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Decision-Making Techniques Based on Complex Intuitionistic Fuzzy Power Interaction Aggregation Operators and Their Applications

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ABSTRACT

In this manuscript, we evaluate the interaction operational laws for complex intuitionistic fuzzy (CIF) information. Furthermore, we derive the CIF power interaction averaging (CIFPIA) operator, CIF power interaction weighted averaging (CIFPIWA) operator, CIF power interaction geometric (CIFPIG) operator, and CIF power interaction weighted geometric (CIFPIWG) operator. Some fundamental properties are also examined for the evaluated operators. Additionally, to simplify the above operators, we illustrate a multi-attribute decision-making procedure (MADMP) based on the exposed operators for CIF information to enhance the worth and stability of the presented operators. Finally, we compare the invented operators with some existing operators in the presence of some numerical examples to show the supremacy and effectiveness of the derived approaches.

Keywords: Complex Intuitionistic fuzzy sets, Power interaction aggregation operators, Decision-making problem

1 | INTRODUCTION

To address the valuable and best decision among the collection of decisions, the theory of MADMP is very perfect and strong for examining the best decision from the collection of alternatives. MADMP is used in many applications in the presence of classical set theory, but during the decision-making procedure, we failed to evaluate our target because of limited opinions such as zero or one. Zadeh [1] increased this opinion in the shape of the fuzzy set (FS) which contained many opinions compared to classical set theory. Furthermore, one of the most perfect and valuable extensions of FS theory, called the superior Mandelbrot set was invented by Negi and Rani [2]. In the FS theory, we have membership grade and non-membership grade, but both are dependent on each other. To propose a new one, which has a membership grade and non-membership grade, and both are independent, then what happened, for this, the intuitionistic FS (IFS) was presented by Atanassov [3], [4]. IFS theory has two grades, called truth function "$\equiv^{\Lambda\Xi_{AT}}(\partial_u)$" and falsity function "$\exists^{\Lambda\Xi_{AT}}(\partial_u)$" such as $0 \leq \equiv^{\Lambda\Xi_{AT}}(\partial_u) + \exists^{\Lambda\Xi_{AT}}(\partial_u) \leq 1$. Some applications of IFS have been discussed, for instance, distance measures [5], similarity operators [6], simulation approach [7], similarity-distance measures [8], evaluation of pandemic under uncertainty [9], TOPSIS technique [10],power aggregation operators [11], Schweizer-Sklar prioritized aggregation operators [12], WASPAS techniques

[13], MAIRCA technique [14], distance measures [15], divergence measures [16], and distance measures based on IFS and their application in pattern recognition, medical diagnosis, and clustering analysis [17].

In FS theory, we have only one grade in the shape of a real number, which can easily deal with two-dimension information, but in many real-life cases, we have a problem that isn't dealt with by FS theory, to manage such type of problems, Ramot et al. [18] introduced the complex FS (CFS) which has a truth grade in the shape of a complex number, whose amplitude and phase terms in the shape of fuzzy numbers. Furthermore, Liu et al. [19] exposed the distance measures for CFSs, and Mahmood et al. [20] derived the complex fuzzy coverings. Furthermore, to include the grade of complex-valued falsity, the idea of complex IFS (CIFS) was exposed by Alkouri and Salleh [21]. The major condition of CIFS is stated by: $0 \leq \equiv^{\Lambda\Xi_{AT}}(\partial_u) + \exists^{\Lambda\Xi_{AT}}(\partial_u) \leq 1$ and $0 \leq \equiv^{\Lambda\Xi_{PT}}(\partial_u) + \exists^{\Lambda\Xi_{PT}}(\partial_u) \leq 1$, where $\left(\equiv^{\Lambda\Xi_{AT}}(\partial_u), \equiv^{\Lambda\Xi_{PT}}(\partial_u)\right)$ and $\left(\exists^{\Lambda\Xi_{AT}}(\partial_u), \exists^{\Lambda\Xi_{PT}}(\partial_u)\right)$ are represents the complex-valued of truth value and the complex-valued of falsity value. CIFS is the modified type of FSs, IFSs, and CFSs, and because of this reason, many scholars have utilized it in different fields, for instance, distance measures for IFS were exposed by Rani and Garg [22], information measures for CIFS were invented by Garg and Rani [23], distance measures for CIFSs was evaluated by Ke et al. [24], and robust correlation coefficient based on CIFS was evaluated by Garg and Rani [25].

Yager [26] derived the power averaging operators for aggregating the collection of finite information into singleton sets. Furthermore, interaction aggregating operators (AOs) for IFSs was exposed by Garg [27]. Additionally, geometric interaction AOs for generalized IFSs was evaluated by He et al.[28]. Furthermore, interaction averaging AOs for IFSs was evaluated by He et al. [29]. Jiang et al. [30] presented the power AOs for IFSs. Xu [31] evaluated the simple power AOs for IFSs. Xu [32] derived the AOs for IFSs. Moreover, Xu and Yager [33] exposed the geometric AOs for IFSs. Additionally, generalized AOs for CIFSs were exposed by Garg and Rani [34]. Further, power AOs for CIFSs were invented by Rani and Garg [35]. After a long assessment, we observed that the idea of power interaction operators based on CIFS is not evaluated by anyone. Therefore, keeping the advantages of the power interaction operators and CIFSs, the major contribution of this manuscript is stated below:

1) To evaluate the interaction operational laws for CIF information.
2) To derive the CIFPIA operator, CIFPIWA operator, CIFPIG operator, and CIFPIWG operator. Some fundamental properties are also examined for the evaluated operators.
3) To simplify the above operators, we illustrate a MADMP based on the exposed operators for CIF information to enhance the worth and stability of the presented operators.
4) To compare the invented operators with some existing operators in the presence of some numerical examples to show the supremacy and effectiveness of the derived approaches.

This article is arranged in the shape: In Section 2, we analyzed the idea of CIFSs and their algebraic operational laws. Moreover, we stated the idea of PAOs based on a universal set B_u. In Section 3, we evaluated the interaction operational laws for CIF information. Furthermore, we derived the CIFPIA operator, CIFPIWA operator, CIFPIG operator, and CIFPIWG operator. Some fundamental properties are also examined for the evaluated operators. In Section 4, we illustrated a MADMP based on the exposed operators for CIF information to enhance the worth and stability of the presented operators. In Section 5, we compared the invented operators with some existing operators in the presence of some numerical examples to show the supremacy and effectiveness of the derived approaches. Some concluding information is in Section 6.

2 | PRELIMINARIES

Information In this section, we analyzed the idea of CIFSs and their algebraic operational laws. Moreover, for any finite collection of positive integers, we stated the idea of PAOs based on a universal set B_u.

Definition 1: [21] An ordinary finite universal set B_u, a CIFS $\Lambda\Xi_{\mathrm{II}}$ in B_u is a representation:

$$\Lambda\Xi_{\mathrm{II}} = \left\{\left(\equiv^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u)e^{i2\pi\left(\equiv^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)\right)}, \exists^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u)e^{i2\pi\left(\exists^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)\right)}\right) : \partial_u \in \mathrm{B}_u\right\}$$

$$= \left\{\left(\left(\equiv^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u), \equiv^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)\right), \left(\exists^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u), \exists^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)\right)\right) : \partial_u \in \mathrm{B}_u\right\}$$

Where $\equiv^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u)e^{i2\pi\left(\equiv^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)\right)}$ and $\exists^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u)e^{i2\pi\left(\exists^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)\right)}$ represents or shows the complex-valued truth and falsity function with $0 \leq \equiv^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u) + \exists^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u) \leq 1$ and $0 \leq \equiv^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u) + \exists^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u) \leq 1$. Noticed that $\equiv^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u), \equiv^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u), \exists^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u)$ and $\exists^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)$ are stated the amplitude term and phase term of truth and falsity grade with a neutral grade, such as: $℃^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u)e^{i2\pi\left(℃^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)\right)} = \left(1-\left(\equiv^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u) + \exists^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u)\right)\right)e^{i2\pi\left(1-\left(\equiv^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)+\exists^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)\right)\right)}$. Finally, the structure of CIFN is represented by: $\Lambda\Xi_{\mathrm{II}}^{\partial} = \left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} e^{i2\pi\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)}, \exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} e^{i2\pi\left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)}\right) = \left(\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right), \left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right), \partial = 1,2,\ldots,\alpha.$

Definition 2: [21]Let $\Lambda\Xi_{\mathrm{II}}^{\partial} = \left(\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right), \left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right), \partial = 1,2$ be any two CIFNs. Then

$$\Lambda\Xi_{\mathrm{II}}^{1} \oplus \Lambda\Xi_{\mathrm{II}}^{2} = \begin{pmatrix} \left(\equiv_{1}^{\Lambda\Xi_{\mathrm{AT}}} + \equiv_{2}^{\Lambda\Xi_{\mathrm{AT}}} - \equiv_{1}^{\Lambda\Xi_{\mathrm{AT}}}\equiv_{2}^{\Lambda\Xi_{\mathrm{AT}}}, \equiv_{1}^{\Lambda\Xi_{\mathrm{PT}}} + \equiv_{2}^{\Lambda\Xi_{\mathrm{PT}}} - \equiv_{1}^{\Lambda\Xi_{\mathrm{PT}}}\equiv_{2}^{\Lambda\Xi_{\mathrm{PT}}}\right), \\ \left(\exists_{1}^{\Lambda\Xi_{\mathrm{AT}}}\exists_{2}^{\Lambda\Xi_{\mathrm{AT}}}, \exists_{1}^{\Lambda\Xi_{\mathrm{PT}}}\exists_{2}^{\Lambda\Xi_{\mathrm{PT}}}\right) \end{pmatrix}$$

$$\Lambda\Xi_{\mathrm{II}}^{1} \oplus \Lambda\Xi_{\mathrm{II}}^{2} = \begin{pmatrix} \left(\equiv_{1}^{\Lambda\Xi_{\mathrm{AT}}}\equiv_{2}^{\Lambda\Xi_{\mathrm{AT}}}, \equiv_{1}^{\Lambda\Xi_{\mathrm{PT}}}\equiv_{2}^{\Lambda\Xi_{\mathrm{PT}}}\right), \\ \left(\exists_{1}^{\Lambda\Xi_{\mathrm{AT}}} + \exists_{2}^{\Lambda\Xi_{\mathrm{AT}}} - \exists_{1}^{\Lambda\Xi_{\mathrm{AT}}}\exists_{2}^{\Lambda\Xi_{\mathrm{AT}}}, \exists_{1}^{\Lambda\Xi_{\mathrm{PT}}} + \exists_{2}^{\Lambda\Xi_{\mathrm{PT}}} - \exists_{1}^{\Lambda\Xi_{\mathrm{PT}}}\exists_{2}^{\Lambda\Xi_{\mathrm{PT}}}\right) \end{pmatrix}$$

$$\widetilde{\Omega_s}\Lambda\Xi_{\mathrm{II}}^{1} = \left(\left(1-\left(1-\equiv_{1}^{\Lambda\Xi_{\mathrm{AT}}}\right)^{\widetilde{\Omega_s}}, 1-\left(1-\equiv_{1}^{\Lambda\Xi_{\mathrm{PT}}}\right)^{\widetilde{\Omega_s}}\right), \left(\left(\exists_{1}^{\Lambda\Xi_{\mathrm{AT}}}\right)^{\widetilde{\Omega_s}}, \left(\exists_{1}^{\Lambda\Xi_{\mathrm{PT}}}\right)^{\widetilde{\Omega_s}}\right)\right)$$

$$\left(\Lambda\Xi_{\mathrm{II}}^{1}\right)^{\widetilde{\Omega_s}} = \left(\left(\left(\equiv_{1}^{\Lambda\Xi_{\mathrm{AT}}}\right)^{\widetilde{\Omega_s}}, \left(\equiv_{1}^{\Lambda\Xi_{\mathrm{PT}}}\right)^{\widetilde{\Omega_s}}\right), \left(1-\left(1-\exists_{1}^{\Lambda\Xi_{\mathrm{AT}}}\right)^{\widetilde{\Omega_s}}, 1-\left(1-\exists_{1}^{\Lambda\Xi_{\mathrm{PT}}}\right)^{\widetilde{\Omega_s}}\right)\right)$$

