ORIGINAL ARTICLE

OPEN ACCESS

A Novel Approach of Picture Fuzzy Sets with Unknown Degree of Weights based on Schweizer-Sklar Aggregation Operators.

Abrar Hussain[1], Sajid Latif[2], Kifayat Ullah[3]

Author's Affiliation:

[1, 2,3]Department of Mathematics, Riphah International University (Lahore campus), Lahore, Pakistan

Article History:

Submitted: August 13, 2022
Revised: November 19, 2022
Accepted: December 23, 2022
Published online: December 30, 2022

Corresponding author(s):

Kifayat Ullah
Kifayat.khan.dr@gmail.com

ABSTRACT

This work aims to present some new aggregation operators (AOs) by generalizing the theory of Schweizer-Sklar (SS) aggregation tools. The picture fuzzy set (PFS) is the modified version of intuitionistic fuzzy sets and fuzzy sets. The PFS can handle such situations with three components of human opinion to mitigate the impact of insufficient information on human opinions. By utilizing concepts of prioritization, several mathematicians proposed different AOs. Some realistic operations of SS are presented under the picture fuzzy (PF) information system. We develop a class of new approaches based on picture fuzzy information, including picture fuzzy Schweizer-Sklar prioritized average (PFSSPA) and picture fuzzy Schweizer-Sklar prioritized geometric (PFSSPG) operators. Moreover, we also proposed a series of AOs with specific degrees of weights, such as picture fuzzy Schweizer-Sklar prioritized weighted average (PFSSPWA) and picture fuzzy Schweizer-Sklar prioritized weighted geometric (PFSSPWG) operators. Some realistic characteristics and special cases of currently developed approaches are also presented. To demonstrate the solution to complicated real-life problems, an algorithm for the MADM problem is established under a system of picture-fuzzy information. To check the potential of proposed aggregation approaches, we gave a numerical example to select desirable optimal options by using invented approaches. To reveal the flexibility and applicability of invented approaches, sensitive analysis, and comparative analysis are illustrated by contrasting the findings of existing approaches with currently developed approaches.

Keywords: Picture fuzzy values, Aggregation operators, Schweizer Sklar aggregation tools, and decision-making process.

1 | INTRODUCTION

By considering preference values offered by experts based on attribute values, multi-attribute decision-making (MADM) challenges primarily affect the evaluation or derivation of the best form of the limited preference. However, due to the fuzziness and complexity of real-world decision-making (DM) issues, experts have a hard time giving concrete numerical representations of people's preferences. To solve the aforementioned problem, Zadeh [1] revised and expanded classical set theory and developed the concept of the fuzzy set (FS). Some experts have displayed and used FS in the surroundings of many fields due to its valuable verity and prominent form. FS is the generalization of a crisp set in which an element can be a member of a universal set or not. Many mathematicians have used the idea of FS in various situations, for example [2], pattern recognition [3], and medical diagnosis [4]. The ideology of FS consists of only one grade, named membership grade (MG), and is defined on a universal set with the restriction to the unit interval $[0, 1]$. However, it is not sufficient to only discuss the MG as human opinion is not unidirectional. It may be the opposite of some ideas. It is necessary to adjust the non-membership grade (NMG). Information is presented in the form of MG and NMG in many valuable and practical contexts. To overcome this problem, Atanassov [5] included the NMG grade into the FS framework, which is named the intuitionistic fuzzy set (IFS) technique. The defining feature of IF information is expressed by $\left(ƀ(ϐ), ƅ(ϐ)\right)$ where the terms $ƀ(ϐ)$ and $ƅ(ϐ)$ represent the MG and NMG. If information is handled extremely conveniently, even when dealing with an issue that FSs cannot give. Additionally, many researchers have made numerous applications in varied and useful disciplines, such as the derivation of several distance-measure-based IF information by Chen et al. [6], Li et al. [7] put forward the theory of Choquet integral-based IF aggregation operators (AOs). Hung and Yang [8] defined similarity measures of IFS based on Hausdorff distance. After studying IFS's fundamental ideas, Atanassov generalized the MD and NMD to interval numbers. Atanassov [9] introduced interval-valued (IV) IFS (IVIFS) and established its operational principles and comparison techniques. An IFS cannot deal with all the issues when the sum of MG and NMG exceeds the limits of the unit interval. For example, if the value of MG is 0.7 and NMG is 0.5, then their sum is $0.7 + 0.5 = 1.2$ exceeding the limit of IFS.

