ORIGINAL ARTICLE

OPEN ACCESS

A Pythagorean Fuzzy CRITIC MAUT Decision Framework for Disease Diagnosis under Uncertain Clinical Data

Naima [1], Khazaima Safdar [2], Sidra Farooq [3], Saeid Jafari [4]

Authors' Affiliation:

[1,3]Department of Mathematics, Quid-i-Azam University (QAU), Islamabad, Pakistan.
Email: naimashabbbirfrfu@gmail.com
[2]Department of Mathematics, University of Management and Technology Lahore, Pakistan.
Email: F2025109014@umt.edu.pk
[4]Mathematical and Physical Science Foundation, Slagelse, Vestsjaelland South, Denmark
Email: jafaripersia@gmail.com

Article History:

Submitted: March 08, 2026
Revised: June 16, 2026
Accepted: June 27, 2026
Published online: June 30, 2026

Corresponding author(s):

Khazaima Safda & Saeid Jafari
Email: F2025109014@umt.edu.pk; jafaripersia@gmail.com

ABSTRACT

Clinical information is often incomplete, vague, and uncertain, and the diagnosis of disease is a difficult task for accurate decision making. Multi-Criteria Decision-making (MCDM) is a range of decision-making methods that have been extensively used in healthcare, but most methods have the following problems: They use subjective criterion weights, they are not good at expressing hesitation and uncertainty, and they do not have integrated objective criterion weights and a utility-based ranking mechanism. To resolve these drawbacks, this research introduces a novel Pythagorean Fuzzy CRITIC MAUT decision framework for the diagnosis of a disease in the case of uncertainty in clinical conditions. The proposed framework incorporates Pythagorean fuzzy sets (PFS) to effectively model uncertainty and hesitation, uses the CRITIC method to determine the weights of the criteria objective based on the variability of the data and inter-criterion correlation, and finally applies the multi-attribute utility theory (MAUT) to assess and rank the disease alternatives based on their utility values. The proposed framework is applied to a numerical case study with four disease alternatives. The results indicate that the alternative $(\mho_2)$ has the highest value of the integrated utility (0.7664) and is the most preferred disease alternative, followed by $(\mho_3)$ (0.3970) and $(\mho_1)$ (0.3841) and $(\mho_4)$ (0.1565) respectively. The results showed that the proposed framework can effectively distinguish between the various disease options, minimize subjective influences, and offer a clear, objective, and accurate decision-support system. Thus, the proposed model is very promising in helping clinical decisions in uncertain clinical settings.

Keywords: CRITIC method, MCDM, MAUT method, PFS, Uncertain clinical data

(MSC2020) Codes: 90B50, 90C29, 03E72, 90C70, 62P10

1. | INTRODUCTION

Intelligent clinical decision support systems, such as artificial intelligence (AI), machine learning (ML), probabilistic reasoning, and fuzzy decision making, have helped in the diagnosis of diseases. Although these developments have been made, there are still some significant challenges with the current diagnostic frameworks. Numerous studies use subjective approaches for criterion weighting, offer limited ability to accommodate various types of clinical uncertainty, fail to include explicit and easily understood decision-making steps, and may give inferior diagnostic results when clinical information is incomplete or inconsistent. To overcome these shortcomings, this paper introduces a novel Pythagorean Fuzzy CRITIC–MAUT model in which PFS are used to model uncertainty, the CRITIC method is used to determine objective weights of the criteria, and the MAUT method is used to rank the disease alternatives using utilities. This integrated framework offers a more transparent, objective, and reliable decision-support mechanism to diagnose the disease in the uncertain clinical environment.

While artificial intelligence (AI) has proven to be beneficial in identifying diseases, there are AI models that seem like a "black box" and are hard for physicians to understand how the decision is reached. Probabilistic reasoning methods require sufficient and reliable clinical data, but in real healthcare settings, such information is often incomplete or unavailable. Likewise, conventional fuzzy decision-support models are not able to capture various types of uncertainties and doubts. However, most of the studies available today do not incorporate the use of objective criterion weighting, uncertainty modelling, and utility-based ranking in a single decision-making process. To overcome these drawbacks, the present study proposed a Pythagorean Fuzzy CRITIC–MAUT framework to solve the disease diagnosis problem, which has been solved using PFS to represent uncertainties, the CRITIC method to objectively calculate the criterion weights, and MAUT method to rank the disease alternatives according to their overall utility values, and thus to obtain a more transparent and reliable decision support framework for diagnosis of diseases.

Although there have been significant advancements in the use of intelligent clinical decision support systems, there remain a number of drawbacks. Existing methods lack objective criteria for the importance of the criteria used to diagnose a disease, which may lead to variations in the diagnosis. Furthermore, a large number of models cannot effectively deal with different types of uncertainty, e.g., vagueness, ambiguity, and incomplete clinical information. Other methods that are based on artificial intelligence are also not interpretable, meaning it is challenging to determine how a diagnosis is made. In addition, many current models may be less effective if clinical data are incomplete, inconsistent, or uncertain, making the models less robust and reliable. These constraints could lead to misdiagnosis and some loss of clinician confidence in the process. It is therefore crucial that a framework is established which integrates objective weighting, effective modelling of the uncertainty, and systematic ranking, to enhance the accuracy, transparency and consistency of the diagnosis of disease.

Various types of uncertainty affect clinical disease diagnosis. Vagueness develops if the symptoms are not clearly defined, and ambiguity when the same clinical information can be interpreted in a number of ways. Approximate/imprecise measurements cause imprecision, while missing patient records or unavailable laboratory results cause incompleteness. Inconsistency is when the information from different clinical sources is different. Further, epistemic uncertainty can also happen from a lack of knowledge by the medical community or limited evidence concerning the patient's condition. These uncertainties are different and create a challenging disease diagnosis decision making process, which needs effective modelling of uncertainty.

The proposed Pythagorean Fuzzy CRITIC–MAUT framework is quite different from the existing disease diagnosis models. The primary objective of an AI-based diagnosis system is to maximize prediction accuracy, but most of the time, it is a black-box system, making its predictions hard to explain. Probabilistic reasoning techniques rely on large amounts of statistical data and might not work well if there is insufficient or incomplete clinical information. Many traditional fuzzy decision-support models can represent uncertainty, but most of these models have subjective weight on the criteria or lack an integrated mechanism to assign objective weight and rank based on utilities. The proposed approach, however, integrates PFS to handle uncertainty, the CRITIC method to objectively obtain the criterion weights, and the MAUT method to systematically rank the disease alternatives. This integration offers a clear, objective, and dependable decision support tool for disease diagnosis in a complex clinical scenario when uncertainty exists.

Disease diagnosis is a complex multi-criteria decision-making (MCDM) problem because clinicians must simultaneously evaluate multiple diagnostic criteria, such as patient symptoms, laboratory test results, medical history, imaging findings, and clinical observations. These criteria often have different levels of importance and may contain uncertain or incomplete information. Considering only a single criterion may lead to inaccurate diagnostic decisions. Therefore, an MCDM framework provides a systematic approach to evaluate all relevant diagnostic criteria simultaneously and supports more consistent and reliable clinical decision-making. A Fermatean fuzzy MCDM method for selection and ranking Problems: Case studies [1] A fuzzy MCDM model for supplier selection [2] MCDM methods and FS [3] An extension to fuzzy MCDM [4] Fuzzy MCDM based on ideal and anti-ideal concepts [5] Evaluating sustainable fishing development strategies using fuzzy MCDM approach [6].

Diagnosing a disease, when the clinical data provided are not absolutely clear, is crucial, as physicians are often required to diagnose a disease with poor medical evidence. Uncertainty can come about due to the non-availability of patient records, overlaps between disease symptoms, inconsistencies in laboratory test results, or limitations of medical imaging and diagnostic equipment. Controlling this uncertainty can minimize diagnostic mistakes, enable the early diagnosis of diseases, facilitate timely treatment, thereby enhancing therapeutic effectiveness, and ultimately benefit the patients. The clinical analysis of uncertain data, the more reliable its use in diagnosis and efficient decision support, is facilitated to an increasing degree by modern diagnostic systems, based on artificial intelligence, machine learning, fuzzy logic, and probabilistic models, which enable more accurate and consistent diagnoses from the clinician. These technologies are helpful to healthcare providers in discovering patterns they wouldn't be able to observe using a traditional diagnosis. Moreover, by mitigating uncertainty in clinical information, the quality of health services improves, as well as long-term care and disease prevention for patients.