Definition 3: [21] Let $\Lambda\Xi_{\mathrm{II}}^{\partial} = \left(\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right), \left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right), \partial = 1,2$ be any two CIFNs. Then

$$SCO\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right) = \frac{1}{2}\left(\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} + \equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right) - \left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} + \exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right) \in [-1,1]$$

$$ACC\left(\Lambda\Xi_{\text{II}}^{\partial}\right)=\frac{1}{2}\left(\left(\equiv_{\partial}^{\Lambda\Xi_{\text{AT}}}+\equiv_{\partial}^{\Lambda\Xi_{\text{PT}}}\right)+\left(\exists_{\partial}^{\Lambda\Xi_{\text{AT}}}+\exists_{\partial}^{\Lambda\Xi_{\text{PT}}}\right)\right)\in[0,1]$$

Where, $SCO\left(\Lambda\Xi_{\text{II}}^{\partial}\right)$ and $ACC\left(\Lambda\Xi_{\text{II}}^{\partial}\right)$ shows the score and accuracy values with some interpretations, such as: if $SCO(\Lambda\Xi_{\text{II}}^{1})>SCO(\Lambda\Xi_{\text{II}}^{2})\Rightarrow\Lambda\Xi_{\text{II}}^{1}>\Lambda\Xi_{\text{II}}^{2}$, if $SCO(\Lambda\Xi_{\text{II}}^{1})<SCO(\Lambda\Xi_{\text{II}}^{2})\Rightarrow\Lambda\Xi_{\text{II}}^{1}<\Lambda\Xi_{\text{II}}^{2}$, if $SCO(\Lambda\Xi_{\text{II}}^{1})=SCO(\Lambda\Xi_{\text{II}}^{2})\Rightarrow$, thus if $ACC(\Lambda\Xi_{\text{II}}^{1})>ACC(\Lambda\Xi_{\text{II}}^{2})\Rightarrow\Lambda\Xi_{\text{II}}^{1}>\Lambda\Xi_{\text{II}}^{2}$, if $ACC(\Lambda\Xi_{\text{II}}^{1})<ACC(\Lambda\Xi_{\text{II}}^{2})\Rightarrow\Lambda\Xi_{\text{II}}^{1}<\Lambda\Xi_{\text{II}}^{2}$.

Definition 3: [26] Let $\Lambda\Xi_{\text{II}}^{\partial},\partial=1,2,\dots,\alpha$ be any collection of positive numbers. Then

$$PAO\left(\Lambda\Xi_{\text{II}}^{1},\Lambda\Xi_{\text{II}}^{2},\dots,\Lambda\Xi_{\text{II}}^{\alpha}\right)=\sum_{\partial=1}^{\alpha}\left(\frac{\left(1+{}^{\circ}\text{C}^{\circ}\text{C}\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)\Lambda\Xi_{\text{II}}^{\partial}}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}\text{C}^{\circ}\text{C}\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}\right)$$

Further, ${}^{\circ}\text{C}^{\circ}\text{C}\left(\Lambda\Xi_{\text{II}}^{\partial}\right)=\sum_{\substack{\partial=1,\\ \partial\neq k}}^{\alpha}SUPP\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)$, where $SUPP\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)=1-DIS\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)$ represents the support degree between $\Lambda\Xi_{\text{II}}^{\partial}$ and $\Lambda\Xi_{\text{II}}^{k}$ with the following conditions:

1) $SUPP\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)\in[0,1]$.
2) $SUPP\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)=SUPP\left(\Lambda\Xi_{\text{II}}^{k},\Lambda\Xi_{\text{II}}^{\partial}\right)$.
3) If $DIS\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)<DIS\left(\Lambda\Xi_{\text{II}}^{l},\Lambda\Xi_{\text{II}}^{m}\right)$, then $SUPP\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)\geq SUPP\left(\Lambda\Xi_{\text{II}}^{l},\Lambda\Xi_{\text{II}}^{m}\right)$, where $DIS\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)$ represents the distance between $\Lambda\Xi_{\text{II}}^{\partial}$ and $\Lambda\Xi_{\text{II}}^{k}$.

3 | POWER INTERACTION AGGREGATION OPERATORS FOR CIFSS

In this section, we address the idea of interaction operational laws and further with the help of these laws, we evaluate the idea of the CIFPIA operator, CIFPIWA operator, CIFPIG operator, CIFPIWG operator, and their basic properties such as idempotency, monotonicity, and boundedness.

Definition 4: Let $\Lambda\Xi_{\text{II}}^{\partial}=\left(\left(\equiv_{\partial}^{\Lambda\Xi_{\text{AT}}},\equiv_{\partial}^{\Lambda\Xi_{\text{PT}}}\right),\left(\exists_{\partial}^{\Lambda\Xi_{\text{AT}}},\exists_{\partial}^{\Lambda\Xi_{\text{PT}}}\right)\right),\partial=1,2$ be any two CIFNs. Then

$$\Lambda\Xi_{\text{II}}^{1}\oplus\Lambda\Xi_{\text{II}}^{2}=\begin{pmatrix}\left(\equiv_{1}^{\Lambda\Xi_{\text{AT}}}+\equiv_{2}^{\Lambda\Xi_{\text{AT}}}-\equiv_{1}^{\Lambda\Xi_{\text{AT}}}\equiv_{2}^{\Lambda\Xi_{\text{AT}}},\equiv_{1}^{\Lambda\Xi_{\text{PT}}}+\equiv_{2}^{\Lambda\Xi_{\text{PT}}}-\equiv_{1}^{\Lambda\Xi_{\text{PT}}}\equiv_{2}^{\Lambda\Xi_{\text{PT}}}\right),\\ \begin{pmatrix}\exists_{1}^{\Lambda\Xi_{\text{AT}}}+\exists_{2}^{\Lambda\Xi_{\text{AT}}}-\exists_{1}^{\Lambda\Xi_{\text{AT}}}\exists_{2}^{\Lambda\Xi_{\text{AT}}}-\equiv_{1}^{\Lambda\Xi_{\text{AT}}}\exists_{2}^{\Lambda\Xi_{\text{AT}}}-\exists_{1}^{\Lambda\Xi_{\text{AT}}}\equiv_{2}^{\Lambda\Xi_{\text{AT}}},\\ \exists_{1}^{\Lambda\Xi_{\text{PT}}}+\exists_{2}^{\Lambda\Xi_{\text{PT}}}-\exists_{1}^{\Lambda\Xi_{\text{PT}}}\exists_{2}^{\Lambda\Xi_{\text{PT}}}-\equiv_{1}^{\Lambda\Xi_{\text{PT}}}\exists_{2}^{\Lambda\Xi_{\text{PT}}}-\exists_{1}^{\Lambda\Xi_{\text{PT}}}\equiv_{2}^{\Lambda\Xi_{\text{PT}}}\end{pmatrix}\end{pmatrix}$$

$$\Lambda\Xi_{\text{II}}^{1}\oplus\Lambda\Xi_{\text{II}}^{2}=\begin{pmatrix}\begin{pmatrix}\equiv_{1}^{\Lambda\Xi_{\text{AT}}}+\equiv_{2}^{\Lambda\Xi_{\text{AT}}}-\equiv_{1}^{\Lambda\Xi_{\text{AT}}}\equiv_{2}^{\Lambda\Xi_{\text{AT}}}-\exists_{1}^{\Lambda\Xi_{\text{AT}}}\equiv_{2}^{\Lambda\Xi_{\text{AT}}}-\equiv_{1}^{\Lambda\Xi_{\text{AT}}}\exists_{2}^{\Lambda\Xi_{\text{AT}}},\\ \equiv_{1}^{\Lambda\Xi_{\text{PT}}}+\equiv_{2}^{\Lambda\Xi_{\text{PT}}}-\equiv_{1}^{\Lambda\Xi_{\text{PT}}}\equiv_{2}^{\Lambda\Xi_{\text{PT}}}-\exists_{1}^{\Lambda\Xi_{\text{PT}}}\equiv_{2}^{\Lambda\Xi_{\text{PT}}}-\equiv_{1}^{\Lambda\Xi_{\text{PT}}}\exists_{2}^{\Lambda\Xi_{\text{PT}}}\end{pmatrix},\\ \left(\exists_{1}^{\Lambda\Xi_{\text{AT}}}+\exists_{2}^{\Lambda\Xi_{\text{AT}}}-\exists_{1}^{\Lambda\Xi_{\text{AT}}}\exists_{2}^{\Lambda\Xi_{\text{AT}}},\exists_{1}^{\Lambda\Xi_{\text{PT}}}+\exists_{2}^{\Lambda\Xi_{\text{PT}}}-\exists_{1}^{\Lambda\Xi_{\text{PT}}}\exists_{2}^{\Lambda\Xi_{\text{PT}}}\right)\end{pmatrix}$$

$$\widetilde{\Omega}_{s}\Lambda\Xi_{\text{II}}^{1}=\begin{pmatrix}\left(1-\left(1-\equiv_{1}^{\Lambda\Xi_{\text{AT}}}\right)^{\widetilde{\Omega}_{s}},1-\left(1-\equiv_{1}^{\Lambda\Xi_{\text{PT}}}\right)^{\widetilde{\Omega}_{s}}\right),\\ \begin{pmatrix}\left(1-\equiv_{1}^{\Lambda\Xi_{\text{AT}}}\right)^{\widetilde{\Omega}_{s}}-\left(1-\left(\equiv_{1}^{\Lambda\Xi_{\text{AT}}}+\exists_{1}^{\Lambda\Xi_{\text{AT}}}\right)\right)^{\widetilde{\Omega}_{s}},\\ \left(1-\equiv_{1}^{\Lambda\Xi_{\text{PT}}}\right)^{\widetilde{\Omega}_{s}}-\left(1-\left(\equiv_{1}^{\Lambda\Xi_{\text{PT}}}+\exists_{1}^{\Lambda\Xi_{\text{PT}}}\right)\right)^{\widetilde{\Omega}_{s}}\end{pmatrix}\end{pmatrix}$$

$$\left(\Lambda\Xi_{\text{II}}^{1}\right)^{\widetilde{\Omega}_{s}}=\begin{pmatrix}\begin{pmatrix}\left(1-\exists_{1}^{\Lambda\Xi_{\text{AT}}}\right)^{\widetilde{\Omega}_{s}}-\left(1-\left(\exists_{1}^{\Lambda\Xi_{\text{AT}}}+\equiv_{1}^{\Lambda\Xi_{\text{AT}}}\right)\right)^{\widetilde{\Omega}_{s}},\\ \left(1-\exists_{1}^{\Lambda\Xi_{\text{PT}}}\right)^{\widetilde{\Omega}_{s}}-\left(1-\left(\exists_{1}^{\Lambda\Xi_{\text{PT}}}+\equiv_{1}^{\Lambda\Xi_{\text{PT}}}\right)\right)^{\widetilde{\Omega}_{s}}\end{pmatrix},\\ \left(1-\left(1-\exists_{1}^{\Lambda\Xi_{\text{AT}}}\right)^{\widetilde{\Omega}_{s}},1-\left(1-\exists_{1}^{\Lambda\Xi_{\text{PT}}}\right)^{\widetilde{\Omega}_{s}}\right)\end{pmatrix}$$

The above information is called interaction operational laws for the CIFNs. Furthermore, we derive some operators based on the above interaction operational laws.

