented in Table 13.

Table 13. Normalized DM

Ⱥ/Ꞓ	$Ꞓ_1$	$Ꞓ_2$	$Ꞓ_3$	$Ꞓ_4$	$Ꞓ_5$
$Ⱥ_1$	0.32	0.63	0.95	0.70	0.53
$Ⱥ_2$	1.00	0.97	0.63	1.00	0.49
$Ⱥ_3$	0.93	0.58	1.00	0.70	0.78
$Ⱥ_4$	0.66	1.00	0.61	0.76	0.47
$Ⱥ_5$	0.99	0.58	0.94	0.77	1.00

Step 5.2: Using the prioritized weights of the features listed in Table 12, integrate the normalized DM displayed in Table 13 using the initiated aggregation operators given in Eq. (22). Table 14 displays the $Q_{\text{Ï}}^1$ aggregated DM.

Table 14. Aggregated DM

Alternatives	$Q_{\text{Ï}}^1$
$Ⱥ_1$	0.66
$Ⱥ_2$	0.81
$Ⱥ_3$	0.79
$Ⱥ_4$	0.72
$Ⱥ_5$	0.83

Step 5.3: Using the prioritized weights of the features listed in Table 12, integrate the normalized DM displayed in Table 13 using the initiated aggregation operators given in Eq. (23). Table 15 displays the $Q_{\text{Ï}}^2$ aggregated DM.

Table 15. Aggregated DM

Alternatives	$Q_{\text{Ï}}^2$
$Ⱥ_1$	0.63
$Ⱥ_2$	0.78
$Ⱥ_3$	0.77
$Ⱥ_4$	0.70
$Ⱥ_5$	0.81

Step 5.4: Using the prioritized weights of the features listed in Table 12, integrate the normalized DM displayed in Table 13 using the initiated aggregation operators given in Eq. (24). Table 16 displays the $Q_{\text{Ï}}$ aggregated DM.

Table 16. Aggregated DM

Alternatives	$\boldsymbol{Q}_{\text{Ï}}$
$Ⱥ_1$	0.64
$Ⱥ_2$	0.79
$Ⱥ_3$	0.78
$Ⱥ_4$	0.71
$Ⱥ_5$	0.82

Step 5.5: As a consequence, we discovered that in each case alternative ₳_5, which reflects "Digital Career Guidance Platforms," is the best alternative to career planning and employment guidance. The score values and ranking of alternatives are shown in Table 17.

Table 17. Score values and ranking

Proposed	₳_1	₳_2	₳_3	₳_4	₳_5	Ranking result	Optimal
$Q_{\ddot{I}}^{1}$	0.66	0.81	0.79	0.72	0.83	$\text{₳}_5 > \text{₳}_2 > \text{₳}_3 > \text{₳}_4 > \text{₳}_1$	₳_5
$Q_{\ddot{I}}^{2}$	0.63	0.78	0.77	0.70	0.81	$\text{₳}_5 > \text{₳}_2 > \text{₳}_3 > \text{₳}_4 > \text{₳}_1$	₳_5
$Q_{\ddot{I}}$	0.64	0.79	0.78	0.71	0.82	$\text{₳}_5 > \text{₳}_2 > \text{₳}_3 > \text{₳}_4 > \text{₳}_1$	₳_5

This suggests that ₳_5, which stands for "Digital Career Guidance Platforms," is the optimal option in each scenario. Furthermore, Figure 2 provides a geometrical representation of the score values and ranking of CTSFWA (represented by $Q_{\ddot{I}}^{1}$), CTSFWG (represented by $Q_{\ddot{I}}^{2}$), and WASPAS (represented by $Q_{\ddot{I}}$).