Under highly uncertain clinical data, decision making is an important part of disease diagnosis: clinicians need to consider many sources of evidence and consider the likelihood of several conditions in making their judgment, and to choose between the various diagnoses and treatment options. Within sound decision-making processes, clinical experience is integrated with patient history, diagnostic test results, and decision support in computing programs to minimize the implications of uncertainty while minimizing the risk of misdiagnosis. Good decision-making not only ensures patient safety and correct diagnosis but also reduces the use of health resources, preventing unnecessary tests and treatments and therefore improving resource use. With increasing complexity and diversity of healthcare data, solid frameworks are needed for making decisions to provide consistent, evidence-based, and patient-centred health care. What's more, higher-tier decision support tools can help prioritize high-risk cases and help to make more informed and timely clinical decisions. This ultimately boosts the efficiency with which health care services are delivered and enhances a patient's satisfaction and healthcare quality.

1.1. | Extension of Fuzzy Set

In classical set theory, information is represented in a binary fashion: an element is either a member of a set or it is not. In the real world, however, many decision-making problems involve uncertain, vague, or incomplete information that cannot be accurately represented by binary logic, particularly in health care. This limitation was overcome by the introduction of FS theory by Zadeh [7], which assigns a degree of membership to each element ranging from 0 to 1. Although FS effectively represent uncertainty, they consider only the degree of membership and do not explicitly model non-membership.

To provide a better representation of uncertainty, Atanassov [8] introduced Intuitionistic FS (IFS) by incorporating both membership and non-membership degrees, along with a hesitation margin. Intuitionistic FS (IFS). The mathematical requirements of IFS make it very difficult to capture situations of high uncertainty, however. The concept is extended to PFSs proposed by Yager [9], which provide a more flexible relationship between membership and non-membership degrees, yielding a larger decision space. PFS can thus be used more effectively than traditional fuzzy models in expressing uncertainty, hesitation, and ambiguity. PFS: state of the art and future directions [10], A framework for choosing an appropriate FS extension in modeling [11], and optimizing industrial robot selection using novel trigonometric Pythagorean fuzzy normal aggregation operators [12]. Hamacher aggregation operators for Pythagorean fuzzy sets and their application in a multi-attribute decision-making problem [13]. As shown in Table 1, PFS provides a larger feasible decision space than FS and IFS because of the constraint $0 \leq \check{a}^2 + \acute{c} \leq 1$. This allows better representation of uncertainty and hesitation, making PFS more suitable for disease diagnosis under uncertain clinical conditions. The comparison between FS, IFS, and PFS is provided in Table 1.

Table 1. Numerical Payoff structure for the Prisoner's Dilemma between Assad and HTS

Characteristics	**FS**	**IFS**	**PFS**
Mathematical Constraint	$\check{a}$	$0 \leq \check{a} + \acute{c} \leq 1$	$0 \leq \check{a}^2 + \acute{c}^2 \leq 1$
Uncertainty Representation	Limited	Moderate	High
Handling Uncertainty	Not supported	Supported	Highly supported
Main Limitations	Cannot represent hesitation	Linear constraint	Slightly higher computational complexity
Disease's diagnosis suitability	Low	Moderate	High

PFS, on the other hand, offers a good compromise between uncertainty representation, mathematical simplicity, and computational efficiency. The quadratic constrains ext*ends th*e decision space of IFSs without increasing the computational burden of more recent uncertainty models. Moreover, the application of PFS has been widely tested in healthcare and medical decision-making, proving its capability of effectively modeling the uncertain clinical information and of facilitating transparent and reliable decision-making. Hence, the uncertainty modelling framework for the proposed hybrid CRITIC MAUT approach is adopted as PFS. The comparison between Fermatean FS (FFS), q-rung orthopair FS (q-ROFS), and Spherical FS (SFS) and PFS is given in Table 2.

Table 2. Shows the Basic Comparison of FFS, q-ROFS, SFS, and PFS

Model	Strength	Limitations	Reason for not selecting
FFS	High uncertainty representation	Higher computational complexity	More complex than required for this study
Q-ROFS	Very flexible	Parameter selection increases complexity	Reduced understandability
SFS	Handles multiple uncertainty dimensions	Complex aggregation operators	Increased execution complexity
PFS	Good balance of flexibility, simplicity, and computational efficiency	Moderate computational cost	Choose for this study

1.2. | Novelty of the CRITIC MAUT Method

CRITIC MAUT is a hybrid MCDM method that is a combination of the benefits of the CRITIC and MAUT methods. The most interesting feature is that it makes use of the data available and determines the importance (weights) of each criterion and then ranks the alternatives in terms of their overall utility values in accordance with their weights. The CRITIC method gives greater weights to the criteria with more useful information and less similarity between criteria, and the MAUT method incorporates the weights and the performance of each alternative to generate a final ranking. This combination gives the decision-making process a more systematic, transparent, and reliable process, taking into account the criteria importance and alternative performances. So, the CRITIC MAUT method is a useful and efficient method for solving multi-criteria problems for comparing and prioritizing alternatives. Moreover, the approach diminishes the impact of overstated criteria and thus produces more significant evaluation results. It can also be used in other decision-making situations in which a ranking of alternatives is needed that is objective and consistent. Moreover, it is a straightforward technique and can efficiently work with a huge number of criteria and alternatives. It provides a logical, easy-to-follow structure for comparing options based on their performance. This means the CRITIC MAUT can be used to guide more accurate, reliable decision making in a variety of practical applications.

1.3. | Problem Statement and Literature Review

Disease diagnosis is critical for optimal patient treatment, but physicians may be uncertain due to overlapping symptoms and inconsistent laboratory findings. These difficulties can compromise diagnostic accuracy and affect clinical decisions. Hence, there is a greater need for reliable decision-support tools that can handle uncertainty.

The CRITIC method has been identified as an objective weighting method that assigns weights to criteria based on data variability and inter-criteria correlation, and the MAUT method is a systematic, utility-based alternative ranking method.

While there have been some successful applications of Pythagorean fuzzy theory, CRITIC, and MAUT individually or in combination, very little effort has been devoted to studying how these three methods can be integrated into an integrated framework for diagnosing disease. Thus, a comprehensive decision-support model that can manage the uncertainty in the clinical information, provide a systematic ranking of disease alternatives, and effectively determine criterion weights is needed.

To address this gap, a Pythagorean fuzzy CRITIC MAUT model is proposed in this study for disease diagnosis. The proposed model offers a transparent, reliable, and systematic framework to support decision-making in healthcare applications by integrating the uncertainty-handling features of PFSs with the objective-weighting system of CRITIC and the ranking approach of MAUT. Enhanced CRITIC-REGIME method for decision making based on Pythagorean fuzzy rough number [14], A Hybrid LDA and Fuzzy CRITIC-MABAC Model for Smart City Ranking [15], and Internal control risk assessment using interval valued spherical fuzzy CRITIC EDAS [16]. MAUT: Multi-attribute utility theory method for multi-attribute decision-making [17] and A hierarchical elicitation process based on the combination of different weighting methods within multi-attribute utility theory in a group decision-making context [18]. Potential Pythagorean fuzzy soft set: A novel approach for medical diagnosis techniques [19]. Pythagorean fuzzy soft RMS approach to decision making and medical diagnosis [20]. A hybrid decision-making framework with Pythagorean fuzzy information for childhood Cancer prognosis [21]. An approach to MCGDM based on multi-granulation Pythagorean fuzzy rough set over two universes and its application to the medical decision problem [22]. Improved composite relation for PFSs and its application to medical diagnosis [23] and the prognosis of allergy-based diseases using Pythagorean fuzzy Hypersoft mapping structures, and recommending medication [24]. Complex Pythagorean normal interval-valued fuzzy aggregation operators for solving medical diagnosis problem [25]; Pythagorean fuzzy cognitive analysis for medical care and treatment decisions [26] FS FS-based approaches for improved medical diagnosis: An analysis and overview of major research directions [27].