Definition 5: Let $\Lambda\Xi_{\text{II}}^{\partial}=\left(\left(\equiv_{\partial}^{\Lambda\Xi_{\text{AT}}},\equiv_{\partial}^{\Lambda\Xi_{\text{PT}}}\right),\left(\exists_{\partial}^{\Lambda\Xi_{\text{AT}}},\exists_{\partial}^{\Lambda\Xi_{\text{PT}}}\right)\right),\partial=1,2,\ldots,\alpha$ be any collection of CIFNs. Then

$$CIFPIA\left(\Lambda\Xi_{\text{II}}^{1},\Lambda\Xi_{\text{II}}^{2},\ldots,\Lambda\Xi_{\text{II}}^{\alpha}\right)=\oplus_{\partial=1}^{\alpha}\left(\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)\Lambda\Xi_{\text{II}}^{\partial}}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}\right) \quad (1)$$

Called CIFPIA operator with ${}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)=\sum_{\substack{\partial=1,\\ \partial\neq k}}^{\alpha}SUPP\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)$, where $SUPP\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)=1-DIS\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)$ represents the support degree between $\Lambda\Xi_{\text{II}}^{\partial}$ and $\Lambda\Xi_{\text{II}}^{k}$ with the following conditions:

1) $SUPP\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)\in[0,1]$.
2) $SUPP\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)=SUPP\left(\Lambda\Xi_{\text{II}}^{k},\Lambda\Xi_{\text{II}}^{\partial}\right)$.
3) If $DIS\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)<DIS\left(\Lambda\Xi_{\text{II}}^{l},\Lambda\Xi_{\text{II}}^{m}\right)$, then $SUPP\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)\geq SUPP\left(\Lambda\Xi_{\text{II}}^{l},\Lambda\Xi_{\text{II}}^{m}\right)$, where $DIS\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)$ represents the distance between $\Lambda\Xi_{\text{II}}^{\partial}$ and $\Lambda\Xi_{\text{II}}^{k}$.

Where,

$$DIS\left(\Lambda\Xi_{\text{II}}^{\partial},\Lambda\Xi_{\text{II}}^{k}\right)=\frac{1}{4}\left(\left|\equiv_{\partial}^{\Lambda\Xi_{\text{AT}}}-\equiv_{k}^{\Lambda\Xi_{\text{AT}}}\right|+\left|\equiv_{\partial}^{\Lambda\Xi_{\text{PT}}}-\equiv_{k}^{\Lambda\Xi_{\text{PT}}}\right|+\left|\exists_{\partial}^{\Lambda\Xi_{\text{AT}}}-\exists_{k}^{\Lambda\Xi_{\text{AT}}}\right|+\left|\exists_{\partial}^{\Lambda\Xi_{\text{PT}}}-\exists_{k}^{\Lambda\Xi_{\text{PT}}}\right|\right)$$

Theorem 1: Let $\Lambda\Xi_{\text{II}}^{\partial}=\left(\left(\equiv_{\partial}^{\Lambda\Xi_{\text{AT}}},\equiv_{\partial}^{\Lambda\Xi_{\text{PT}}}\right),\left(\exists_{\partial}^{\Lambda\Xi_{\text{AT}}},\exists_{\partial}^{\Lambda\Xi_{\text{PT}}}\right)\right),\partial=1,2,\ldots,\alpha$ be any collection of CIFNs. Then we expose that the aggregated value of Equation 1 is also a CIFN, such as

$$CIFPIA\left(\Lambda\Xi_{\text{II}}^{1},\Lambda\Xi_{\text{II}}^{2},\ldots,\Lambda\Xi_{\text{II}}^{\alpha}\right)=\left(\begin{array}{c}\left(1-\prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{\text{AT}}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}},1-\prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{\text{PT}}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}}\right),\\ \left(\begin{array}{c}\prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{\text{AT}}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\alpha}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{\text{AT}}}+\exists_{\partial}^{\Lambda\Xi_{\text{AT}}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}},\\ \prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{\text{PT}}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\alpha}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{\text{PT}}}+\exists_{\partial}^{\Lambda\Xi_{\text{PT}}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}}\end{array}\right)\end{array}\right) \quad (2)$$

Proof: Here, we use the procedure of mathematical induction and derive the data in Equation 2. For this, we prove it for $\alpha=2$, such as

$$\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}\Lambda\Xi_{\text{II}}^{1}=\left(\begin{array}{c}\left(1-\left(1-\equiv_{1}^{\Lambda\Xi_{\text{AT}}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}},1-\left(1-\equiv_{1}^{\Lambda\Xi_{\text{PT}}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}}\right),\\ \left(\begin{array}{c}\left(1-\equiv_{1}^{\Lambda\Xi_{\text{AT}}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}}-\left(1-\left(\equiv_{1}^{\Lambda\Xi_{\text{AT}}}+\exists_{1}^{\Lambda\Xi_{\text{AT}}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}},\\ \left(1-\equiv_{1}^{\Lambda\Xi_{\text{PT}}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}}-\left(1-\left(\equiv_{1}^{\Lambda\Xi_{\text{PT}}}+\exists_{1}^{\Lambda\Xi_{\text{PT}}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}}\end{array}\right)\end{array}\right)$$

$$\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}\Lambda\Xi_{II}^{2}=\left(\begin{array}{c}\left(1-\left(1-\equiv_{2}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},1-\left(1-\equiv_{2}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\right),\\ \left(\begin{array}{c}\left(1-\equiv_{2}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\left(1-\left(\equiv_{2}^{\Lambda\Xi_{AT}}+\exists_{2}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},\\ \left(1-\equiv_{2}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\left(1-\left(\equiv_{2}^{\Lambda\Xi_{PT}}+\exists_{2}^{\Lambda\Xi_{PT}}\right)\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\end{array}\right)\end{array}\right)$$

Thus,

$$CIFPIA\left(\Lambda\Xi_{II}^{1},\Lambda\Xi_{II}^{2}\right)=\oplus_{\partial=1}^{2}\left(\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)\Lambda\Xi_{II}^{\partial}}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}\right)$$

$$=\left(\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{1}\right)\right)\Lambda\Xi_{II}^{1}}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}\right)\oplus\left(\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)\Lambda\Xi_{II}^{2}}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}\right)$$

$$=\left(\begin{array}{c}\left(1-\left(1-\equiv_{1}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},1-\left(1-\equiv_{1}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\right),\\ \left(\begin{array}{c}\left(1-\equiv_{1}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\left(1-\left(\equiv_{1}^{\Lambda\Xi_{AT}}+\exists_{1}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},\\ \left(1-\equiv_{1}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\left(1-\left(\equiv_{1}^{\Lambda\Xi_{PT}}+\exists_{1}^{\Lambda\Xi_{PT}}\right)\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{1}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\end{array}\right)\end{array}\right)$$

$$\oplus\left(\begin{array}{c}\left(1-\left(1-\equiv_{2}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},1-\left(1-\equiv_{2}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\right),\\ \left(\begin{array}{c}\left(1-\equiv_{2}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\left(1-\left(\equiv_{2}^{\Lambda\Xi_{AT}}+\exists_{2}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},\\ \left(1-\equiv_{2}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\left(1-\left(\equiv_{2}^{\Lambda\Xi_{PT}}+\exists_{2}^{\Lambda\Xi_{PT}}\right)\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{2}\right)\right)}{\sum_{\partial=1}^{2}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\end{array}\right)\end{array}\right)$$

$$=\left(\begin{array}{c}\left(1-\prod_{\partial=1}^{2}\left(1-\equiv_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},1-\prod_{\partial=1}^{2}\left(1-\equiv_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\right),\\ \left(\begin{array}{c}\prod_{\partial=1}^{2}\left(1-\equiv_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\prod_{\partial=1}^{2}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}+\exists_{\partial}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},\\ \prod_{\partial=1}^{2}\left(1-\equiv_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\prod_{\partial=1}^{2}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{PT}}+\exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right)^{\frac{\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^\circ C^\circ C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\end{array}\right)\end{array}\right)$$

The theory in Equation 2 is correct for $\alpha=2$, we assume that it is also correct for $\alpha=\mathfrak{r}$, then

$$
\begin{aligned}
&CIFPIA\left(\Lambda\Xi_{II}^{1},\Lambda\Xi_{II}^{2},\dots,\Lambda\Xi_{II}^{\mathfrak{r}}\right)\\
&=\left(\begin{array}{c}
\left(1-\prod_{\partial=1}^{\mathfrak{r}}\left(1-\equiv_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},1-\prod_{\partial=1}^{\mathfrak{r}}\left(1-\equiv_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\right),\\
\left(\begin{array}{c}
\prod_{\partial=1}^{\mathfrak{r}}\left(1-\equiv_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\mathfrak{r}}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}+\exists_{\partial}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},\\
\prod_{\partial=1}^{\mathfrak{r}}\left(1-\equiv_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\mathfrak{r}}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{PT}}+\exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}
\end{array}\right)
\end{array}\right)
\end{aligned}
$$

Thus, we prove it for $\alpha=\mathfrak{r}+1$, such as

$$
CIFPIA\left(\Lambda\Xi_{II}^{1},\Lambda\Xi_{II}^{2}\right)=\bigoplus_{\partial=1}^{\mathfrak{r}+1}\left(\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)\Lambda\Xi_{II}^{\partial}}{\sum_{\partial=1}^{\mathfrak{r}+1}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}\right)
$$

$$
=\bigoplus_{\partial=1}^{\mathfrak{r}}\left(\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)\Lambda\Xi_{II}^{\partial}}{\sum_{\partial=1}^{\mathfrak{r}+1}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}\right)\oplus\left(\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\mathfrak{r}+1}\right)\right)\Lambda\Xi_{II}^{\mathfrak{r}+1}}{\sum_{\partial=1}^{\mathfrak{r}+1}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}\right)
$$