To overcome these issues, Yager [10] gave the idea of the Pythagorean fuzzy set (PyFS) that deals with ambiguous data in various fuzzy environments and deals with the problems in a better way than IFS. A PyFS has the condition $ƀ^2(ϐ) + ƅ^2(ϐ)$ which is more general than IFS and easy to tackle the problems faced during IFS. Hussain et al. [11] worked on the PyF soft set and described their application in DM. Peng et al. [12] developed PyF soft and explained their application. Some cases arise when PyFS fails as the condition $ƀ^2(ϐ) + ƅ^2(ϐ)$ does not meet the required data, for instance, if MG is 0.8 and NMG is 0.9. After applying the PyFS condition $(0.8)^2 + (0.9)^2 = 1.45 \notin [0,1]$ which does not meet the restriction of PyFS. Yager [13] proposed a robust theory of q-rung orthopair fuzzy set (q-ROFS) by the relaxing condition of PyFSs with the sum of q^{th} power of the MG and NMG bounded on $[0, 1]$. Later on, Ali et al. [14] put forward the theory of q-ROFS to handle vague information about human opinions. The mathematical structure of the q-ROFS is as follows $ƀ^q(ϐ) + ƅ^q(ϐ), \forall, q \in Z^+$. Wang et al. [15] viewed some similarity measures based on the cosine function for q-ROFS and found their applications. Tian et al. [16] extended a novel approach of the q-rung orthopair fuzzy system to select a suitable green supplier enterprise with an advanced decision making procedure. Liu et al. [17] also proposed cosine similarity measures and some distance measures between q-ROFS. According to the findings of the preceding investigations, all the above-mentioned decision-making and aggregation operators (AOs) fall within the purview of the IFS or IVIF set theories.

Although these theories have been effectively utilized in a variety of fields, certain real-world circumstances cannot be described by IFSs. For example, in the case of voting, human views include more yes, abstain, no, and refuse responses, which cannot be effectively represented in IFSs. Also, if an expert solicits an opinion from a specific individual concerning a specific thing, a person may respond that 0.3 indicates the probability that the statement is true, 0.4 indicates that the statement is untrue, and 0.2 indicates that he or she is unsure. This is also not addressed by the FSs or IFSs. Cuong [18] introduced the ideology of a picture fuzzy set (PFS) by generalizing the idea of an IFS. Cuong and Pham [19] developed some fuzzy logic operators for PFS. Datta and Ganju [20] acknowledged some new aspects of PFS. Cuong [21] made a representable classification of the t-norm for PFS. Garg [22] developed AOs for PFS using the multi-criteria DM technique. Ahmad and Dai [23] proposed PF rough sets and rough PFSs and found their applications. Hussain et al. [24] elaborated the q-rung orthopair fuzzy structure theory to reduce the sparking rate between wires crossing in a grid station. Khan et al. [25] projected a generalized PF soft set and found their uses in decision support systems. Boran and Akay [26] extended the theory of similarity measures based on intuitionistic fuzzy information to reduce the influence of incomplete information on human opinions. Senapati et al. [27]–[29] extended the theoretic concepts of triangular norms aggregation tools and developed a class of new approaches for solving real-life applications.

Some particular properties of Hamacher aggregation tools explored based on Pythagorean fuzzy information proposed and suggested a class of new approaches by Lu et al. [30]. Riaz et al. [31] discussed an application for sustainable energy resources and classified different characteristics of q-ROF information. Liu and Jiang [32] discussed some realistic aggregation tools to express distance among different objects by extending the concepts of IFSs with interval-valued intuitionistic fuzzy information. The decision making process and some robust mathematical approaches under a system of q-rung orthopair fuzzy situations presented by Riaz et al. [33]. Liu and Gao [34] utilized appropriate features of McLaurin symmetric mean operators and developed a class of new approaches to handle unpredictable situations. Due to fluctuations in the prices of fuel and gas, there are many advantages of electric vehicles, Zhao and Li [35] introduced a robust aggregation model to choose a reliable electric motor car under a system of decision making process. Several mathematicians demonstrated numerous aggregation models under consideration of different fuzzy domains.