1.4. | Objectives and Contribution

The aims of this study are:

To build a hybrid Pythagorean Fuzzy CRITIC MAUT model for diagnosis of diseases in uncertain clinical conditions.

- To assess the importance of the diagnostic criteria objectively, according to the CRITIC weighing method.
- To model various types of clinical uncertainties by PFS.
- To use the MAUT approach to rank the disease alternatives according to their overall utility values.
- To test the effectiveness and applicability of the proposed framework by conducting a disease diagnosis case study.

1.5. | Scientific Contribution

The results of the study are presented here with methodological, theoretical, and practical implications for disease diagnosis in a context of uncertainty.

Methodological contribution: The proposed novel hybrid Pythagorean Fuzzy CRITIC MAUT framework is a combination of an uncertainty model, objective criteria weighting, and ranking based on a utility model into a single disease diagnosis model.

Theoretical contribution: The study not only makes a theoretical contribution to the adoption of PFS over the conventional FS but also shows that it can model clinical uncertainty better than the conventional fuzzy methods in a hybrid MCDM framework.

Practical contribution: The suggested framework provides a clear, objective, and trustworthy decision-support system for disease diagnosis to help to enhance the quality and uniformity of clinical decision-making in the uncertain context.

1.6. | Layout

The rest of this paper is organized as follows: In Section 2, the basic definitions and the score function are discussed, which are required for the proposed decision-making frameworks. In Section 3, the Pythagorean fuzzy CRITIC MAUT methodology is presented in detail, which includes the calculation of the objective criterion weights using the CRITIC method and the ranking of the disease alternatives using the MAUT method. A case study of disease diagnosis is presented in Section 4. In Section 5, the proposed model is numerically evaluated by calculating criterion weights, utility values, integrated utility values, and the final ranking of the disease alternatives. In Section 6, the proposed framework is qualitatively compared with other fuzzy frameworks. Lastly, Section 7 presents the solid conclusion. The pictorial format of the layout is given in Figure 1.

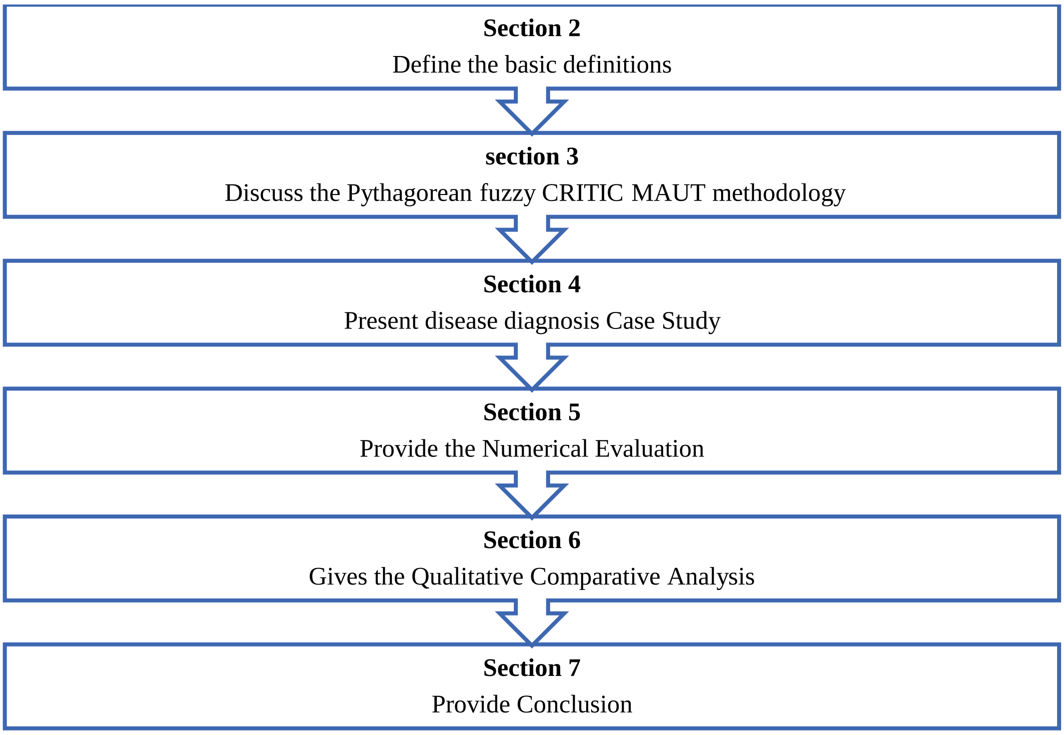

Figure 1. Shows the Structure of the Proposed Theory

2. | PRELIMINARIES

In this section, some basic definitions of fuzzy set (FS), IFS, and PFS along with their score functions are presented.

Definition 1: [7]: Let Ğ be a fixed set. Then, an FS is defined as:

$$\text{Ğ} = \{(\text{ă}(ø))|ø \in \text{Ğ}\} \quad (1)$$

Where ă (ø) ∈ [0,1] represents the membership degree.

Definition 2: [8]: Let Ğ be a fixed set. Then, an IFS is defined as:

$$\text{Ğ} = \{(\text{ă}(ø), \text{ć}(ø)|\, ø \in \text{Ğ}\} \quad (2)$$

Where ă (ø) ∈ [0,1] represents the membership degree and ć(ø) ∈ [0,1] represents the non-membership degree, and satisfy the conditions $0 \leq \text{ă}(ø) + \text{ć}(ø) \geq 1$. For ease, (ă (ø), ć (ø)) is identified as an intuitionistic fuzzy value (IFVS).

Definition 3: [9] Let Ğ be a fixed set. Then, a PFS is explained as:

$$\text{Ğ} = \{(\text{ă}(ø), \text{ć}(ø))|\, ø \in \text{Ğ}\} \quad (3)$$

Where ă(ø) ∈ [0,1] represents the membership degree and ć(ø) ∈ [0,1] represents the non-membership degree, and satisfy the conditions $0 \leq \text{ă}(ø)^2 + \text{ć}(ø)^2 \geq 1$. Where the hesitancy degree is $\text{Ɣ}(ø) = \sqrt{1 - \text{ă}^2 - \text{ć}^2}$. For ease, ((ă (ø), ć(ø)) is identified as PF values (PFVs).

Membership degree in the context of clinical diagnosis is the extent of the evidence available to support a particular disease, while non-membership degree is the extent of evidence that contradicts a clinical diagnosis. The hesitation degree is due to insufficient patient information, uncertain laboratory test results, unclear information from medical imaging, or conflicting health care viewpoints. The PFS can be used to model supporting evidence, opposing evidence, and diagnostic uncertainty together, which can be used to model more realistically the uncertainty in the healthcare environment and thus improve the reliability of the clinical diagnosis, providing a more flexible and realistic framework for modeling clinical decision-making in the uncertain healthcare environment.

Definition 4:[28] The score function for PFS is illustrated

As:

$$\zeta(\o) = \breve{a}(\o)^2 - \acute{c}(\o)^2 \tag{4}$$

3. | PROPOSED GAME-THEORETIC MODEL FOR CONFLICT ANALYSIS

3.1. | Calculation of Criteria Importance Weights by the CRITIC Method

In this study, the objective weights of the diagnostic criteria are obtained using the CRITIC method. The CRITIC method differs from subjective weighting methods, which calculate criterion weights based on the opinions of decision-makers. The CRITIC method was chosen because it is capable of obtaining the weights of the criteria without any prior personal judgements and takes into account two important aspects of the data, namely its information variability and the inter-criteria correlation. The standard deviation measures the information variability, that is, the discriminating power of each criterion, and the correlation coefficient measures the degree of redundancy between criteria. If a criterion shows high variation and low correlation with the other criteria, it brings more unique information and thus receives a higher weight. The CRITIC method considers both information amounts and uniqueness at the same time, while several other objective weighting methods only consider the dispersion of data, thus yielding more reliable and objective criterion weights.