$$
\begin{aligned}
&=\left(\begin{array}{c}
\left(1-\prod_{\partial=1}^{\mathfrak{r}}\left(1-\equiv_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},1-\prod_{\partial=1}^{\mathfrak{r}}\left(1-\equiv_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\right),\\
\left(\begin{array}{c}
\prod_{\partial=1}^{\mathfrak{r}}\left(1-\equiv_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\mathfrak{r}}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}+\exists_{\partial}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},\\
\prod_{\partial=1}^{\mathfrak{r}}\left(1-\equiv_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\mathfrak{r}}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{PT}}+\exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}
\end{array}\right)
\end{array}\right)\\
&\oplus\left(\begin{array}{c}
\left(1-\left(1-\equiv_{\mathfrak{r}+1}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\mathfrak{r}+1}\right)\right)}{\sum_{\partial=1}^{\mathfrak{r}+1}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},1-\left(1-\equiv_{\mathfrak{r}+1}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\mathfrak{r}+1}\right)\right)}{\sum_{\partial=1}^{\mathfrak{r}+1}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\right),\\
\left(\begin{array}{c}
\left(1-\equiv_{\mathfrak{r}+1}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\mathfrak{r}+1}\right)\right)}{\sum_{\partial=1}^{\mathfrak{r}+1}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\left(1-\left(\equiv_{\mathfrak{r}+1}^{\Lambda\Xi_{AT}}+\exists_{\mathfrak{r}+1}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\mathfrak{r}+1}\right)\right)}{\sum_{\partial=1}^{\mathfrak{r}+1}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},\\
\left(1-\equiv_{\mathfrak{r}+1}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\mathfrak{r}+1}\right)\right)}{\sum_{\partial=1}^{\mathfrak{r}+1}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\left(1-\left(\equiv_{\mathfrak{r}+1}^{\Lambda\Xi_{PT}}+\exists_{\mathfrak{r}+1}^{\Lambda\Xi_{PT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\mathfrak{r}+1}\right)\right)}{\sum_{\partial=1}^{\mathfrak{r}+1}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}
\end{array}\right)
\end{array}\right)
\end{aligned}
$$

$$
=\left(\begin{array}{c}
\left(1-\prod_{\partial=1}^{\mathfrak{r}+1}\left(1-\equiv_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},1-\prod_{\partial=1}^{\mathfrak{r}+1}\left(1-\equiv_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\right),\\
\left(\begin{array}{c}
\prod_{\partial=1}^{\mathfrak{r}+1}\left(1-\equiv_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\mathfrak{r}+1}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}+\exists_{\partial}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}},\\
\prod_{\partial=1}^{\mathfrak{r}+1}\left(1-\equiv_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\mathfrak{r}+1}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{PT}}+\exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}
\end{array}\right)
\end{array}\right)
$$

Hence, we effectively proved our main results for all positive integers α. Moreover, we derive the idempotency, monotonicity, and boundedness.

Property 1: (Idempotency) If $\Lambda\Xi_{\mathrm{II}}^{\partial} = \Lambda\Xi_{\mathrm{II}} = \left((\equiv^{\Lambda\Xi_{\mathrm{AT}}}, \equiv^{\Lambda\Xi_{\mathrm{PT}}}), (\exists^{\Lambda\Xi_{\mathrm{AT}}}, \exists^{\Lambda\Xi_{\mathrm{PT}}})\right), \partial = 1,2,\dots,\alpha,$ then

$$CIFPIA(\Lambda\Xi_{\mathrm{II}}^{1}, \Lambda\Xi_{\mathrm{II}}^{2}, \dots, \Lambda\Xi_{\mathrm{II}}^{\alpha}) = \Lambda\Xi_{\mathrm{II}}$$

Proof: Consider $\Lambda\Xi_{\mathrm{II}}^{\partial} = \Lambda\Xi_{\mathrm{II}} = \left((\equiv^{\Lambda\Xi_{\mathrm{AT}}}, \equiv^{\Lambda\Xi_{\mathrm{PT}}}), (\exists^{\Lambda\Xi_{\mathrm{AT}}}, \exists^{\Lambda\Xi_{\mathrm{PT}}})\right), \partial = 1,2,\dots,\alpha,$ then

$$CIFPIA(\Lambda\Xi_{\mathrm{II}}^{1}, \Lambda\Xi_{\mathrm{II}}^{2}, \dots, \Lambda\Xi_{\mathrm{II}}^{\alpha})$$

$$= \left(\begin{array}{c} \left(1-\prod_{\partial=1}^{\alpha}(1-\equiv^{\Lambda\Xi_{\mathrm{AT}}})^{\frac{\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}}, 1-\prod_{\partial=1}^{\alpha}(1-\equiv^{\Lambda\Xi_{\mathrm{PT}}})^{\frac{\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}}\right), \\ \left(\begin{array}{c} \prod_{\partial=1}^{\alpha}(1-\equiv^{\Lambda\Xi_{\mathrm{AT}}})^{\frac{\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}} - \prod_{\partial=1}^{\alpha}\left(1-(\equiv^{\Lambda\Xi_{\mathrm{AT}}}+\exists^{\Lambda\Xi_{\mathrm{AT}}})\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}}, \\ \prod_{\partial=1}^{\alpha}(1-\equiv^{\Lambda\Xi_{\mathrm{PT}}})^{\frac{\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}} - \prod_{\partial=1}^{\alpha}\left(1-(\equiv^{\Lambda\Xi_{\mathrm{PT}}}+\exists^{\Lambda\Xi_{\mathrm{PT}}})\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}} \end{array}\right) \end{array}\right)$$

$$= \left(\begin{array}{c} \left(1-(1-\equiv^{\Lambda\Xi_{\mathrm{AT}}})^{\frac{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}}, 1-(1-\equiv^{\Lambda\Xi_{\mathrm{PT}}})^{\frac{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}}\right), \\ \left(\begin{array}{c} (1-\equiv^{\Lambda\Xi_{\mathrm{AT}}})^{\frac{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}} - \left(1-(\equiv^{\Lambda\Xi_{\mathrm{AT}}}+\exists^{\Lambda\Xi_{\mathrm{AT}}})\right)^{\frac{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}}, \\ (1-\equiv^{\Lambda\Xi_{\mathrm{PT}}})^{\frac{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}} - \left(1-(\equiv^{\Lambda\Xi_{\mathrm{PT}}}+\exists^{\Lambda\Xi_{\mathrm{PT}}})\right)^{\frac{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}} \end{array}\right) \end{array}\right)$$

$$= \left(\begin{array}{c} \left(1-(1-\equiv^{\Lambda\Xi_{\mathrm{AT}}}), 1-(1-\equiv^{\Lambda\Xi_{\mathrm{PT}}})\right), \\ \left(\begin{array}{c} (1-\equiv^{\Lambda\Xi_{\mathrm{AT}}}) - \left(1-(\equiv^{\Lambda\Xi_{\mathrm{AT}}}+\exists^{\Lambda\Xi_{\mathrm{AT}}})\right), \\ (1-\equiv^{\Lambda\Xi_{\mathrm{PT}}}) - \left(1-(\equiv^{\Lambda\Xi_{\mathrm{PT}}}+\exists^{\Lambda\Xi_{\mathrm{PT}}})\right) \end{array}\right) \end{array}\right), \frac{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)} = 1$$

$$= \left(\begin{array}{c} (1-1+\equiv^{\Lambda\Xi_{\mathrm{AT}}}, 1-1+\equiv^{\Lambda\Xi_{\mathrm{PT}}}), \\ \left(\begin{array}{c} (1-\equiv^{\Lambda\Xi_{\mathrm{AT}}}) - (1-\equiv^{\Lambda\Xi_{\mathrm{AT}}}-\exists^{\Lambda\Xi_{\mathrm{AT}}}), \\ (1-\equiv^{\Lambda\Xi_{\mathrm{PT}}}) - (1-\equiv^{\Lambda\Xi_{\mathrm{PT}}}-\exists^{\Lambda\Xi_{\mathrm{PT}}}) \end{array}\right) \end{array}\right)$$

$$= \left((\equiv^{\Lambda\Xi_{\mathrm{AT}}}, \equiv^{\Lambda\Xi_{\mathrm{PT}}}), (\exists^{\Lambda\Xi_{\mathrm{AT}}}, \exists^{\Lambda\Xi_{\mathrm{PT}}})\right) = \Lambda\Xi_{\mathrm{II}}.$$

Property 2: (Monotonicity) If $\Lambda\Xi_{\mathrm{II}}^{\partial} \leq \Lambda\Xi\%_{\mathrm{II}}^{\partial}$, then

$$CIFPIA(\Lambda\Xi_{\mathrm{II}}^{1}, \Lambda\Xi_{\mathrm{II}}^{2}, \dots, \Lambda\Xi_{\mathrm{II}}^{\alpha}) \leq CIFPIA\left(\Lambda\Xi\%_{\mathrm{II}}^{1}, \Lambda\Xi\%_{\mathrm{II}}^{2}, \dots, \Lambda\Xi\%_{\mathrm{II}}^{\alpha}\right)$$

Proof: Let$\Lambda\Xi_{\mathrm{II}}^{\partial} \leq \Lambda\Xi\%_{\mathrm{II}}^{\partial}$, thus $\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} \leq \equiv\%_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}} \leq \equiv\%_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}$ and $\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} \geq \exists\%_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}} \geq \exists\%_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}$, then

$$\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} \leq \equiv\%_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} \Rightarrow 1-\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} \geq 1-\equiv\%_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}$$

$$\Rightarrow (1-\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}})^{\frac{\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}} \geq \left(1-\equiv\%_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}}$$

$$\Rightarrow \prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}} \geq \prod_{\partial=1}^{\alpha}\left(1-\equiv\%_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}$$

$$\Rightarrow -\prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}} \leq -\prod_{\partial=1}^{\alpha}\left(1-\equiv\%_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}$$

$$\Rightarrow 1-\prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}} \leq 1-\prod_{\partial=1}^{\alpha}\left(1-\equiv\%_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}$$

Further, to follow the same procedure, we have

$$\equiv_{\partial}^{\Lambda\Xi_{PT}} \leq \equiv\%_{\partial}^{\Lambda\Xi_{PT}} \Rightarrow 1-\prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}} \leq 1-\prod_{\partial=1}^{\alpha}\left(1-\equiv\%_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}$$

Moreover, we evaluate the amplitude term of non-membership information, such as

$$\exists_{\partial}^{\Lambda\Xi_{AT}} \geq \exists\%_{\partial}^{\Lambda\Xi_{AT}} \Rightarrow \equiv_{\partial}^{\Lambda\Xi_{AT}}+\exists_{\partial}^{\Lambda\Xi_{AT}} \geq \equiv\%_{\partial}^{\Lambda\Xi_{AT}}+\exists\%_{\partial}^{\Lambda\Xi_{AT}}$$

$$\Rightarrow \left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}+\exists_{\partial}^{\Lambda\Xi_{AT}}\right)\right) \leq \left(1-\left(\equiv\%_{\partial}^{\Lambda\Xi_{AT}}+\exists\%_{\partial}^{\Lambda\Xi_{AT}}\right)\right)$$

$$\Rightarrow \left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}+\exists_{\partial}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}} \leq \left(1-\left(\equiv\%_{\partial}^{\Lambda\Xi_{AT}}+\exists\%_{\partial}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}$$

$$\Rightarrow \prod_{\partial=1}^{\alpha}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}+\exists_{\partial}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}} \leq \prod_{\partial=1}^{\alpha}\left(1-\left(\equiv\%_{\partial}^{\Lambda\Xi_{AT}}+\exists\%_{\partial}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}$$

$$\begin{aligned}\Rightarrow &\prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\alpha}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}+\exists_{\partial}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\\ &\geq \prod_{\partial=1}^{\alpha}\left(1-\equiv\%_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\alpha}\left(1-\left(\equiv\%_{\partial}^{\Lambda\Xi_{AT}}+\exists\%_{\partial}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\end{aligned}$$

Further, to follow the same procedure, we have

$$\begin{aligned}\exists_{\partial}^{\Lambda\Xi_{PT}} \geq \exists\%_{\partial}^{\Lambda\Xi_{PT}} \Rightarrow &\prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\alpha}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{PT}}+\exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\\ &\geq \prod_{\partial=1}^{\alpha}\left(1-\equiv\%_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\alpha}\left(1-\left(\equiv\%_{\partial}^{\Lambda\Xi_{PT}}+\exists\%_{\partial}^{\Lambda\Xi_{PT}}\right)\right)^{\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{II}^{\partial}\right)\right)}}\end{aligned}$$