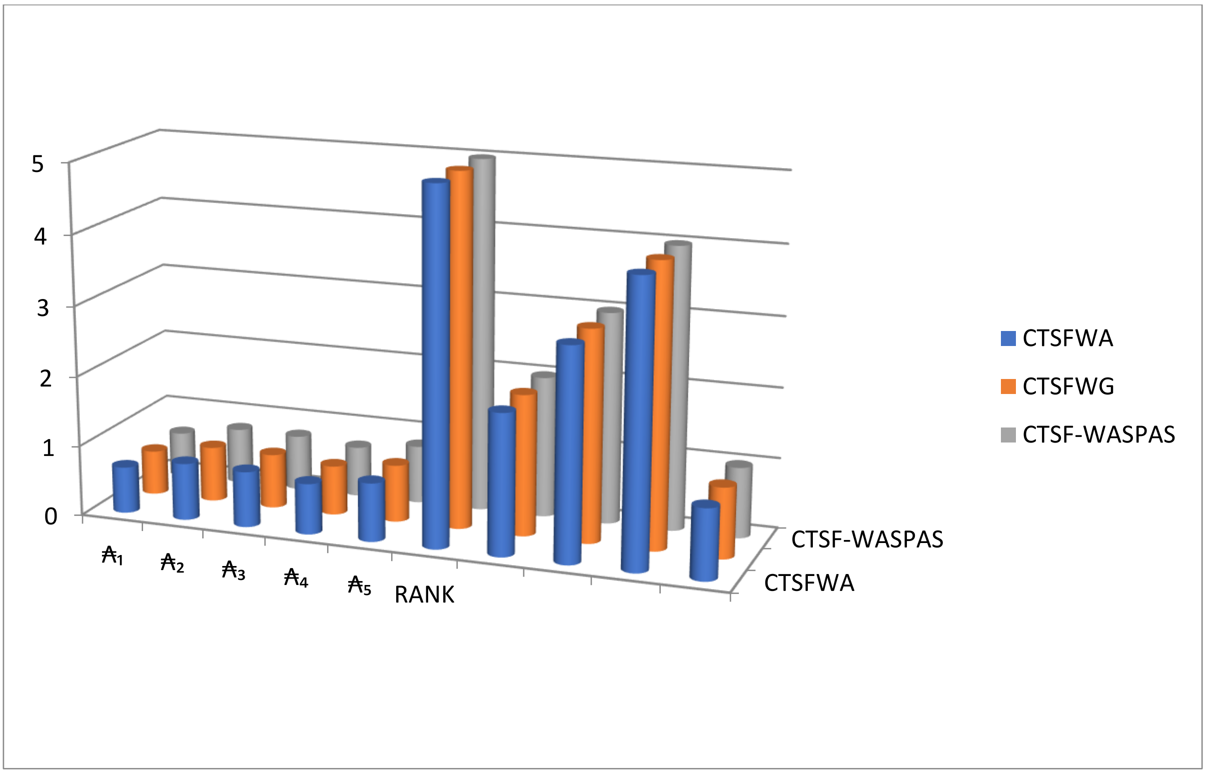

Figure 2. Geometrical representation of alternatives under the proposed work

6 | SENSITIVITY AND COMPUTATIONAL ANALYSIS

This section aims to carry out the sensitivity and computational analysis of our proposed hybrid model of the MAGDM problem. This regulates the prioritized weights of attributes, additive and multiplicative approaches, in order to assess the robustness of the decision-making outcomes. This entails methodically changing λ from 0 to 1, typically in increments of 0.1, and recalculating the total

CRITIC-WASPAS scores for each option at each stage. After that, the rankings are analyzed to see how various aggregation techniques affect them. We investigate whether the best option is the same for every value of λ. As a result, it shows that the decision's consequence is steady and trustworthy.