Step 1: Develop a decision matrix $Z = [\underline{r}_{ij}]$, where $i = 1,2, \dots m$ and $j = 1,2, \dots n$ the respective alternatives and criteria.

$$Z = [\underline{r}_{ij}]_{m\times n} = \begin{bmatrix} \underline{r}_{11} & \underline{r}_{12} & \underline{r}_{1n} \\ \underline{r}_{21} & \underline{r}_{22} & \underline{r}_{2n} \\ \vdots & \vdots \ddots & \vdots \\ \underline{r}_{m1} & \underline{r}_{m2} \cdots & \underline{r}_{nm} \end{bmatrix} \tag{5}$$

Step 2: Find the score value.

$$\zeta(\o) = \breve{a}(\o)^2 - \acute{c}(\o)^2 \tag{6}$$

Step 3: The standard deviation of each criterion is determined as:

$$\Upsilon_j = \sqrt{\frac{1}{\mathrm{n}}\sum_{\mathrm{i}=1}^{\mathrm{n}}\left(r_{ij} - r_j\right)^2} \tag{7}$$

Step 4: It is then established which criteria correlate with each other.

The correlation coefficient δ_{jk} between two criteria j and k is obtained using the score matrix.

$$\delta_{jk} = \frac{\sum_{i=1}^{n}(r_{ij} - r_j)(r_{ik} - r_k)}{\sqrt{(r_{ij} - r_j)^2}\sqrt{(r_{ik} - r)^2}}, \qquad j = 1, 2, \ldots\ldots, n \tag{8}$$

Where,

$$r_j = \frac{1}{\mathrm{n}}\sum_{\mathrm{i=1}}^{\mathrm{n}} r_{\mathrm{ij}}\ , r_{\mathrm{k}} = \frac{1}{\mathrm{n}}\sum_{\mathrm{i=1}}^{\mathrm{n}} r_{\mathrm{ik}} \tag{9}$$

Symbolize the means of the criteria j and k, respectively.

Step 5: To determine the amount of information, the standard deviation, and the correlation coefficient.

$$v_j = \sigma_j . \sum_{k=1}^{n}(1 - Q_{jk}) \tag{10}$$

Step 6: We calculate the objective weight.

$$\dot{\omega}_j = \frac{v_j}{\sum_{j=1}^{n} v_j} \tag{11}$$

3.2. | Ranking by MAUT Method

MAUT technique ranks alternatives based on their utility values to assess options in situations where several criteria have to be taken into consideration concurrently. In MAUT, the performance of both alternatives is used to calculate a utility score by considering the decision-maker's preferences and the weights assigned to the criteria. This change makes it easier to compare and analyze alternatives, especially when the decision problem has many conflicting criteria. MAUT is used as it offers a structured method to translate the performance of the alternatives against each of the criteria into comparable utility values. The utility aggregation mechanism aggregates the utility values of all the criteria to get an overall utility value for each alternative, using the objective CRITIC weights. This allows for ranking disease alternatives transparently and consistently. The proposed framework is based on the assumption that both the evaluation criteria are independent and that the overall utility of an alternative is simply the weighted sum of its individual criterion utilities. These assumptions are usually used in MAUT and are the basis for multi-criteria decision making.

Step 1: The decision problem can be identified and defined first, including all relevant criteria and alternatives.

Step 2: Once the criteria are defined, a weighting process is conducted to establish the relative importance of each criterion. In the research, the objective weights of the criteria are computed using the CRITIC method.

Step 3: They are known as the performance values of all alternatives in relation to each criterion which are presented in the decision matrix.

$$Z = [\underline{r}_{ij}]_{m\times n} = \begin{bmatrix} \underline{r}_{11} & \underline{r}_{12} & \underline{r}_{1n} \\ \underline{r}_{21} & \underline{r}_{22} & \underline{r}_{2n} \\ \vdots & \vdots \ddots & \vdots \\ \underline{r}_{m1} & \underline{r}_{m2} \cdots & \underline{r}_{nm} \end{bmatrix} \tag{13}$$

$\underline{r}_{ij}$ Present the act value of the ith alternative on the jth criterion.

Step 4: The Utility of every criterion is determined. When the criterion is fully met, the highest level is set to 1. The lowest value will be 0. On the other hand, the normalization process yields the following intermediate values: beneficial and non-beneficial criteria. The following equations are performed respectively, equations (14) and (15):

$$\xi_{\mathrm{j}}(\underline{r}_{ij}) = \frac{\underline{r}_{ij} - \min(\underline{r}_{ij})}{\max(\underline{r}_{ij}) - \min(\underline{r}_{ij})} \tag{14}$$

$$\xi_{\mathrm{j}}(\underline{r}_{ij}) = \frac{max(\underline{r}_{ij}) - \underline{r}_{ij}}{\max(\underline{r}_{ij}) - \min(\underline{r}_{ij})} \tag{15}$$

Here, $\xi_{\mathrm{j}}(\underline{r}_{ij})$ denotes the normalized utility value of the alternative i under criterion j.

Step 5: After evaluating the utility of each criterion, the aggregate utility of each alternative is determined.

$$\sigma(\text{₦}_i) = \sum_{\mathrm{j}=1}^{n} \text{đ}_j \underline{r}_j(\underline{r}_{ij}) \tag{16}$$

$\sigma(\text{₦}_i)$ is the total utility of the alternatives i; đ_j indicates the weights of the criterion j. The decision-makers will consider the option with the highest integrated utility value.

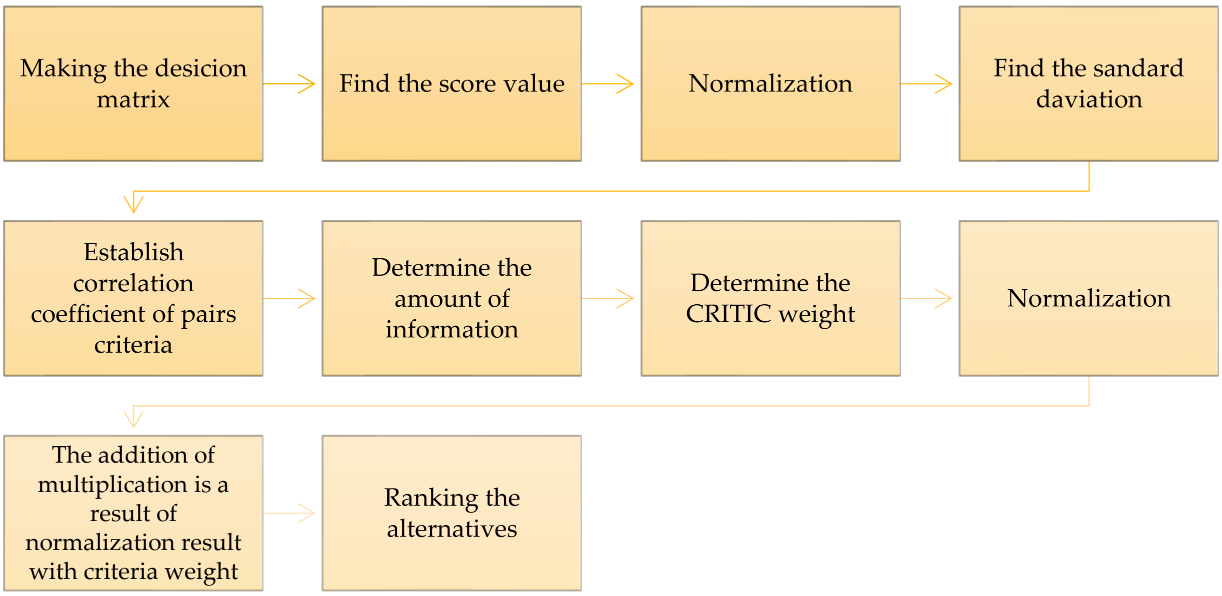

Figure 2. Stepwise Implementation Process of the Proposed Pythagorean Fuzzy CRITIC MAUT Methodology

3.3. | Mathematical Assumptions of PFS

The proposed CRITIC-MAUT framework is based on PFS, where evaluations are made by assigning membership and non-membership degrees that will satisfy the mathematical condition that $0 \leq ă^2 + ć \leq 1$. The hesitation degree is implicitly derived from these values and reflects the uncertainty that remains in connection with each evaluation. These assumptions enable PFS to effectively model uncertainty while maintaining computational efficiency and simplicity in MCDM. More recent fuzzy models, such as Fermatean fuzzy sets (FFS), spherical fuzzy sets (SFS), and q-rung orthopair fuzzy sets (q-ROFS), are more appropriate for applications that are characterized by highly complex uncertainty, conflicting judgments, and/or greater flexibility in the representation of membership and non-membership information than the decision context investigated in this study.