The concepts of Schweizer-Sklar aggregation tools were given by Schweizer-Sklar [36] in 1960. These aggregation tools are more flexible and enable the provision of authentic information during the decision-making process. By using some reliable characters of Schweizer-Sklar aggregation tools, several mathematicians anticipated different aggregation models based on different fuzzy circumstances. For instance, Garg et al. [37] illustrated a series of new aggregation approaches by using basic Hamacher aggregation tools' basic operations under intuitionistic fuzzy information. Correlation among different input arguments is expressed by utilizing characteristics of Schweizer-Sklar aggregation tools, Khan et al. [38] classified a class of new approaches based on intuitionistic fuzzy information. A novel approach to the decision-making process under the system of interval-valued intuitionistic fuzzy information was developed by Zhang [39]. We also studied some robust characteristics and features of Schweizer-Sklar aggregation tools under different fuzzy circumstances seen in references [40]–[42].

The PFS contains extensive information about an object with three components of human opinions and can deal with unpredictable information about human opinions. Sometimes decision makers face a few difficulties during the decision process. The concepts of prioritization among different attributes make decisions easy for a decision-maker. The main objectives and contributions of this research work are organized as follows:

i. To discuss the notion of PFS with appropriate characteristics of PF information.
ii. Some reliable operations of Schweizer-Sklar aggregation tools are presented under the system of PF information.
iii. By utilizing the concepts of prioritization among different attributes, we developed a class of new approaches like PFSSPA, PFSSPG, PFSSPWA, and PFSSPWG operators.
iv. To show the effectiveness and reliability of developed approaches, some realistic features and characteristics are also expressed.
v. To resolve some complicated challenges of real-life situations, we demonstrate an algorithm of the MAMD problem under consideration of our developed methodologies.
vi. The setting of different parametric values of the Schweizer-Sklar aggregation tools discusses the advantages and applicability of invented approaches.

The structure of this article is organized as follows: Section 2 exposed basic concepts of PFSs with some fundamental rules, which are necessary for the development of this research work. Some reliable operations of the Schweizer-Sklar aggregation tools are proposed in section 3. A class of new approaches using basic operations of Schweizer-Sklar aggregation tools, including PFSSPA and PFSSPG operators presented in section 4. Section 5 covered a class of new approaches based on PF information like PFSSPWA and PFSSPWG operators. An algorithm of the MADM problem and a numerical example are illustrated to choose a suitable optimal option during the decision-making process in section 6. Some remarkable comments summarized the complete research work in section 7.

2 | PRELIAMINARIES

In this section, we have improved the quality of certain current data. To generate novel concepts and insightful hypotheses, we first define elementary definitions of IFS and PFS and revise the idea of prioritized AO.

Definition 1: [5] Let an IFS Ŧ on a universal set Ꞧ is expressed as follows:

$$Ŧ = \{(ɞ, ƀ_Ŧ(ɞ), ƀ_Ŧ(ɞ)) | ɞ \in Ʀ\}$$

Where $ƀ_Ŧ(ɞ): Ʀ \rightarrow [0,1]$ denotes the MG and $Ƀ_Ŧ(ɞ)$ denotes NMG of ɞ in Ʀ, respectively. An IFS must satisfy the following conditions:

$$0 \leq ƀ_Ŧ(ɞ) + Ƀ_Ŧ(ɞ) \leq 1$$

Definition 2: [43] Let a PFS Ŧ on a universal set Ʀ is expressed as follows:

$$Ŧ = \{(ɞ, ƀ_Ŧ(ɞ), Ч_Ŧ(ɞ), Ƀ_Ŧ(ɞ)) | ɞ \in Ʀ\}$$

Where $ƀ_Ŧ(ɞ): Ʀ \rightarrow [0,1]$, $Ч_Ŧ(ɞ): Ʀ \rightarrow [0,1]$, and $Ƀ_Ŧ(ɞ): Ʀ \rightarrow [0,1]$ denote the MG, the AG, and the NMG of ɞ in Ʀ, respectively. A PFS must satisfy the following conditions:

$$0 \leq ƀ_Ŧ(ɞ) + Ч_Ŧ(ɞ) + Ƀ_Ŧ(ɞ) \leq 1$$

Furthermore, the refusal grade is indicated by $₡=\left(1 - \left(ƀ_Ŧ(ɞ) + Ч_Ŧ(ɞ) + Ƀ_Ŧ(ɞ)\right)\right)$ and a triplet $ℵ = \left(ƀ_Ŧ(ɞ), Ч_Ŧ(ɞ), Ƀ_Ŧ(ɞ)\right)$ is known as picture fuzzy value (PFV).