Table 18. Score value and ranking of alternatives in various parametric values

λ	A_1	A_2	A_3	A_4	A_5	Ranking	Optimal
$\lambda_1 = 0.1$	0.63	0.78	0.77	0.70	0.81	$A_5 > A_2 > A_3 > A_4 > A_1$	A_5
$\lambda_2 = 0.2$	0.63	0.78	0.77	0.70	0.82	$A_5 > A_2 > A_3 > A_4 > A_1$	A_5
$\lambda_3 = 0.3$	0.64	0.79	0.78	0.71	0.82	$A_5 > A_2 > A_3 > A_4 > A_1$	A_5
$\lambda_4 = 0.4$	0.64	0.79	0.78	0.71	0.82	$A_5 > A_2 > A_3 > A_4 > A_1$	A_5
$\lambda_5 = 0.5$	0.64	0.79	0.78	0.71	0.82	$A_5 > A_2 > A_3 > A_4 > A_1$	A_5
$\lambda_6 = 0.6$	0.65	0.79	0.78	0.71	0.82	$A_5 > A_2 > A_3 > A_4 > A_1$	A_5
$\lambda_7 = 0.7$	0.65	0.80	0.78	0.72	0.83	$A_5 > A_2 > A_3 > A_4 > A_1$	A_5
$\lambda_8 = 0.8$	0.65	0.80	0.79	0.72	0.83	$A_5 > A_2 > A_3 > A_4 > A_1$	A_5
$\lambda_9 = 0.9$	0.66	0.80	0.79	0.72	0.83	$A_5 > A_2 > A_3 > A_4 > A_1$	A_5

As demonstrated in Table 18, sensitivity analysis is therefore essential for verifying the reliability and consistency of the suggested CRITIC-WASPAS model. This table shows the alternative. A_5 consistently ranks as the best option across all parameter values, proving A_5 as the greatest option for maximizing consumer value and corroborating the analytical results. Figure 3 graph illustrates how each parameter affects the suggested work stability and reliability.