4. | CASE STUDY

A numerical case study is performed to show the application of the proposed Pythagorean Fuzzy CRITIC–MAUT framework in the diagnosis of disease in an uncertain clinical condition. Four disease alternatives, namely influenza, Pneumonia, COVID-19, and common cold, are assessed for each of the four diagnostic criteria: Fever severity, cough intensity, shortness of breath, fatigue level, headache severity, Consistency of laboratory tests, resource requirement, and diagnostic uncertainty. Clinicians frequently have limited and/or unclear, or inaccurate information, which makes diagnosis difficult. To overcome this problem, the proposed framework utilizes PFSs for the representation of uncertainty, the CRITIC method for the determination of objective criterion weights, and the MAUT method for the assessment and ranking of the disease alternatives based on their overall utility values. Alternatives to the disease are presented in Table 1, and diagnostic criteria in Table 3.

Table 3. Clinical Descriptions of Disease Alternatives Included in the Case Study

Alternative Code	Alternative Name	Description
$Ƈ_1$	Influenza	A common viral infection that affects the respiratory system and causes fever, cough, fatigue, headache, and muscle pain
$Ƈ_2$	Pneumonia	A lung infection that causes inflammation of air sacs and is often associated with cough, fever, and breathing difficulties.
$Ƈ_3$	COVID-19	A contagious viral disease that affects the respiratory system and may present symptoms ranging from fever to severe respiratory distress
$Ƈ_4$	Common cold	A mild viral infection of the tract usually characterized by cough, sore throat, headache, and nasal congestion.

Table 4. The Criteria Used as a Reference in the Diagnosis of Disease

Criterion Code	Criterion Name	Description
$ƛ_1$	Fever severity	Measures how bad the fever is. How long does it last?
$ƛ_2$	Cough intensity	Look at how the patient coughs.
$ƛ_3$	Shortness of breath	Checks if the patient has trouble breathing and is not getting enough oxygen.
$ƛ_4$	Fatigue level	Measures how tired the patient is.

5. | NUMERICAL EVALUATION

In this section, a numerical evaluation of the suggested Pythagorean fuzzy CRITIC MAUT decision framework is presented by using hypothetical disease data. The uncertainty and imprecision of clinical information are represented by four diagnostic criteria, which are assessed using Pythagorean fuzzy numbers (PFNs) to evaluate four disease alternatives. The Pythagorean fuzzy decision matrix is first constructed, and scores are calculated using this score function. The scores obtained are then standardized so they can be compared on the same basis across criteria. Later, the CRITIC method is used to compute the objective criterion weights of the data using its normalized values, considering the variation and correlation among the data. These weights are then used in the MAUT method to calculate the rank of each disease alternative and to get the overall ranking. The numerical evaluation demonstrates the development of the proposed framework and its effectiveness in supporting clinical disease diagnosis systematically and transparently in the presence of uncertainty. In order to demonstrate the computational procedure of the proposed Pythagorean fuzzy CRITIC MAUT framework, a hypothetical data set was used in this study since the main purpose of this study is to validate the proposed framework. The data set was compiled to represent the clinical characteristics of the common clinical cases of Influenza, Pneumonia, COVID-19, and Common Cold, chosen due to the similar symptoms and the complexity of diagnosis. To simulate uncertainty in clinical assessment, Pythagorean fuzzy numbers were assigned. So, this case study offers methodological evidence for the proposed framework, and further studies will compare its performance to the real clinical data and known diagnostic standards.

5.1. | CRITIC Method

The CRITIC method is used to establish the objective importance of the diagnostic criteria based on information in the decision matrix. At first, the Pythagorean fuzzy decision matrix will be transformed into score values by the corresponding score function. The scores obtained are then normalized to remove differences in the level of measurement of the criteria and make them comparable. Then the standard deviation of each criterion is calculated to indicate the dispersion of the data, and the correlations between the criteria are computed to assess their relationships. The information content of each criterion is then calculated, and the objective criterion weights are calculated by normalization. The weights obtained are indicative of the relative importance of the diagnostic criteria, which are then used as input for the next MAUT evaluation.

Step 1: Hypothetical membership and non-membership values were determined according to the common clinical characteristics of the selected diseases reported in the literature, and the Pythagorean fuzzy decision matrix was developed. The severity and uncertainty of each diagnostic criterion were represented with fuzzy values that meet the conditions of PFS. For methodological validation of the proposed method, these values are used, and not real patient data. The decision matrix is given in Table 5.

Table 5. Initial Pythagorean Fuzzy Decision Matrix for Disease Diagnosis

	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
$\tilde{c}_1$	(0.50,0.20)	(0.35,0.55)	(0.57,0.40)	(0.33,0.20)
$\tilde{c}_2$	(0.78,0.40)	(0.35,0.50)	(0.55,0.25)	(0.50,0.30)
$\tilde{c}_3$	(0.45,0.30)	(0.10,0.30)	(0.45,0.15)	(0.70,0.20)
$\tilde{c}_4$	(0.20,0.10)	(0.10,0.60)	(0.27,0.16)	(0.60,0.30)

Step 2: In this step, we used definition 2 to find the scores of the alternatives provided in Table 6.

Table 6. The Scores Obtained from the Pythagorean Duzzy Decision Matrix

	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
$\tilde{c}_1$	0.1170	−0.1235	0.1212	0.0279
$\tilde{c}_2$	0.4106	−0.0821	0.1508	0.0980
$\tilde{c}_3$	0.0641	−0.0260	0.0878	0.3350
$\tilde{c}_4$	0.0070	−0.2150	0.0156	0.1890

Step 3. We find the normalized value as shown in Table 7.

Table 7. Normalized Decision Matrix

	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
$\tilde{c}_1$	0.2726	0.5159	0.7813	0.0000
$\tilde{c}_2$	1.0000	0.2970	1.0000	0.2282
$\tilde{c}_3$	0.1416	0.0000	0.5339	1.0000
$\tilde{c}_4$	0.0000	1.0000	0.0000	0.5245

Step 4. The standard deviation is calculated and is presented in Table 8

Table 8. Presentation of the Standard Deviation

$\mathbf{0.3855}$	$\mathbf{0.3649}$	$\mathbf{0.3726}$	$\mathbf{0.3739}$

Step 5. At this point, the correlation is computed, and the results are provided in Table 9.

Table 9. Correlation Coefficient Among Diagnostic Criteria

	λ_1	λ_2	λ_3	λ_4
C_1	1.0000	−0.3613	0.8180	−0.4335
C_2	−0.3613	1.0000	−0.6421	−0.3702
C_3	0.8180	−0.6421	1.0000	−0.4529
C_4	−0.4335	−0.3702	−0.4529	1.0000

Step 6. We identify the number of pieces of information that will be between the standard deviation and the correlation coefficient. These findings are presented in Table 10.

Table 10. Information Content of Diagnostic Criteria Computed Using the CRITIC Method.

Quantity of Information			
$\mathbf{0.5347}$	0.5935	0.4051	0.6519

Step 7. The objective weight is obtained, and it is presented in Table 11.

Table 11. Objective Weights of Diagnostic Criteria Obtained Using the CRITIC Method.