Hence, using the above four inequalities, and by using the idea of score and accuracy functions, we can easily obtain our targeted results, such as

$$CIFPIA\left(\Lambda\Xi_{II}^{1},\Lambda\Xi_{II}^{2},\ldots,\Lambda\Xi_{II}^{\alpha}\right) \leq CIFPIA\left(\Lambda\Xi\%_{II}^{1},\Lambda\Xi\%_{II}^{2},\ldots,\Lambda\Xi\%_{II}^{\alpha}\right).$$

Property 3: (Boundedness) If $\Lambda\Xi_{\mathrm{II}}^{-}=\left(\left(\min_{\partial}\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\min_{\partial}\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right),\left(\max_{\partial}\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\max_{\partial}\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right)$ and $\Lambda\Xi_{\mathrm{II}}^{+}=\left(\left(\max_{\partial}\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\max_{\partial}\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right),\left(\min_{\partial}\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\min_{\partial}\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right)$, thus

$$\Lambda\Xi_{\mathrm{II}}^{-}\le CIFPIA\left(\Lambda\Xi_{\mathrm{II}}^{1},\Lambda\Xi_{\mathrm{II}}^{2},\dots,\Lambda\Xi_{\mathrm{II}}^{\alpha}\right)\le\Lambda\Xi_{\mathrm{II}}^{+}.$$

Proof: Consider that $\Lambda\Xi_{\mathrm{II}}^{-}=\left(\left(\min_{\partial}\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\min_{\partial}\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right),\left(\max_{\partial}\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\max_{\partial}\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right)$ and $\Lambda\Xi_{\mathrm{II}}^{+}=\left(\left(\max_{\partial}\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\max_{\partial}\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right),\left(\min_{\partial}\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\min_{\partial}\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right)$, then by using property 1 and property 2, we have

$$CIFPIA\left(\Lambda\Xi_{\mathrm{II}}^{1},\Lambda\Xi_{\mathrm{II}}^{2},\dots,\Lambda\Xi_{\mathrm{II}}^{\alpha}\right)\le CIFPIA\left(\Lambda\Xi^{+}{}_{\mathrm{II}}^{1},\Lambda\Xi^{+}{}_{\mathrm{II}}^{2},\dots,\Lambda\Xi^{+}{}_{\mathrm{II}}^{\alpha}\right)=\Lambda\Xi_{\mathrm{II}}^{+}$$
$$CIFPIA\left(\Lambda\Xi_{\mathrm{II}}^{1},\Lambda\Xi_{\mathrm{II}}^{2},\dots,\Lambda\Xi_{\mathrm{II}}^{\alpha}\right)\ge CIFPIA\left(\Lambda\Xi^{-}{}_{\mathrm{II}}^{1},\Lambda\Xi^{-}{}_{\mathrm{II}}^{2},\dots,\Lambda\Xi^{-}{}_{\mathrm{II}}^{\alpha}\right)=\Lambda\Xi_{\mathrm{II}}^{-}$$

Then

$$\Lambda\Xi_{\mathrm{II}}^{-}\le CIFPIA\left(\Lambda\Xi_{\mathrm{II}}^{1},\Lambda\Xi_{\mathrm{II}}^{2},\dots,\Lambda\Xi_{\mathrm{II}}^{\alpha}\right)\le\Lambda\Xi_{\mathrm{II}}^{+}.$$

Definition 6: Let $\Lambda\Xi_{\mathrm{II}}^{\partial}=\left(\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right),\left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right),\partial=1,2,\dots,\alpha$ be any collection of CIFNs. Then

$$CIFPIWA\left(\Lambda\Xi_{\mathrm{II}}^{1},\Lambda\Xi_{\mathrm{II}}^{2},\dots,\Lambda\Xi_{\mathrm{II}}^{\alpha}\right)=\bigoplus_{\partial=1}^{\alpha}\left(\frac{\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)\Lambda\Xi_{\mathrm{II}}^{\partial}}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}\right)\tag{3}$$

Called CIFPIWA operator with ${}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)=\sum_{\substack{\partial=1,\\ \partial\ne k}}^{\alpha}SUPP\left(\Lambda\Xi_{\mathrm{II}}^{\partial},\Lambda\Xi_{\mathrm{II}}^{k}\right)$, where $SUPP\left(\Lambda\Xi_{\mathrm{II}}^{\partial},\Lambda\Xi_{\mathrm{II}}^{k}\right)=1-DIS\left(\Lambda\Xi_{\mathrm{II}}^{\partial},\Lambda\Xi_{\mathrm{II}}^{k}\right)$ represents the support degree between $\Lambda\Xi_{\mathrm{II}}^{\partial}$ and $\Lambda\Xi_{\mathrm{II}}^{k}$ with the following conditions:

1) $SUPP\left(\Lambda\Xi_{\mathrm{II}}^{\partial},\Lambda\Xi_{\mathrm{II}}^{k}\right)\in[0,1]$.
2) $SUPP\left(\Lambda\Xi_{\mathrm{II}}^{\partial},\Lambda\Xi_{\mathrm{II}}^{k}\right)=SUPP\left(\Lambda\Xi_{\mathrm{II}}^{k},\Lambda\Xi_{\mathrm{II}}^{\partial}\right)$.
3) If $DIS\left(\Lambda\Xi_{\mathrm{II}}^{\partial},\Lambda\Xi_{\mathrm{II}}^{k}\right)<DIS\left(\Lambda\Xi_{\mathrm{II}}^{l},\Lambda\Xi_{\mathrm{II}}^{m}\right)$, then $SUPP\left(\Lambda\Xi_{\mathrm{II}}^{\partial},\Lambda\Xi_{\mathrm{II}}^{k}\right)\ge SUPP\left(\Lambda\Xi_{\mathrm{II}}^{l},\Lambda\Xi_{\mathrm{II}}^{m}\right)$, where $DIS\left(\Lambda\Xi_{\mathrm{II}}^{\partial},\Lambda\Xi_{\mathrm{II}}^{k}\right)$ represents the distance between $\Lambda\Xi_{\mathrm{II}}^{\partial}$ and $\Lambda\Xi_{\mathrm{II}}^{k}$.

Where,

$$DIS\left(\Lambda\Xi_{\mathrm{II}}^{\partial},\Lambda\Xi_{\mathrm{II}}^{k}\right)=\frac{1}{4}\left(\left|\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}-\equiv_{k}^{\Lambda\Xi_{\mathrm{AT}}}\right|+\left|\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}-\equiv_{k}^{\Lambda\Xi_{\mathrm{PT}}}\right|+\left|\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}-\exists_{k}^{\Lambda\Xi_{\mathrm{AT}}}\right|+\left|\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}-\exists_{k}^{\Lambda\Xi_{\mathrm{PT}}}\right|\right)$$

Further, the weight vector is stated by: $\beta_{\partial}^{w}\in[0,1]$ with $\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}=1$.

Theorem 2: Let $\Lambda\Xi_{\mathrm{II}}^{\partial}=\left(\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right),\left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right),\partial=1,2,\dots,\alpha$ be any collection of CIFNs. Then we expose that the aggregated value of Equation 3 is also a CIFN, such as

$$\begin{aligned}&CIFPIWA\left(\Lambda\Xi_{\mathrm{II}}^{1},\Lambda\Xi_{\mathrm{II}}^{2},\dots,\Lambda\Xi_{\mathrm{II}}^{\alpha}\right)\\&=\left(\begin{array}{l}\left(1-\prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}\right)^{\frac{\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}},1-\prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)^{\frac{\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}}\right),\\ \left(\begin{array}{l}\prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}\right)^{\frac{\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\alpha}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}+\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}\right)\right)^{\frac{\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}},\\ \prod_{\partial=1}^{\alpha}\left(1-\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)^{\frac{\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}}-\prod_{\partial=1}^{\alpha}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}+\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right)^{\frac{\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+{}^{\circ}\mathrm{C}^{\circ}\mathrm{C}\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}}\end{array}\right)\end{array}\right)\end{aligned}\tag{4}$$

Property 4: (Idempotency) If $\Lambda\Xi_{\mathrm{II}}^{\partial}=\Lambda\Xi_{\mathrm{II}}=\left(\left(\equiv^{\Lambda\Xi_{\mathrm{AT}}},\equiv^{\Lambda\Xi_{\mathrm{PT}}}\right),\left(\exists^{\Lambda\Xi_{\mathrm{AT}}},\exists^{\Lambda\Xi_{\mathrm{PT}}}\right)\right),\partial=1,2,\dots,\alpha$, then

$$CIFPIWA\left(\Lambda\Xi_{\mathrm{II}}^{1},\Lambda\Xi_{\mathrm{II}}^{2},\dots,\Lambda\Xi_{\mathrm{II}}^{\alpha}\right)=\Lambda\Xi_{\mathrm{II}}$$

Property 5: (Monotonicity) If $\Lambda\Xi_{\mathrm{II}}^{\partial}\le\Lambda\Xi\%_{\mathrm{II}}^{\partial}$, then

$$CIFPIWA\left(\Lambda\Xi_{\mathrm{II}}^{1},\Lambda\Xi_{\mathrm{II}}^{2},\dots,\Lambda\Xi_{\mathrm{II}}^{\alpha}\right)\le CIFPIWA\left(\Lambda\Xi\%_{\mathrm{II}}^{1},\Lambda\Xi\%_{\mathrm{II}}^{2},\dots,\Lambda\Xi\%_{\mathrm{II}}^{\alpha}\right)$$

Property 6: (Boundedness) If $\Lambda\Xi_{\text{II}}^{-} = \left(\left(\min_{\partial} \equiv_{\partial}^{\Lambda\Xi_{AT}}, \min_{\partial} \equiv_{\partial}^{\Lambda\Xi_{PT}}\right), \left(\max_{\partial} \exists_{\partial}^{\Lambda\Xi_{AT}}, \max_{\partial} \exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right)$ and $\Lambda\Xi_{\text{II}}^{+} = \left(\left(\max_{\partial} \equiv_{\partial}^{\Lambda\Xi_{AT}}, \max_{\partial} \equiv_{\partial}^{\Lambda\Xi_{PT}}\right), \left(\min_{\partial} \exists_{\partial}^{\Lambda\Xi_{AT}}, \min_{\partial} \exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right)$, thus

$$\Lambda\Xi_{\text{II}}^{-} \leq CIFPIWA(\Lambda\Xi_{\text{II}}^{1}, \Lambda\Xi_{\text{II}}^{2}, \dots, \Lambda\Xi_{\text{II}}^{\alpha}) \leq \Lambda\Xi_{\text{II}}^{+}.$$

Definition 7: Let $\Lambda\Xi_{\text{II}}^{\partial} = \left(\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}, \equiv_{\partial}^{\Lambda\Xi_{PT}}\right), \left(\exists_{\partial}^{\Lambda\Xi_{AT}}, \exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right), \partial = 1,2,\dots,\alpha$ be any collection of CIFNs. Then

$$CIFPIG(\Lambda\Xi_{\text{II}}^{1}, \Lambda\Xi_{\text{II}}^{2}, \dots, \Lambda\Xi_{\text{II}}^{\alpha}) = \otimes_{\partial=1}^{\alpha} \left(\Lambda\Xi_{\text{II}}^{\partial}\right)^{\frac{\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}} \tag{5}$$