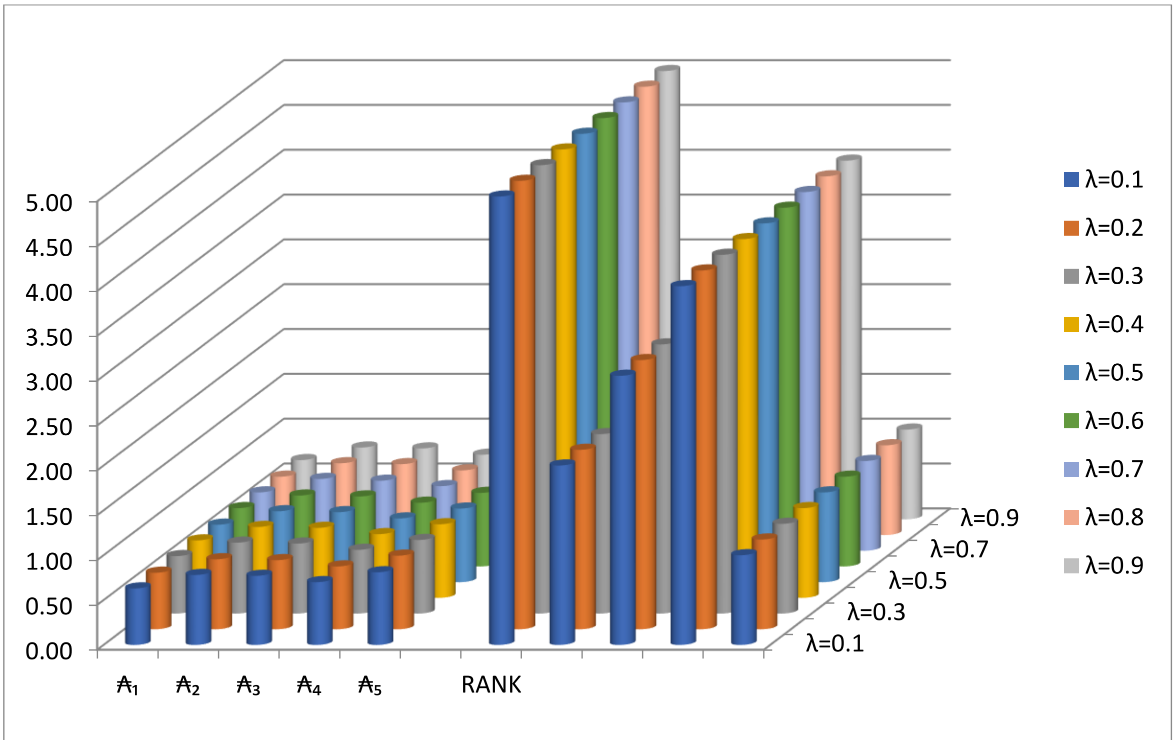

Figure 3. Sensitivity graph with respect to variations in input parameters

7 | COMPARATIVE ANALYSIS

The comparison study shows how the assessed approaches differ in terms of performance, efficiency, and dependability. It is clear from comparing their results under the same circumstances that each approach has distinct advantages and disadvantages. In contrast to the current approaches, which typically exhibit decreased stability and responsiveness, the suggested methodology consistently produces improved accuracy and resilience, especially in settings marked by uncertainty and complicated decision factors. Overall, the comparison highlights the superior effectiveness of the proposed method and justifies its adoption over the traditional techniques. We summarized the in-depth conversation in Table 19.

Table 19. Comparative analysis of the proposed hybrid model in the CTSFS environment

Characteristic	**Traditional FS IFS**	**PFS / SFS / TSF**	**Circular FS Models (CIFS, CPFS, CSFS)**	**Proposed CTSFS–CRITIC–WASPAS Model**
Uncertainty representation	Partial membership only (FS); membership & non-membershi (IFS)	Membership, abstinence, and non-membershir with fixed values	Circular region arounc fixed fuzzy values	Circular region with power-based flexibility
Hesitation modeling	Not explicitly modeled	Modeled, but point-based	Modeled with limited circular variation	Modeled with radius-based dynamic fluctuation
Directional uncertainty	Not supported	Not supported	Partially supported	Fully supported through a circular structure
Domain flexibility	Very limited	Moderate	Restricted by strict constraints	Highly flexible via q-rung power constraint
Handling expert opinion variability	Weak	Moderate	Good	Excellent
Representation of humar reasoning	Low	Moderate	Good	Very high
Weight determination	Subjective	Mostly subjective	Mostly subjective	Objective (CRITIC method)
Bias reduction	Poor	Moderate	Moderate	Strong
Decision aggregation	Single aggregation method	Single or limited hybrid methods	Limited hybrid methods	Hybrid WASPAS (additive + multiplicative)
Ranking stability	Sensitive to parameter changes	Moderately stabl	Moderately stable	Highly stable (validated by

				sensitivity analysis)
Computational robustness	Low	Moderate	Moderate	High
Applicability to real-world problems	Limited	Moderate	Limited	Strong (career planning case study)
Empirical validation	Rare	Limited	Very limited	Comprehensive case study
Sensitivity analysis	Rarely applied	Occasionally applied	Rare	Fully performed
Overall decision reliability	Low	Moderate	Moderate	High

7.1. Compatibility and Validation

The ability of current research to handle complex fuzzy data is limited. Comparison research is conducted to verify the superiority and compatibility of the proposed hypothesis. The recommended process is applied to the data in Table 4, and the ranking results are compared with those of alternative methods, including those by [26], who worked on IFS and PFS, which are a special case of CTSFS and lack circular behavior. The geometric and weighted averaging operators in IFS were employed by [27], but because of CTSFS's bigger domain and circular behavior, these operators are insufficient. Yusoff et al. [23] elaborate on the idea of Cq-ROFS, which is a special case of CTSFS, due to a limited domain, and also has no information about the abstinence degree. The CPFS was adopted by [24], which can be regarded as a particular instance of CTSFS, but it has also imitated domain. CSFS is defined by [25], which is the extension of CPFS. These drawbacks are addressed by our suggested theory, which offers a more sophisticated and thorough method.