CRITIC Weight			
$\mathbf{0.2447}$	0.2716	0.1854	0.2983

5.2. | MAUT Method

Once the objective criterion weights are obtained using the CRITIC method, the MAUT method is used to rank the disease alternatives. The weights of each criterion are involved in calculating the utility value of each criterion for each disease alternative. The CRITIC weights are used in the calculation of the utility value of each criterion for each disease alternative. These weighted utility values are then combined to get the overall utility score of each alternative. Lastly, the disease alternatives are ranked by their integrated utility values, with the highest value indicating the most suitable diagnosis. This evaluation shows that the proposed Pythagorean Fuzzy CRITIC MAUT approach applies to disease diagnosis in an uncertain clinical setting and is systematic and transparent.

Step 8: Initially, the utility of each criterion is evaluated. The normalization procedure is applied to intermediate values. The findings are given in Table 12.

Table 12. Utility Values of Disease Alternatives Computed Using the MAUT Approach

	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
$\tilde{C}_1$	0.2726	0.5159	0.7813	0.0000
$\tilde{C}_2$	1.0000	0.2970	1.0000	0.5245
$\tilde{C}_3$	0.1416	1.0000	0.0000	1.0000
$\tilde{C}_4$	0.0000	0.0000	0.0000	0.5245

Step 9: The integrated utility of each choice is calculated by using the utility function of every criterion. The weights of the criteria obtained by the CRITIC approach are used in these calculations. Lastly, Table 13 presents the integrated utility of each alternative.

Table 13. Integrated Utility Scores and Final Ranking of Disease Alternatives

	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$	**Total Utility**
$\tilde{C}_1$	0.0667	0.1262	0.1912	0.0000	0.3841
$\tilde{C}_2$	0.2716	0.0807	0.2716	0.1425	0.7664
$\tilde{C}_3$	0.0262	0.1854	0.0000	0.1854	0.3970
$\tilde{C}_4$	0.0000	0.0000	0.0000	0.1565	0.1565

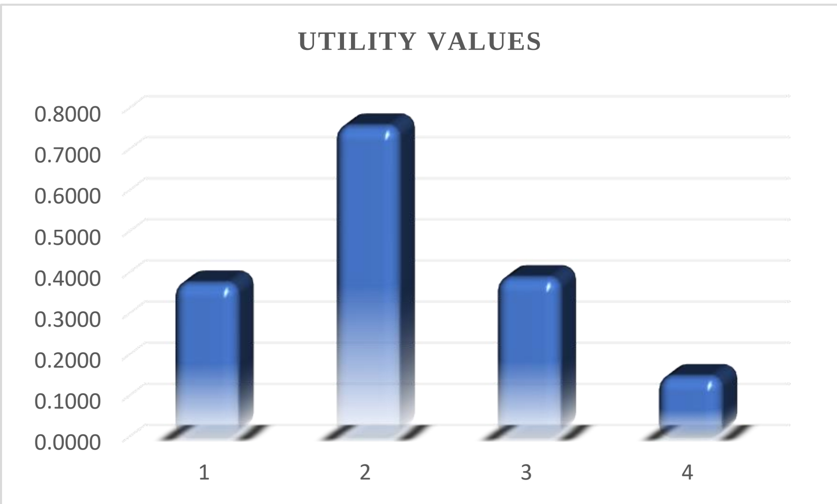

Figure 3. Disease Alternatives Ranking Using the Proposed Pythagorean Fuzzy CRITIC MAUT Framework. The Horizontal Axis Shows the Range of Alternatives, and the Vertical Axis Shows the Range of Score Values.

5.3. | Results Discussion

As shown in Table 11, the objective weight (λ_i) that was highest was the weight for fatigue level (λ_4), followed by cough intensity(λ_2), fever severity (λ_1), and shortness of breath (λ_3). A higher weight in the CRITIC method means that a criterion will give more useful and distinctive information to distinguish the disease alternatives. The high weight given to fatigue level indicates that this symptom is very different in the different diseases evaluated and may offer the most unique information in the diagnostic process. Conversely, shortness of breath had the lowest weight, meaning that it is less discriminatory for differentiating the diseases that were included in this study. The weights obtained are further analyzed in the correlation analysis given in Table 9. There was a strong positive correlation between the severity of fever (λ_1) and the development of shortness of breath (λ_3) suggesting that these two parameters can give similar information for diagnosis. The CRITIC method gives reduced weight to redundant information, and therefore the shortness of breath (λ_3) was weighed lower. By contrast, the fatigue level (λ_4) correlations with other criteria were relatively low, indicating that it provides relatively independent and discriminative information. Consequently, the objective weight for fatigue was the highest, and it was the most important factor in the final disease ranking.

Pneumonia ($\mho_2$) was the most preferred disease choice and the disease with the highest total utility score emerged in the MAUT evaluation. This finding suggests that pneumonia meets more weighted diagnostic criteria with a higher level of weight, specifically the highly weighted criteria of fatigue level and cough intensity. The overall diagnostic performance of the rest of the diseases was comparable, with COVID-19 ($\mho_3$) taking second place with a utility score, followed closely by Influenza ($\mho_1$). Common cold ($\mho_4$) obtained the lowest utility score; hence, the performance of common cold, in terms of the weighted clinical criteria, was not as good as the other evaluated alternatives, and therefore common cold was the least preferred diagnosis.

The results of this study indicate that the proposed Pythagorean Fuzzy CRITIC–MAUT is effective in identifying the most informative diagnostic criteria and in ranking the disease alternatives objectively with respect to their overall utility values. Objective criterion weighting combined with Pythagorean fuzzy uncertainty modelling helps to make a clinical decision that is transparent, consistent, and reliable in the case of diagnostic uncertainty.

6. | QUALITATIVE COMPARISON ANALYSIS

The qualitative comparison of the proposed Pythagorean fuzzy CRITIC MAUT approach for disease diagnosis in uncertain clinical information. FS, IFS, and PFS can deal with uncertainty in medical diagnosis, but they do not have an integrated mechanism to determine the importance of the diagnostic criteria and systematically rank the alternatives of diseases objectively. The proposed framework can resolve these drawbacks, using a PFS to represent uncertainty, a CRITIC methodology to weight the objective criteria, and a MAUT methodology to rank the approaches. The integration allows for the handling of uncertainty and hesitation in clinical evaluations. Moreover, the proposed approach helps to more reliably and transparently assess the competing disease alternatives by taking into account the interrelationships and variability between the diagnostic criteria. The resulting utility ranking helps health care providers select the most appropriate alternative disease diagnosis, based on the overall performance of each alternative for several clinical criteria. The proposed Pythagorean Fuzzy CRITIC MAUT framework is, therefore, superior to the existing fuzzy approaches in terms of giving a more comprehensive and effective decision support framework for disease diagnosis in uncertain clinical environments. The qualitative comparison analysis of the proposed approach with the existing framework is given in Table 14.

Table 14. Qualitative Comparison of the Proposed Framework with Existing Fuzzy Decision-Making Approaches

Features	FS	IFS	PFS	Proposed PFS CRITIC MAUT
Handles uncertain clinical data	✓	✓	✓	✓
Represents hesitation	✗	✓	✓	✓
Larger uncertainty representation space	✗	✗	✓	✓
Supports objective criterion weighting	✗	✗	✗	✓
Considers inter-criteria correlation	✗	✗	✗	✓
Reduces subjective bias	✗	✗	✗	✓
Supports utility-based ranking	✗	✓	✓	✓
Supports multi-criteria evaluation	✓	✓	✓	✓
Provides a transparent decision process	✓	✓	✓	✓
Ensures consistent evaluation	✓	✓	✓	✓
Suitable for an uncertain medical diagnosis	✓	✓	✓	✓
Applicable to disease diagnosis	✓	✓	✓	✓

7. | SENSITIVITY ANALYSIS

To test the strength and stability of the proposed decision-making process, four different weighting cases ($S_1$to S_4) were considered, in addition to the equal-weight case. The results show that C_2 always secured the best performance value in all the scenarios regardless of the changes in the weights of the criterion. The performance value of C_3 was generally the second best, but it depended on the weights allocated. In all scenarios, C_1 consistently had the third highest position, and C_4had the lowest performance values in the analysis. Only under the scenario S_4 did the ranking of the alternatives change slightly, with the change being that C_2 and C_3 swapped positions, and C_2 remained the best alternative, while C_4 remained the worst. This minor change is in line with the strength, stability, and reliability of the suggested decision-making model in the two weightings. Moreover, the small number of criterion changes needed to obtain the various ranking results indicates the insensitivity of the proposed approach to criterion changes. This robustness suggests that the model can consistently and reliably deliver the decision results that decision makers would like even if they give different weights to their preferences. Thus, through the sensitivity analysis, the reliability of the proposed model for multi-criteria decision making in practice is confirmed. The results of the detailed sensitivity analysis are summarized in Table 15, and the graphical comparison of the performance values is shown in Figure 4.