Definition 3: [44] For any $ℵ = \left(ƀ_Ŧ(ɞ), Ч_Ŧ(ɞ), Ƀ_Ŧ(ɞ)\right)$ PFV, we can find the score function and accuracy function in the following way:

$$₵(ℵ) = \frac{1}{3}(2 + ƀ_Ŧ - Ч_Ŧ - Ƀ_Ŧ), ₵(ℵ) \in [0, 1]$$

Similarly, accuracy functions can also be found as:

$$\tilde{A}(ℵ) = \frac{1}{3}(2 + ƀ_Ŧ + Ч_Ŧ + Ƀ_Ŧ), \tilde{A}(ℵ) \in [0, 1]$$

Definition 4: For any two $ℵ_1 = \left(ƀ_{Ŧ_1}(ɞ), Ч_{Ŧ_1}(ɞ), Ƀ_{Ŧ_1}(ɞ)\right)$ and $ℵ_2 = \left(ƀ_{Ŧ_2}(ɞ), Ч_{Ŧ_2}(ɞ), Ƀ_{Ŧ_2}(ɞ)\right)$ PFVs. By using score values defined in Definition 3, we can compare which PFV is superior to the other in the following way:

$₵(ℵ_1) > ₵(ℵ_2)$ then, $ℵ_1 > ℵ_2$

$₵(ℵ_1) < ₵(ℵ_2)$ then, $ℵ_1 < ℵ_2$

If $₵(ℵ_1) = ₵(ℵ_2)$ then,

$\tilde{A}(ℵ_1) > \tilde{A}(ℵ_2)$ then, $ℵ_1 > ℵ_2$

$\tilde{A}(ℵ_1) < \tilde{A}(ℵ_2)$ then, $ℵ_1 < ℵ_2$

Definition 5: [45] Let $₡ = (₡_1, ₡_2, \ldots, ₡_ꞩ)$ be a class of attributes, prioritization among different attributes expressed by linear ordering $₡_1 > ₡_2 >, \ldots, ₡_n$ such that $₡_k$ has higher preferences $₡_j$ as $k < j$. The prioritized averaging (PA) operator is defined as follow:

$$PA(₡_1, ₡_2, \ldots, ₡_ꞩ) = \sum_{₱=1}^{ꞩ} ₲_ꞩ ℵ_{Ŧ_ꞩ}$$

Remember that $₲_₱ = \frac{\varepsilon_₱}{\sum_{₱=1}^{ꞩ} \varepsilon_₱}, \varepsilon_₱ = \prod_{k=1}^{₱-1} ₵(ℵ_k), ₱ = 2, 3, \ldots, ꞩ$. The initial values are $\varepsilon_1 = 1$ and $₵(ℵ_k)$ denotes score values of k^{th} PFVs. Then, PA is called the prioritized averaging (PA) operator.

3 | BASIC OPERATIONS OF SCHWEIZER-SKLAR AGGREGAITON TOOLS BASED ON PF INFORMAITON

Schweizer-Sklar operations contain Schweizer-Sklar product and sum, which are the special cases of algebraic t-norm and t-conorm, respectively. These norms are chosen to represent the intersection and union of two PFS.