Table 20. Comparison with existing literature

References	Input	Optimal result
Albaity et al. [26]	The data in Table 4 cannot be assessed	The methods are restricted
Xu [27]	The data in Table 4 cannot be assessed	The methods are restricted
Yusoff et al. [23]	The data in Table 4 cannot be assessed	The methods are restricted
Xu and Zhang [24]	The data in Table 4 cannot be assessed	The methods are restricted
Ashraf and Chohan [25]	The data in Table 4 cannot be assessed	The methods are restricted
$Q_{\ddot{I}}^{1}$	$\text{Ⱥ}_2 > \text{Ⱥ}_3 > \text{Ⱥ}_4 > \text{Ⱥ}_5 > \text{Ⱥ}_1$	Ⱥ_5
$Q_{\ddot{I}}^{2}$	$\text{Ⱥ}_2 > \text{Ⱥ}_3 > \text{Ⱥ}_4 > \text{Ⱥ}_5 > \text{Ⱥ}_1$	Ⱥ_5
$Q_{\ddot{I}}$	$\text{Ⱥ}_2 > \text{Ⱥ}_3 > \text{Ⱥ}_4 > \text{Ⱥ}_5 > \text{Ⱥ}_1$	Ⱥ_5

Table 20 demonstrates that the advanced features of the suggested strategy make it both possible and powerful, while the data shown in Table 4 cannot be solved by the limited techniques now in use. Current methods have limited frameworks based on algebraic properties, procedures, extensions, and operators, which is their main flaw. IFS, PyFS, PFS, and circular frameworks are a few examples of these frameworks. In contrast to previous work, the initiated solution acts on the CTSF information, providing flexibility with a wider scope and circular behavior. This highlights the shortcomings of previous established work and makes current procedures a specific case of the proposed work.

7.2. Key Observations

Some key observations of this manuscript are given below:

- The proposed model outperforms existing fuzzy frameworks in modeling hesitation, uncertainty, and directional ambiguity.
- The integration of CRITIC objective weighting significantly reduces expert bias.
- The WASPAS hybrid aggregation improves ranking accuracy and stability.
- Sensitivity analysis confirms robust and consistent decision outcomes.
- The model demonstrates strong real-world applicability in career planning and employment guidance.

8 | CONCLUSION

To solve the complex multi-attribute group decision-making (MAGDM) problems in the context of uncertainty, this research suggested a new CRITICWASPAS decision-making model that is Circular T-Spherical Fuzzy Set (CTSFS) based. The suggested solution incorporates the objective attribute weighting of the CRITIC method with the capability of the hybrid ranking of the WASPAS method, and simultaneously, uncertainty, hesitation, and directional ambiguity are represented using the CTSFS structure. The suggested model can be applied in a more realistic and adaptable model of human judgment in ambiguous situations of decision by extending the traditional fuzzy environment to a circular T-spherical framework, which enables the expert judgments to vary in a specified radius. Incorporation of membership, abstinence, non-membership, and radius parameters contributes greatly to the level of expressiveness of evaluations as opposed to classical fuzzy, intuitionistic, picture, and spherical fuzzy models. It was shown that the proposed model is applicable and effective using a career planning and employment guidance case study in a vocational training situation. Digital Career Guidance Platforms were the most appropriate alternative that was consistently reached. In addition, the sensitivity analysis also guaranteed the strength and stability of the decision results with different values of the WASPAS balance parameter, which proves the reliability of the proposed approach.

8.1. Constraints and Limitations

The developed fuzzy aggregation models are applicable in practice. They, however, have some limitations. Information presented in the form provided in Definition 2 can only be handled in the proposed work. It does not favor complex fuzzy information or bipolar fuzzy information. Notwithstanding these shortcomings, the suggested model can be significant in the area of fuzzy decision-making.

8.2. Future Direction

Larger and more varied datasets can be added to this study in subsequent studies to further confirm the robustness of the suggested methodology. To improve accuracy under different degrees of uncertainty, more fuzzy frameworks or hybrid decision-making models can be investigated. This direction can be further extended by exploring variants like some averaging aggregation operators of TSFS [28], Aczel–Alsina power aggregation operator [29], some circular Pythagorean fuzzy Muirhead means operators [30], Dombi hamy mean operators [31], circular q-rung orthopair fuzzy information [32], and Aczel-Alsina hamy mean operators [33].

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