Table 15. Sensitivity Analysis Ranking Results with Different Weight Scenarios

Scenarios	Ranking results	Best alternative
Original weight	$C_2 > C_3 > C_1 > C_4$	C_2
$\boldsymbol{S_1}$	$C_2 > C_3 > C_1 > C_4$	C_2
$\boldsymbol{S_1}$	$C_2 > C_3 > C_1 > C_4$	C_2
$\boldsymbol{S_1}$	$C_2 > C_3 > C_1 > C_4$	C_2
$\boldsymbol{S_1}$	$C_2 > C_3 > C_1 > C_4$	C_2
Equal weight	$C_2 > C_3 > C_1 > C_4$	C_2

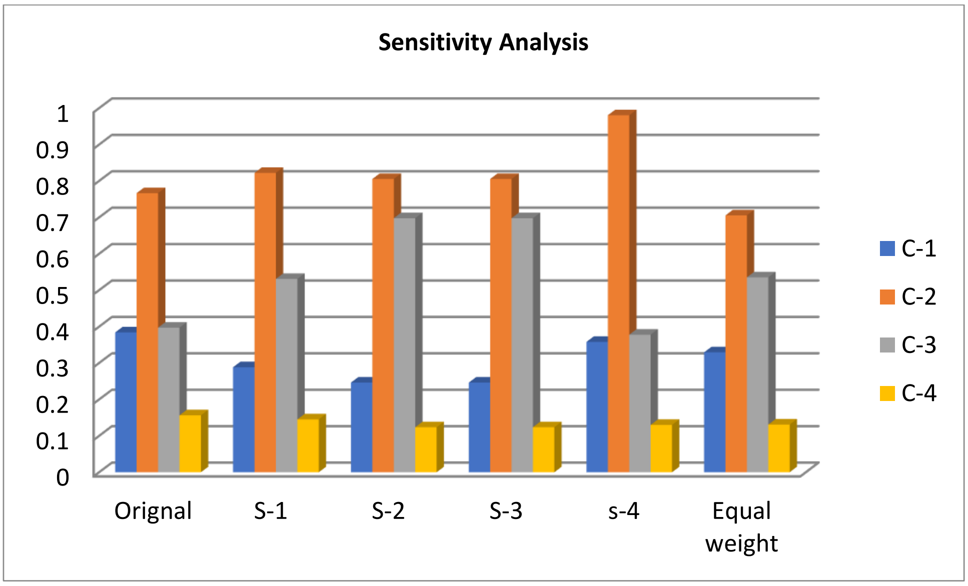

Figure 4. Shows the Impact of Weight Change on Ranking. The Horizontal Axis Shows the Different Weight Scenarios ($S_1 - S_4$) and the vertical axis shows the Range of Score Values. The alternative is Represented by ($C_1 - C_4$).

8. | CONCLUSION

A new Pythagorean Fuzzy CRITIC MAUT decision-making model was proposed for the diagnosis of diseases with incomplete medical information. The framework was successfully developed to combine the PFS, CRITIC method, and MAUT method to effectively deal with uncertainty, identify objective criteria weights, and rank the disease alternatives. The proposed model can reduce subjectivity, that is, from the position of minimizing subjective bias, to give a systematic and reliable decision-support mechanism that enhances diagnostic accuracy. The case study showed that the framework can effectively identify the most probable disease even when multiple symptoms are present and when the situation is uncertain. The results show that the framework proposed in this study provides more flexibility, objectivity, and reliability than the traditional decision-making process. Thus, it may be a helpful instrument to assist medical practitioners in disease diagnosis. This paper can be extended by implementing more sophisticated fuzzy techniques or incorporating clinical and real-world datasets, which could further improve diagnostic performance.

8.1. | Limitations and Future Directions

This study proposes a disease diagnosis decision model based on Pythagorean fuzzy CRITIC MAUT for uncertain clinical data. The proposed framework is capable of performing well with respect to uncertainty management and objectively ranking disease alternatives, but it has some drawbacks.

First of all, the numerical evaluation is not conducted on a real clinical case, but on a hypothetical case study. Hence, the proposed framework should be tested on real patient records from hospitals or other healthcare institutions to assess its effectiveness in practice. Second, there is a limited number of disease alternatives and diagnostic criteria that are taken into consideration. The number of diseases, symptoms, laboratory test results, and individual patient factors involved in diagnosis is greater in "real" clinical settings and may make the diagnostic process more complex. Third, the proposed model assumes that the diagnostic criteria are independent, but interactions between medical attributes could affect the final diagnosis.

In the future, we will extend our work with different fuzzy frameworks, like Generalized q-ROFS [29]. Similarity measures based on T-spherical fuzzy information with applications to pattern recognition and decision making [30] and A novel decision-making algorithm enhancing career planning and employment guidance on the circular T-spherical fuzzy CRITIC WASPAS method [31].

Acknowledgment: The authors thank the anonymous reviewers for their valuable comments and constructive suggestions, which helped improve the quality of this manuscript.

Author Contributions: Conceptualization, Naima and S.J.; methodology, K.S. and S.J.; software, S.F.; validation, Naima, K.S., and S.J.; formal analysis, K.S.; investigation, Naima and K.S.; resources, S.J.; data curation, S.F.; writing—original draft preparation, Naima and S.F.; writing—review and editing, S.J. and K.S.; visualization, S.F.; supervision, S.J.; project administration, S.J. All authors have read and agreed to the published version of the manuscript.

Declaration of Conflicting Interest: The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding: The author(s) received no financial support for the research, authorship, and/or publication of this article.

Data Availability Statement: The data supporting the findings of this study are synthetically generated and are fully included within this article. No additional datasets were generated or analyzed during the current study.

Plagiarism Statement: This manuscript was screened for textual similarity using Turnitin. The similarity results were found to be within the plagiarism thresholds prescribed by the Higher Education Commission (HEC) of Pakistan and the journal's editorial policy.

Artificial Intelligence (AI) Statement: The authors declare that no generative artificial intelligence tools or AI-assisted technologies were used in the preparation of this manuscript.

REFERENCES

[1] H. Aydoğan and V. Ozkir, "A Fermatean fuzzy MCDM method for selection and ranking Problems: Case studies," *Expert Syst. Appl.*, vol. 237, p. 121628, Mar. 2024, doi: 10.1016/j.eswa.2023.121628.

[2] D. Dalalah, M. Hayajneh, and F. Batieha, "A fuzzy multi-criteria decision making model for supplier selection," *Expert Syst. Appl.*, vol. 38, no. 7, pp. 8384–8391, Jul. 2011, doi: 10.1016/j.eswa.2011.01.031.

[3] C. Kahraman, "Multi-Criteria Decision Making Methods and Fuzzy Sets," in *Fuzzy Multi-Criteria Decision Making*, vol. 16, C. Kahraman, Ed., in Springer Optimization and Its Applications, vol. 16. , Boston, MA: Springer US, 2008, pp. 1–18. doi: 10.1007/978-0-387-76813-7_1.