Definition 6: [36] Let $A = (\text{ƀ}_A, \text{Ч}_A, \text{ƅ}_A)$ and $B = (\text{ƀ}_B, \text{Ч}_B, \text{ƅ}_B)$ be the two PFVs, then generalized intersection and union is defined as:

$$A \cap_{T,T^*} B = \left\{\left(\text{ϐ}, T\left(\text{ƀ}_A(\text{ϐ}), \text{ƀ}_B(\text{ϐ})\right), T^*\left(\text{Ч}_A(\text{ϐ}), \text{Ч}_B(\text{ϐ})\right), T^*\left(\text{ƅ}_A(\text{ϐ}), \text{ƅ}_B(\text{ϐ})\right)\right) | \text{ϐ} \in \text{Ꞧ}\right\}$$

$$A \cup_{T,T^*} B = \left\{\left(\text{ϐ}, T^*\left(\text{ƀ}_A(\text{ϐ}), \text{ƀ}_B(\text{ϐ})\right), T\left(\text{Ч}_A(\text{ϐ}), \text{Ч}_B(\text{ϐ})\right), T\left(\text{ƅ}_A(\text{ϐ}), \text{ƅ}_B(\text{ϐ})\right)\right) | \text{ϐ} \in \text{Ꞧ}\right\}$$

Where T and T^* denote the t-norm and t-conorm, respectively. However, Schweizer-Sklar t-norm and t-conorm (SSTT) are defined as:

$$T_{SS,\zeta}(x, y) = \left(x^{\zeta} + y^{\zeta} - 1\right)^{\frac{1}{\zeta}}$$

$$T^*{}_{SS,\zeta}(x, y) = 1 - \left((1 - x)^{\zeta} + (1 - y)^{\zeta} - 1\right)^{\frac{1}{\zeta}}$$

Where $\zeta < 0$ and $(x, y) \in [0, 1]$. Furthermore, when $\zeta = 0$ then, $T_{\zeta}(x, y) = xy$ and $T^*{}_{\zeta}(x, y) = x + y - xy$. These are algebraic t-norm and t-conorm (ATT), respectively.

Based on SST, we can write some SS operations for PFVs.

Definition 7: [46] Let $\aleph_1 = \left(\text{ƀ}_{\text{Ŧ}_1}(\text{ϐ}), \text{Ч}_{\text{Ŧ}_1}(\text{ϐ}), \text{ƅ}_{\text{Ŧ}_1}(\text{ϐ})\right)$ and $\aleph_2 = \left(\text{ƀ}_{\text{Ŧ}_2}(\text{ϐ}), \text{Ч}_{\text{Ŧ}_2}(\text{ϐ}), \text{ƅ}_{\text{Ŧ}_2}(\text{ϐ})\right)$ be the two PFVs. Based on these PFVs, we derive some fundamental Schweizer-Sklar (SS) operational laws, such as:

i. $$\aleph_1 \oplus \aleph_2 = \begin{pmatrix} 1 - \left(\left(1 - \text{ƀ}_{\text{Ŧ}_1}\right)^{\text{Ȼ}} + \left(1 - \text{ƀ}_{\text{Ŧ}_2}\right)^{\text{Ȼ}} - 1\right)^{\frac{1}{\text{Ȼ}}}, \\ \left(\text{Ч}_{\text{Ŧ}_1}^{\text{Ȼ}} + \text{Ч}_{\text{Ŧ}_2}^{\text{Ȼ}} - 1\right)^{\frac{1}{\text{Ȼ}}}, \left(\text{ƅ}_{\text{Ŧ}_1}^{\text{Ȼ}} + \text{ƅ}_{\text{Ŧ}_2}^{\text{Ȼ}} - 1\right)^{\frac{1}{\text{Ȼ}}} \end{pmatrix}$$

ii. $$\aleph_1 \otimes \aleph_2 = \begin{pmatrix} \left(\text{ƀ}_{\text{Ŧ}_1}^{\text{Ȼ}} + \text{ƀ}_{\text{Ŧ}_2}^{\text{Ȼ}} - 1\right)^{\frac{1}{\text{Ȼ}}}, \\ 1 - \left(\left(1 - \text{Ч}_{\text{Ŧ}_1}\right)^{\text{Ȼ}} + \left(1 - \text{Ч}_{\text{Ŧ}_2}\right)^{\text{Ȼ}} - 1\right)^{\frac{1}{\text{Ȼ}}}, \\ 1 - \left(\left(1 - \text{ƅ}_{\text{Ŧ}_1}\right)^{\text{Ȼ}} + \left(1 - \text{ƅ}_{\text{Ŧ}_2}\right)^{\text{Ȼ}} - 1\right)^{\frac{1}{\text{Ȼ}}} \end{pmatrix}$$