Called CIFPIG operator with $°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right) = \sum_{\substack{\partial=1, \\ \partial\neq k}}^{\alpha} SUPP\left(\Lambda\Xi_{\text{II}}^{\partial}, \Lambda\Xi_{\text{II}}^{k}\right)$, where $SUPP\left(\Lambda\Xi_{\text{II}}^{\partial}, \Lambda\Xi_{\text{II}}^{k}\right) = 1 - DIS\left(\Lambda\Xi_{\text{II}}^{\partial}, \Lambda\Xi_{\text{II}}^{k}\right)$ represents the support degree between $\Lambda\Xi_{\text{II}}^{\partial}$ and $\Lambda\Xi_{\text{II}}^{k}$ with the following conditions:

1) $SUPP\left(\Lambda\Xi_{\text{II}}^{\partial}, \Lambda\Xi_{\text{II}}^{k}\right) \in [0,1]$.
2) $SUPP\left(\Lambda\Xi_{\text{II}}^{\partial}, \Lambda\Xi_{\text{II}}^{k}\right) = SUPP\left(\Lambda\Xi_{\text{II}}^{k}, \Lambda\Xi_{\text{II}}^{\partial}\right)$.
3) If $DIS\left(\Lambda\Xi_{\text{II}}^{\partial}, \Lambda\Xi_{\text{II}}^{k}\right) < DIS\left(\Lambda\Xi_{\text{II}}^{l}, \Lambda\Xi_{\text{II}}^{m}\right)$, then $SUPP\left(\Lambda\Xi_{\text{II}}^{\partial}, \Lambda\Xi_{\text{II}}^{k}\right) \geq SUPP\left(\Lambda\Xi_{\text{II}}^{l}, \Lambda\Xi_{\text{II}}^{m}\right)$, where $DIS\left(\Lambda\Xi_{\text{II}}^{\partial}, \Lambda\Xi_{\text{II}}^{k}\right)$ represents the distance between $\Lambda\Xi_{\text{II}}^{\partial}$ and $\Lambda\Xi_{\text{II}}^{k}$.

Where,

$$DIS\left(\Lambda\Xi_{\text{II}}^{\partial}, \Lambda\Xi_{\text{II}}^{k}\right) = \frac{1}{4}\left(\left|\equiv_{\partial}^{\Lambda\Xi_{AT}} - \equiv_{k}^{\Lambda\Xi_{AT}}\right| + \left|\equiv_{\partial}^{\Lambda\Xi_{PT}} - \equiv_{k}^{\Lambda\Xi_{PT}}\right| + \left|\exists_{\partial}^{\Lambda\Xi_{AT}} - \exists_{k}^{\Lambda\Xi_{AT}}\right| + \left|\exists_{\partial}^{\Lambda\Xi_{PT}} - \exists_{k}^{\Lambda\Xi_{PT}}\right|\right)$$

Theorem 3: Let $\Lambda\Xi_{\text{II}}^{\partial} = \left(\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}, \equiv_{\partial}^{\Lambda\Xi_{PT}}\right), \left(\exists_{\partial}^{\Lambda\Xi_{AT}}, \exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right), \partial = 1,2,\dots,\alpha$ be any collection of CIFNs. Then we expose that the aggregated value of Equation 5 is also a CIFN, such as

$$\begin{aligned} &CIFPIG(\Lambda\Xi_{\text{II}}^{1}, \Lambda\Xi_{\text{II}}^{2}, \dots, \Lambda\Xi_{\text{II}}^{\alpha}) \\ &= \left(\begin{array}{c} \left(\begin{array}{c} \prod_{\partial=1}^{\alpha}\left(1-\exists_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}} - \prod_{\partial=1}^{\alpha}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}+\exists_{\partial}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}}, \\ \prod_{\partial=1}^{\alpha}\left(1-\exists_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}} - \prod_{\partial=1}^{\alpha}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{PT}}+\exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right)^{\frac{\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}} \end{array}\right), \\ \left(1-\prod_{\partial=1}^{\alpha}\left(1-\exists_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}}, 1-\prod_{\partial=1}^{\alpha}\left(1-\exists_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+°C°C\left(\Lambda\Xi_{\text{II}}^{\partial}\right)\right)}}\right) \end{array}\right) \end{aligned} \tag{6}$$

Property 7: (Idempotency) If $\Lambda\Xi_{\text{II}}^{\partial} = \Lambda\Xi_{\text{II}} = \left(\left(\equiv^{\Lambda\Xi_{AT}}, \equiv^{\Lambda\Xi_{PT}}\right), \left(\exists^{\Lambda\Xi_{AT}}, \exists^{\Lambda\Xi_{PT}}\right)\right), \partial = 1,2,\dots,\alpha$, then

$$CIFPIG(\Lambda\Xi_{\text{II}}^{1}, \Lambda\Xi_{\text{II}}^{2}, \dots, \Lambda\Xi_{\text{II}}^{\alpha}) = \Lambda\Xi_{\text{II}}$$

Property 8: (Monotonicity) If $\Lambda\Xi_{\text{II}}^{\partial} \leq \Lambda\Xi\%_{\text{II}}^{\partial}$, then

$$CIFPIG(\Lambda\Xi_{\text{II}}^{1}, \Lambda\Xi_{\text{II}}^{2}, \dots, \Lambda\Xi_{\text{II}}^{\alpha}) \leq CIFPIG\left(\Lambda\Xi\%_{\text{II}}^{1}, \Lambda\Xi\%_{\text{II}}^{2}, \dots, \Lambda\Xi\%_{\text{II}}^{\alpha}\right)$$

Property 9: (Boundedness) If $\Lambda\Xi_{\mathrm{II}}^{-} = \left(\left(\min_{\partial} \equiv_{\partial}^{\Lambda\Xi_{AT}}, \min_{\partial} \equiv_{\partial}^{\Lambda\Xi_{PT}}\right), \left(\max_{\partial} \exists_{\partial}^{\Lambda\Xi_{AT}}, \max_{\partial} \exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right)$ and $\Lambda\Xi_{\mathrm{II}}^{+} = \left(\left(\max_{\partial} \equiv_{\partial}^{\Lambda\Xi_{AT}}, \max_{\partial} \equiv_{\partial}^{\Lambda\Xi_{PT}}\right), \left(\min_{\partial} \exists_{\partial}^{\Lambda\Xi_{AT}}, \min_{\partial} \exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right)$, thus

$$\Lambda\Xi_{\mathrm{II}}^{-} \leq CIFPIG(\Lambda\Xi_{\mathrm{II}}^{1}, \Lambda\Xi_{\mathrm{II}}^{2}, \dots, \Lambda\Xi_{\mathrm{II}}^{\alpha}) \leq \Lambda\Xi_{\mathrm{II}}^{+}.$$

Definition 8: Let $\Lambda\Xi_{\mathrm{II}}^{\partial} = \left(\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}, \equiv_{\partial}^{\Lambda\Xi_{PT}}\right), \left(\exists_{\partial}^{\Lambda\Xi_{AT}}, \exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right), \partial = 1,2, \dots, \alpha$ be any collection of CIFNs. Then

$$CIFPIWG(\Lambda\Xi_{\mathrm{II}}^{1}, \Lambda\Xi_{\mathrm{II}}^{2}, \dots, \Lambda\Xi_{\mathrm{II}}^{\alpha}) = \otimes_{\partial=1}^{\alpha} \left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)^{\frac{\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}} \tag{7}$$

Called CIFPIWG operator with $℃(\Lambda\Xi_{\mathrm{II}}^{\partial}) = \sum_{\substack{\partial=1,\\ \partial\neq k}}^{\alpha} SUPP(\Lambda\Xi_{\mathrm{II}}^{\partial}, \Lambda\Xi_{\mathrm{II}}^{k})$, where $SUPP(\Lambda\Xi_{\mathrm{II}}^{\partial}, \Lambda\Xi_{\mathrm{II}}^{k}) = 1 - DIS(\Lambda\Xi_{\mathrm{II}}^{\partial}, \Lambda\Xi_{\mathrm{II}}^{k})$ represents the support degree between $\Lambda\Xi_{\mathrm{II}}^{\partial}$ and $\Lambda\Xi_{\mathrm{II}}^{k}$ with the following conditions:

1) $SUPP(\Lambda\Xi_{\mathrm{II}}^{\partial}, \Lambda\Xi_{\mathrm{II}}^{k}) \in [0,1]$.
2) $SUPP(\Lambda\Xi_{\mathrm{II}}^{\partial}, \Lambda\Xi_{\mathrm{II}}^{k}) = SUPP(\Lambda\Xi_{\mathrm{II}}^{k}, \Lambda\Xi_{\mathrm{II}}^{\partial})$.
3) If $DIS(\Lambda\Xi_{\mathrm{II}}^{\partial}, \Lambda\Xi_{\mathrm{II}}^{k}) < DIS(\Lambda\Xi_{\mathrm{II}}^{l}, \Lambda\Xi_{\mathrm{II}}^{m})$, then $SUPP(\Lambda\Xi_{\mathrm{II}}^{\partial}, \Lambda\Xi_{\mathrm{II}}^{k}) \geq SUPP(\Lambda\Xi_{\mathrm{II}}^{l}, \Lambda\Xi_{\mathrm{II}}^{m})$, where $DIS(\Lambda\Xi_{\mathrm{II}}^{\partial}, \Lambda\Xi_{\mathrm{II}}^{k})$ represents the distance between $\Lambda\Xi_{\mathrm{II}}^{\partial}$ and $\Lambda\Xi_{\mathrm{II}}^{k}$.

Where,

$$DIS(\Lambda\Xi_{\mathrm{II}}^{\partial}, \Lambda\Xi_{\mathrm{II}}^{k}) = \frac{1}{4}\left(\left|\equiv_{\partial}^{\Lambda\Xi_{AT}} - \equiv_{k}^{\Lambda\Xi_{AT}}\right| + \left|\equiv_{\partial}^{\Lambda\Xi_{PT}} - \equiv_{k}^{\Lambda\Xi_{PT}}\right| + \left|\exists_{\partial}^{\Lambda\Xi_{AT}} - \exists_{k}^{\Lambda\Xi_{AT}}\right| + \left|\exists_{\partial}^{\Lambda\Xi_{PT}} - \exists_{k}^{\Lambda\Xi_{PT}}\right|\right)$$

Further, the weight vector is stated by: $\beta_{\partial}^{w} \in [0,1]$ with $\sum_{\partial=1}^{\alpha} \beta_{\partial}^{w} = 1$.