[4] T.-C. Chu and Y. Lin, "An extension to fuzzy MCDM," *Comput. Math. Appl.*, vol. 57, no. 3, pp. 445–454, Feb. 2009, doi: 10.1016/j.camwa.2008.10.076.

[5] G.-S. Liang, "Fuzzy MCDM based on ideal and anti-ideal concepts," *Eur. J. Oper. Res.*, vol. 112, no. 3, pp. 682–691, Feb. 1999, doi: 10.1016/S0377-2217(97)00410-4.

[6] H.-K. Chiou, G.-H. Tzeng, and D.-C. Cheng, "Evaluating sustainable fishing development strategies using fuzzy MCDM approach," *Omega*, vol. 33, no. 3, pp. 223–234, Jun. 2005, doi: 10.1016/j.omega.2004.04.011.

[7] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965, doi: 10.1016/S0019-9958(65)90241-X.

[8] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, no. 1, pp. 87–96, Aug. 1986, doi: 10.1016/S0165-0114(86)80034-3.

[9] R. R. Yager, "Properties and Applications of Pythagorean Fuzzy Sets," in *Imprecision and Uncertainty in Information Representation and Processing: New Tools Based on Intuitionistic Fuzzy Sets and Generalized Nets*, P. Angelov and S. Sotirov, Eds., Cham: Springer International Publishing, 2016, pp. 119–136. doi: 10.1007/978-3-319-26302-1_9.

[10] X. Peng and G. Selvachandran, "Pythagorean fuzzy set: state of the art and future directions," *Artif. Intell. Rev.*, vol. 52, no. 3, pp. 1873–1927, Oct. 2019, doi: 10.1007/s10462-017-9596-9.

[11] G. Işık, "A framework for choosing an appropriate fuzzy set extension in modeling," *Appl. Intell.*, vol. 53, no. 11, pp. 14345–14370, Jun. 2023, doi: 10.1007/s10489-022-04244-2.

[12] M. Palanikumar, N. Kausar, D. Pamucar, and V. Simic, "Optimizing industrial robot selection using novel trigonometric Pythagorean fuzzy normal aggregation operators," *Complex Intell. Syst.*, vol. 11, no. 10, p. 453, Sep. 2025, doi: 10.1007/s40747-025-02083-5.

[13] M. Asif, U. Ishtiaq, and I. K. Argyros, "Hamacher aggregation operators for pythagorean fuzzy set and its application in multi-attribute decision-making problem," *Spectr. Oper. Res.*, vol. 2, no. 1, pp. 27–40, 2025.

[14] M. Akram, S. Zahid, and M. Deveci, "Enhanced CRITIC-REGIME method for decision making based on Pythagorean fuzzy rough number," *Expert Syst. Appl.*, vol. 238, p. 122014, Mar. 2024, doi: 10.1016/j.eswa.2023.122014.

[15] İ. M. Eligüzel and N. Eligüzel, "A Hybrid LDA and Fuzzy CRITIC-MABAC Model for Smart City Ranking," *IEEE Access*, vol. 13, pp. 51255–51267, 2025, doi: 10.1109/ACCESS.2025.3553674.

[16] F. Sahin and A. Menekse, "Internal control risk assessment using interval valued spherical fuzzy CRITIC EDAS," *Sci. Rep.*, vol. 15, no. 1, p. 21505, Jul. 2025, doi: 10.1038/s41598-025-07852-3.

[17] M. Khazaei, A. Hafezalkotob, and R. Kia, "Chapter 50 - MAUT: Multi-attribute utility theory method for multi-attribute decision-making," in *Encyclopedia of Multi-Attribute Decision Making (MADM)*, G. Haseli, M. Deveci, and M. Hajiaghaei-Keshteli, Eds., Morgan Kaufmann, 2026, pp. 701–712. doi: 10.1016/B978-0-443-33275-3.00069-5.

[18] A. Gómez-Jiménez, A. Jiménez-Martín, Z. Chergui, and S. Marín, "A hierarchical elicitation process based on the combination of different weighting methods within multi-attribute utility theory in a group decision-making context," *Ann. Oper. Res.*, Mar. 2026, doi: 10.1007/s10479-026-07146-5.

[19] D. Wangmo and R. Singh, "Potential Pythagorean fuzzy soft set: A novel approach for medical diagnosis techniques," *J. Fuzzy Ext. Appl.*, vol. 7, no. 2, pp. 615–632, Jun. 2026, doi: 10.22105/jfea.2025.460302.1487.

[20] A. Dey, T. Senapati, M. Pal, and G. Chen, "Pythagorean fuzzy soft RMS approach to decision making and medical diagnosis," *Afr. Mat.*, vol. 33, no. 4, p. 97, Nov. 2022, doi: 10.1007/s13370-022-01031-7.

[21] N. Kumar and J. Mahanta, "A Hybrid Decision Making Framework with Pythagorean Fuzzy Information for Childhood Cancer Prognosis," *Sahand Commun. Math. Anal.*, vol. 22, no. 2, pp. 109–141, Apr. 2025, doi: 10.22130/scma.2024.2038238.1851.

[22] B. Sun, S. Tong, W. Ma, T. Wang, and C. Jiang, "An approach to MCGDM based on multi-granulation Pythagorean fuzzy rough set over two universes and its application to medical decision problem," *Artif. Intell. Rev.*, vol. 55, no. 3, pp. 1887–1913, Mar. 2022, doi: 10.1007/s10462-021-10048-6.

[23] P. A. Ejegwa, “Improved composite relation for pythagorean fuzzy sets and its application to medical diagnosis,” *Granul. Comput.*, vol. 5, no. 2, pp. 277–286, Apr. 2020, doi: 10.1007/s41066-019-00156-8.

[24] M. Saeed, M. Ahsan, M. H. Saeed, A. Mehmood, H. A. E.-W. Khalifa, and I. Mekawy, “The Prognosis of Allergy-Based Diseases Using Pythagorean Fuzzy Hypersoft Mapping Structures and Recommending Medication,” *IEEE Access*, vol. 10, pp. 5681–5696, 2022, doi: 10.1109/ACCESS.2022.3141092.

[25] M. Palanikumar, N. Kausar, D. Pamucar, S. Khan, and M. A. Shah, “Complex Pythagorean Normal Interval-Valued Fuzzy Aggregation Operators for Solving Medical Diagnosis Problem,” *Int. J. Comput. Intell. Syst.*, vol. 17, no. 1, p. 118, May 2024, doi: 10.1007/s44196-024-00504-w.

[26] S. Habib, S. Shahzadi, and M. Deveci, “Pythagorean fuzzy cognitive analysis for medical care and treatment decisions,” *Granul. Comput.*, vol. 8, no. 6, pp. 1887–1906, Nov. 2023, doi: 10.1007/s41066-023-00407-9.

[27] A. K. Shukla, P. Mehra, and P. K. Muhuri, “Fuzzy Sets-Based Approaches for Improved Medical Diagnosis: An Analysis and Overview of Major Research Directions,” *ACM Comput. Surv.*, Jul. 2025, doi: 10.1145/3757058.

[28] G.-W. Wei, “Pythagorean Fuzzy Hamacher Power Aggregation Operators in Multiple Attribute Decision Making,” *Fundam. Informaticae*, vol. 166, no. 1, pp. 57–85, Mar. 2019, doi: 10.3233/FI-2019-1794.

[29] R. R. Yager, “Generalized Orthopair Fuzzy Sets,” *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 5, pp. 1222–1230, Oct. 2017, doi: 10.1109/TFUZZ.2016.2604005.

[30] M. N. Abid, M.-S. Yang, H. Karamti, K. Ullah, and D. Pamucar, “Similarity Measures Based on T-Spherical Fuzzy Information with Applications to Pattern Recognition and Decision Making,” *Symmetry*, vol. 14, no. 2, p. 410, 2022.

[31] Z. Ahmad and E. Rak, “A Novel Decision-Making Algorithm Enhancing Career Planning and Employment Guidance on Circular T-Spherical Fuzzy CRITIC WASPAS Method,” *J. Innov. Res. Math. Comput. Sci.*, vol. 4, no. 2, pp. 51–78, 2025.