Theorem 4: Let $\Lambda\Xi_{\mathrm{II}}^{\partial} = \left(\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}, \equiv_{\partial}^{\Lambda\Xi_{PT}}\right), \left(\exists_{\partial}^{\Lambda\Xi_{AT}}, \exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right), \partial = 1,2, \dots, \alpha$ be any collection of CIFNs. Then we expose that the aggregated value of Equations 7 is also a CIFN, such as

$$\begin{aligned}
&CIFPIWG(\Lambda\Xi_{\mathrm{II}}^{1}, \Lambda\Xi_{\mathrm{II}}^{2}, \dots, \Lambda\Xi_{\mathrm{II}}^{\alpha}) \\
&= \left(\begin{array}{c}
\left(\begin{array}{c}
\prod_{\partial=1}^{\alpha}\left(1-\exists_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}} - \prod_{\partial=1}^{\alpha}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{AT}}+\exists_{\partial}^{\Lambda\Xi_{AT}}\right)\right)^{\frac{\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}}, \\
\prod_{\partial=1}^{\alpha}\left(1-\exists_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}} - \prod_{\partial=1}^{\alpha}\left(1-\left(\equiv_{\partial}^{\Lambda\Xi_{PT}}+\exists_{\partial}^{\Lambda\Xi_{PT}}\right)\right)^{\frac{\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}}
\end{array}\right), \\
\left(1-\prod_{\partial=1}^{\alpha}\left(1-\exists_{\partial}^{\Lambda\Xi_{AT}}\right)^{\frac{\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}}, 1-\prod_{\partial=1}^{\alpha}\left(1-\exists_{\partial}^{\Lambda\Xi_{PT}}\right)^{\frac{\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+℃(\Lambda\Xi_{\mathrm{II}}^{\partial})\right)}}\right)
\end{array}\right)
\end{aligned} \tag{8}$$

Property 10: (Idempotency) If $\Lambda\Xi_{\mathrm{II}}^{\partial} = \Lambda\Xi_{\mathrm{II}} = \left(\left(\equiv^{\Lambda\Xi_{AT}}, \equiv^{\Lambda\Xi_{PT}}\right), \left(\exists^{\Lambda\Xi_{AT}}, \exists^{\Lambda\Xi_{PT}}\right)\right), \partial = 1,2, \dots, \alpha$, then

$$CIFPIWG(\Lambda\Xi_{\mathrm{II}}^{1}, \Lambda\Xi_{\mathrm{II}}^{2}, \dots, \Lambda\Xi_{\mathrm{II}}^{\alpha}) = \Lambda\Xi_{\mathrm{II}}$$

Property 11: (Monotonicity) If $\Lambda\Xi_{\mathrm{II}}^{\partial} \leq \Lambda\Xi\%_{\mathrm{II}}^{\partial}$, then

$$CIFPIWG(\Lambda\Xi_{\mathrm{II}}^{1}, \Lambda\Xi_{\mathrm{II}}^{2}, \dots, \Lambda\Xi_{\mathrm{II}}^{\alpha}) \leq CIFPIWG\left(\Lambda\Xi\%_{\mathrm{II}}^{1}, \Lambda\Xi\%_{\mathrm{II}}^{2}, \dots, \Lambda\Xi\%_{\mathrm{II}}^{\alpha}\right)$$

Property 12: (Boundedness) If $\Lambda\Xi_{\mathrm{II}}^{-} = \left(\left(\min_{\partial} \equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \min_{\partial} \equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right), \left(\max_{\partial} \exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \max_{\partial} \exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right)$ and $\Lambda\Xi_{\mathrm{II}}^{+} = \left(\left(\max_{\partial} \equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \max_{\partial} \equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right), \left(\min_{\partial} \exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \min_{\partial} \exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right)$, thus

$$\Lambda\Xi_{\mathrm{II}}^{-} \leq CIFPIWG(\Lambda\Xi_{\mathrm{II}}^{1}, \Lambda\Xi_{\mathrm{II}}^{2}, \dots, \Lambda\Xi_{\mathrm{II}}^{\alpha}) \leq \Lambda\Xi_{\mathrm{II}}^{+}.$$

We observed that our invented operators based on CIFNs are modified versions of the interaction operators, power operators, and power interaction operators based on FSs, IFSs, CFSs, and CIFSs.

4 | MADM PROCEDURE FOR DERIVED APPROACHES

This section evaluates the implementations of the CIFPIA operator, CIFPIWA operator, CIFPIG operator, and CIFPIWG operator in the environment of MADMP. For this, firstly, we compute the algorithm for evaluating the MADMP and then try to evaluate some real-life problems.

For this, we have a finite collection of alternatives $\Lambda\Xi_{\mathrm{II}}^{1}, \Lambda\Xi_{\mathrm{II}}^{2}, \dots, \Lambda\Xi_{\mathrm{II}}^{\alpha}$, where for each alternative we have the collection of finite attributes $\Lambda\Xi_{\mathrm{AT}}^{1}, \Lambda\Xi_{\mathrm{AT}}^{2}, \dots, \Lambda\Xi_{\mathrm{AT}}^{l}$. Furthermore, we consider the weight vector for the attributes, where the range of attributes and weight vectors are the same, such as $\beta_{\partial}^{w} \in [0,1]$ with $\sum_{\partial=1}^{l} \beta_{\partial}^{w} = 1$. Furthermore, we compute or construct the matrix by including their values in the shape of CIFNs, where $\equiv^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u) e^{i2\pi\left(\equiv^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)\right)}$ and $\exists^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u) e^{i2\pi\left(\exists^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)\right)}$ represents or shows the complex-valued truth and falsity function with $0 \leq \equiv^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u) + \exists^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u) \leq 1$ and $0 \leq \equiv^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u) + \exists^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u) \leq 1$. Noticed that $\equiv^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u), \equiv^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u), \exists^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u)$ and $\exists^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)$ are stated the amplitude term and phase term of truth and falsity grade with a neutral grade, such as: ${}^{\circ}\mathrm{C}^{\circ}\mathrm{C}^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u) e^{i2\pi\left({}^{\circ}\mathrm{C}^{\circ}\mathrm{C}^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)\right)} = \left(1 - \left(\equiv^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u) + \exists^{\Lambda\Xi_{\mathrm{AT}}}(\partial_u)\right)\right) e^{i2\pi\left(1 - \left(\equiv^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u) + \exists^{\Lambda\Xi_{\mathrm{PT}}}(\partial_u)\right)\right)}$. Finally, the structure of CIFN is represented by: $\Lambda\Xi_{\mathrm{II}}^{\partial} = \left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} e^{i2\pi\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)}, \exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} e^{i2\pi\left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)}\right) = \left(\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right), \left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right), \partial = 1,2,\dots,\alpha.$ Finally, for addressing the above problems, we used the following procedure, such as

1) Compute the matrix by including their entries in the shape of CIFNs, where the information is contained cost type of data, then we aim to normalize it, such as

$$N = \begin{cases} \left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} e^{i2\pi\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)}, \exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} e^{i2\pi\left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)}\right) = \left(\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right), \left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right) & for\ benefit \\ \left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} e^{i2\pi\left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)}, \equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}} e^{i2\pi\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)}\right) = \left(\left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right), \left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}, \equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right) & for\ cost \end{cases}$$

But if we have a benefit type of data, then we do not aim to normalize it.

2) Perform the aggregation values of the data in the matrix by considering the CIFPIA operator, CIFPIWA operator, CIFPIG operator, and CIFPIWG operator.
3) Based on the aggregated values, we perform the score values of the aggregated values.
4) Based on the score values, we perform the ranking values of the finite alternatives for evaluating valuable and dominant alternatives between all alternatives.

Finally, we aim to justify the above theory with the help of some numerical examples. For this, we consider the problem of goldmines and try to evaluate it with the help of invented four different techniques to show the reliability and supremacy of the presented approaches.

Assessment of Goldmines

Assessing the capability and potential viability of gold mines contains some major factors related to geology, environmental impact, and economic considerations. The major theme of this problem is to consider some major aspects that are assessed when assessing gold mines, such as:

1) *Geological Factors* "$\Lambda\Xi_{\mathrm{II}}^{1}$"
   i) Ore Grade,
   ii) Ore Type,
   iii) Mineralogy,
   iv) Deposit Size.
2) *Economic Factors* "$\Lambda\Xi_{\mathrm{II}}^{2}$"
   i) Gold Price,
   ii) Operating Costs,
   iii) Capital Expenditure,
   iv) Return on Investment.
3) *Mining Methods* "$\Lambda\Xi_{\mathrm{II}}^{3}$"
   i) Open-Pit vs. Underground,
   ii) Extraction Techniques.
4) *Environmental and Regulatory Considerations* "$\Lambda\Xi_{\mathrm{II}}^{4}$"
   i) *Environmental Impact,*
   ii) Permits and Regulations.
5) *Market Demand and Trends* "$\Lambda\Xi_{\mathrm{II}}^{5}$"
   i) Gold demand,
   ii) Supply and Demand Dynamics.

Where the above five factors are represented the collection of alternatives and to address or select the best or perfect one, we use the following features, such as $\Lambda\Xi_{\mathrm{AT}}^{1}$: Growth analysis, $\Lambda\Xi_{\mathrm{AT}}^{2}$: Social impact, $\Lambda\Xi_{\mathrm{AT}}^{3}$: environmental impact, and $\Lambda\Xi_{\mathrm{AT}}^{4}$: political impact. These four features are stated as the collection of attributes with weight vectors, such as $(0.2,0.1,0.6,0.1)^{T}$. Finally, for addressing the above problems, we used the following procedure, such as

1) Compute the matrix (see Table 1) by including their entries in the shape of CIFNs, where the information is contained cost type of data, then we aim to normalize it, such as

$$N=\begin{cases}\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}e^{i2\pi\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)},\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}e^{i2\pi\left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)}\right)=\left(\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right),\left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right) & for\ benefit\\ \left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}e^{i2\pi\left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)},\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}}e^{i2\pi\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)}\right)=\left(\left(\exists_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\exists_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right),\left(\equiv_{\partial}^{\Lambda\Xi_{\mathrm{AT}}},\equiv_{\partial}^{\Lambda\Xi_{\mathrm{PT}}}\right)\right) & for\ cost\end{cases}$$

But if we have a benefit type of data, then we do not aim to normalize it. Anyhow, the data in Table 1 does not need to be normalized.

Table 1: CIF decision matrix.

$\Lambda\Xi_{\mathrm{II}}^{1}$	$((0.3,0.4),(0.1,0.2))$	$((0.6,0.1),(0.2,0.1))$
$\Lambda\Xi_{\mathrm{II}}^{2}$	$((0.7,0.8),(0.2,0.1))$	$((0.5,0.4),(0.3,0.2))$
$\Lambda\Xi_{\mathrm{II}}^{3}$	$((0.4,0.5),(0.2,0.3))$	$((0.5,0.2),(0.4,0.1))$
$\Lambda\Xi_{\mathrm{II}}^{4}$	$((0.3,0.4),(0.1,0.2))$	$((0.5,0.2),(0.1,0.2))$
$\Lambda\Xi_{\mathrm{II}}^{5}$	$((0.3,0.4),(0.1,0.2))$	$((0.6,0.2),(0.2,0.1))$
$\Lambda\Xi_{\mathrm{II}}^{1}$	$((0.6,0.2),(0.2,0.3))$	$((0.5,0.3),(0.4,0.2))$
$\Lambda\Xi_{\mathrm{II}}^{2}$	$((0.5,0.2),(0.1,0.2))$	$((0.6,0.8),(0.3,0.1))$
$\Lambda\Xi_{\mathrm{II}}^{3}$	$((0.5,0.3),(0.4,0.2))$	$((0.6,0.4),(0.1,0.3))$
$\Lambda\Xi_{\mathrm{II}}^{4}$	$((0.6,0.8),(0.3,0.1))$	$((0.5,0.3),(0.4,0.2))$
$\Lambda\Xi_{\mathrm{II}}^{5}$	$((0.4,0.5),(0.2,0.3))$	$((0.5,0.2),(0.4,0.1))$

then we do not aim to normalize it Furthermore, by using the data in Table 1, we find the weight vectors by using the power aggregation operators, see Table 2. This information is valid for the weighted vector formula "$\frac{\beta_{\partial}^{w}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\beta_{\partial}^{w}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}$" and also for without weight vector formulas "$\frac{\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}{\sum_{\partial=1}^{\alpha}\left(1+{}^{\circ}C^{\circ}C\left(\Lambda\Xi_{\mathrm{II}}^{\partial}\right)\right)}$".

Table 2. Representing the information that is used as a weight vector.

	Weight vectors		Weight vectors	
$\Lambda\Xi_{\mathrm{II}}^{1}$	0.3512	0.2935	0.2202	0.092
$\Lambda\Xi_{\mathrm{II}}^{2}$	0.3333	0.2609	0.2111	0.0826
$\Lambda\Xi_{\mathrm{II}}^{3}$	0.3605	0.3069	0.2209	0.0941
$\Lambda\Xi_{\mathrm{II}}^{4}$	0.3261	0.2765	0.2065	0.5253
$\Lambda\Xi_{\mathrm{II}}^{5}$	0.3466	0.442	0.2102	0.1341
	Weight vectors		Weight vectors	
$\Lambda\Xi_{\mathrm{II}}^{1}$	0.2083	0.5224	0.2202	0.092
$\Lambda\Xi_{\mathrm{II}}^{2}$	0.2444	0.5739	0.2111	0.0826
$\Lambda\Xi_{\mathrm{II}}^{3}$	0.1977	0.505	0.2209	0.0941
$\Lambda\Xi_{\mathrm{II}}^{4}$	0.2609	0.1106	0.2065	0.0876
$\Lambda\Xi_{\mathrm{II}}^{5}$	0.2216	0.2826	0.2216	0.1413

1) Perform the aggregation values of the data in the matrix by considering the CIFPIA operator, CIFPIWA operator, CIFPIG operator, and CIFPIWG operator, see Table 3.

Table 3. Representing the values of aggregations.

	CIFPIA operator	CIFPIG operator
$\Lambda\Xi_{\text{II}}^{1}$	$((0..4886,0.2794),(0.3589,0.2079))$	$((0.5301,0.2858),(0.2174,0.2015))$
$\Lambda\Xi_{\text{II}}^{2}$	$((0.5977,0.6461),(0.2399,0.1463))$	$((0.6158,0.6453),(0.2218,0.147))$
$\Lambda\Xi_{\text{II}}^{3}$	$((0.4917,0.3828),(0.2982,0.2714))$	$((0.5178,0.414),(0.2721,0.2402))$
$\Lambda\Xi_{\text{II}}^{4}$	$((0.4736,0.5065),(0.2876,0.1763))$	$((0.5364,0.5077),(0.2248,0.175))$
$\Lambda\Xi_{\text{II}}^{5}$	$((0.4418,0.3475),(0.2655,0.2156))$	$((0.4892,0.3804),(0.2181,0.1828))$
	CIFPIWA operator	CIFPIWG operator
$\Lambda\Xi_{\text{II}}^{1}$	$((0.5188,0.2658),(0.2221,0.2452))$	$((0.5475,0.2652),(0.1935,0.2458))$
$\Lambda\Xi_{\text{II}}^{2}$	$((0.5704,0.5147),(0.195,0.1717))$	$((0.6026,0.5195),(0.1627,0.167))$
$\Lambda\Xi_{\text{II}}^{3}$	$((0.4822,0.3699),(0.3481,0.2588))$	$((0.5112,0.3955),(0.3191,0.2333))$
$\Lambda\Xi_{\text{II}}^{4}$	$((0.4646,0.3736),(0.1954,0.1935))$	$((0.5048,0.3775),(0.1552,0.1895))$
$\Lambda\Xi_{\text{II}}^{5}$	$((0.4071,0.3832),(0.2344,0.2332))$	$((0.4507,0.4121),(0.1908,0.2042))$

2) Based on the aggregated values, we perform the score values of the aggregated values, see Table 4.

Table 4. Score values of the aggregated values.

	CIFPIA operator	CIFPIG operator	CIFPIWA operator	CIFPIWG operator
$\Lambda\Xi_{\text{II}}^{1}$	0.1506	0.1985	0.1587	0.1867
$\Lambda\Xi_{\text{II}}^{2}$	0.4288	0.4462	0.3592	0.3962
$\Lambda\Xi_{\text{II}}^{3}$	0.1524	0.2097	0.1226	0.1771
$\Lambda\Xi_{\text{II}}^{4}$	0.2581	0.3221	0.2247	0.2688
$\Lambda\Xi_{\text{II}}^{5}$	0.1541	0.2343	0.1614	0.2339

3) Based on the score values, we perform the ranking values of the finite alternatives for evaluating valuable and dominant alternatives between all alternatives, see Table 5.

Table 5. Ranking information.

Methods	Ranking values	Best optimal
CIFPIA Operator	$\Lambda\Xi_{\text{II}}^{2} > \Lambda\Xi_{\text{II}}^{4} > \Lambda\Xi_{\text{II}}^{5} > \Lambda\Xi_{\text{II}}^{3} > \Lambda\Xi_{\text{II}}^{1}$	$\Lambda\Xi_{\text{II}}^{2}$
CIFPIG Operator	$\Lambda\Xi_{\text{II}}^{2} > \Lambda\Xi_{\text{II}}^{4} > \Lambda\Xi_{\text{II}}^{5} > \Lambda\Xi_{\text{II}}^{3} > \Lambda\Xi_{\text{II}}^{1}$	$\Lambda\Xi_{\text{II}}^{2}$
CIFPIWA Operator	$\Lambda\Xi_{\text{II}}^{2} > \Lambda\Xi_{\text{II}}^{4} > \Lambda\Xi_{\text{II}}^{5} > \Lambda\Xi_{\text{II}}^{1} > \Lambda\Xi_{\text{II}}^{3}$	$\Lambda\Xi_{\text{II}}^{2}$
CIFPIWG Operator	$\Lambda\Xi_{\text{II}}^{2} > \Lambda\Xi_{\text{II}}^{4} > \Lambda\Xi_{\text{II}}^{5} > \Lambda\Xi_{\text{II}}^{1} > \Lambda\Xi_{\text{II}}^{3}$	$\Lambda\Xi_{\text{II}}^{2}$

The most favorable decision is $\Lambda\Xi_{\text{II}}^{2}$, according to the theory of CIFPIA operator, CIFPIG operator, CIFPIWA operator, and CIFPIWG operator. Finally, we show the superiority and effectiveness of the invented theory by comparing the results with some prevailing information.

5 | COMPARATIVE ANALYSIS

In this section, we briefly discussed the comparative analysis between proposed techniques and some prevailing methods based on the data in Table 1. During the comparison, we are needed some existing operators based on IFSs and CIFSs, because, with the help of comparison, we can easily derive the supremacy and effectiveness of the invented techniques. For this, we used the following existing techniques, such as interaction aggregating operators (AOs) for IFSs exposed by Garg [27]. Additionally, geometric interaction AOs for generalized IFSs was evaluated by He et al. [28]. Furthermore, interaction averaging AOs for IFSs was evaluated by He et al. [29]. Jiang et al. [30] presented the power AOs for IFSs. Xu [31] evaluated the simple power AOs for IFSs. Xu [32] derived the AOs for IFSs. Moreover, Xu and Yager [33] exposed the geometric AOs for IFSs. Additionally, generalized AOs for CIFSs were exposed by Garg and Rani [24]. Further, power AOs for CIFSs were invented by Rani and Garg [35] After a long assessment, we briefly described the comparative analysis in the shape of Table 6.

Table 6. Comparative analysis between proposed and existing techniques.

Methods	Score values	Ranking values
Garg [27]	Limited features	Limited features
He et al. [28]	Limited features	Limited features
He et al. [29]	Limited features	Limited features
Jiang et al. [30]	Limited features	Limited features
Xu [31]	Limited features	Limited features
Xu [32]	Limited features	Limited features
Xu and Yager [33]	Limited features	Limited features
Garg and Rani [34]	0.1819,0.3701,0.1629,0.2567,-0.091	$\Lambda\Xi_{\text{II}}^{2} > \Lambda\Xi_{\text{II}}^{4} > \Lambda\Xi_{\text{II}}^{1} > \Lambda\Xi_{\text{II}}^{3} > \Lambda\Xi_{\text{II}}^{5}$
Rani and Garg [35]	0.1396,0.2671,0.1274,0.1934,0.4238	$\Lambda\Xi_{\text{II}}^{5} > \Lambda\Xi_{\text{II}}^{2} > \Lambda\Xi_{\text{II}}^{4} > \Lambda\Xi_{\text{II}}^{1} > \Lambda\Xi_{\text{II}}^{3}$
CIFPIA Operator	0.1506,0.4288,0.1524,0.2581,0.1541	$\Lambda\Xi_{\text{II}}^{2} > \Lambda\Xi_{\text{II}}^{4} > \Lambda\Xi_{\text{II}}^{5} > \Lambda\Xi_{\text{II}}^{3} > \Lambda\Xi_{\text{II}}^{1}$
CIFPIG Operator	0.1985,0.4462,0.2097,0.3221,0.2343	$\Lambda\Xi_{\text{II}}^{2} > \Lambda\Xi_{\text{II}}^{4} > \Lambda\Xi_{\text{II}}^{5} > \Lambda\Xi_{\text{II}}^{3} > \Lambda\Xi_{\text{II}}^{1}$
CIFPIWA Operator	0.1587,0.3592,0.1226,0.2247,0.1614	$\Lambda\Xi_{\text{II}}^{2} > \Lambda\Xi_{\text{II}}^{4} > \Lambda\Xi_{\text{II}}^{5} > \Lambda\Xi_{\text{II}}^{1} > \Lambda\Xi_{\text{II}}^{3}$
CIFPIWG Operator	0.1867,0.3962,0.1771,0.2688,0.2339	$\Lambda\Xi_{\text{II}}^{2} > \Lambda\Xi_{\text{II}}^{4} > \Lambda\Xi_{\text{II}}^{5} > \Lambda\Xi_{\text{II}}^{1} > \Lambda\Xi_{\text{II}}^{3}$

The most favorable decision is $\Lambda\Xi_{\text{II}}^{2}$, according to the theory of the CIFPIA operator, CIFPIG operator, CIFPIWA operator, CIFPIWG operator, and the existing theory of Garg and Rani [34], but the theory proposed by Rani and Garg [35] given different ranking results, such as $\Lambda\Xi_{\text{II}}^{5}$ which worst decision according to others. Some existing techniques are not able to evaluate the data in Table because of some limitations and restrictions. Hence, the invented theory is more suitable and reliable due to its unlimited features and conditions.

6 | CONCLUSION

In this manuscript, we evaluated the following ideas, such as

1. We evaluated the interaction operational laws for CIF information.
2. We derived the CIFPIA operator, CIFPIWA operator, CIFPIG operator, and CIFPIWG operator. Some fundamental properties are also examined for the evaluated operators.
3. We simplified the above operators, we illustrate a MADMP based on the exposed operators for CIF information to enhance the worth and stability of the presented operators.
4. We compared the invented operators with some existing operators in the presence of some numerical examples to show the supremacy and effectiveness of the derived approaches.

In the future, we will review some old theories and try to develop some new theories by eliminating their deficiencies or implementing some new conditions on it. Further, we also aim to evaluate some new techniques, methods, and measures based on our invented ideas to improve the worth of the proposed techniques.

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