iii. $$\zeta\aleph_1 = \left(1 - \left(\zeta\left(1 - \text{ƀ}_{\text{Ŧ}_1}\right)^{\text{Ȼ}} - (\zeta - 1)\right)^{\frac{1}{\text{Ȼ}}}, \left(\zeta\text{Ч}_{\text{Ŧ}_1}^{\text{Ȼ}} - (\zeta - 1)\right)^{\frac{1}{\text{Ȼ}}}, \left(\zeta\text{ƅ}_{\text{Ŧ}_1}^{\text{Ȼ}} - (\zeta - 1)\right)^{\frac{1}{\text{Ȼ}}}\right)$$

iv. $$\aleph_{\text{Ŧ}_1}^{\zeta} = \left(\left(\zeta\text{ƀ}_{\text{Ŧ}_1}^{\zeta} - (\zeta - 1)\right)^{\frac{1}{\text{Ȼ}}}, 1 - \left(\zeta\left(1 - \text{Ч}_{\text{Ŧ}_1}\right)^{\text{Ȼ}} - (\zeta - 1)\right)^{\frac{1}{\text{Ȼ}}}, 1 - \left(\zeta\left(1 - \text{ƅ}_{\text{Ŧ}_1}\right)^{\text{Ȼ}} - (\zeta - 1)\right)^{\frac{1}{\text{Ȼ}}}\right)$$

4 | PICTURE FUZZY SCHWEIZER-SKLAR PRIORITIZED AGGREGATION OPERATORS

In this section, we developed a class of new approaches based on PF information such as PFSSPA and PFSSPWA operators, with some notable characteristics.

Definition 7: Let $\aleph_{\text{Ŧ}} = \left(\text{ƀ}_{\text{Ŧ}_{\text{Ᵽ}}}(\text{ϐ}), \text{Ч}_{\text{Ŧ}_{\text{Ᵽ}}}(\text{ϐ}), \text{ƅ}_{\text{Ŧ}_{\text{Ᵽ}}}(\text{ϐ})\right), \text{Ᵽ} = 1, 2, 3, \ldots, \text{ẞ}$ be the collection of PFVs. Then, PF Schweizer-Sklar Prioritized Averaging (PFSSPA) is defined as:

$$PFSSPA(\aleph_1,\aleph_2,\dots,\aleph_{ꞩ}) = Ꞓ_1\aleph_{₮_1} \oplus Ꞓ_2\aleph_{₮_2} \oplus \dots \oplus Ꞓ_{ꞩ}\aleph_{₮_{ꞩ}} = \underset{₱=1}{\overset{n}{\oplus}} Ꞓ_{₱}\aleph_{₮_{₱}}$$

Remember that $Ꞓ_{₱} = \frac{\varepsilon_{₱}}{\sum_{₱=1}^{ꞩ}\varepsilon_{₱}}, \varepsilon_{₱} = \prod_{ꝁ=1}^{₱-1} ₾(\aleph_{ꝁ}), ꝁ = 2,3,\dots,ꞩ$. The initial values are $\varepsilon_1 = 1$ and $₾(\aleph_{ꝁ})$ denotes score values of $ꝁ^{th}$ PFVs.

Theorem 1: Let $\aleph_{₮} = \left(ƀ_{₮_{₱}}(ϐ), Ч_{₮_{₱}}(ϐ), Ƀ_{₮_{₱}}(ϐ)\right), ₱ = 1,2,3,\dots,ꞩ$ be the collection of PFVs. Then, PFSSPA can be proved as follows:

$$PFSSPA(\aleph_1,\aleph_2,\dots,\aleph_{ꞩ}) = \begin{pmatrix} 1-\left(\sum_{₱=1}^{ꞩ} Ꞓ_{₱}\left(1-ƀ_{₮_{₱}}\right)^{Ȼ} - \sum_{₱=1}^{ꞩ} Ꞓ_{₱} + 1\right)^{\frac{1}{Ȼ}}, \\ \left(\sum_{₱=1}^{ꞩ} Ꞓ_{₱} Ч_{₮_{₱}}^{Ȼ} - \sum_{₱=1}^{ꞩ} Ꞓ_{₱} + 1\right)^{\frac{1}{Ȼ}}, \left(\sum_{₱=1}^{ꞩ} Ꞓ_{₱} Ƀ_{₮_{₱}}^{Ȼ} - \sum_{₱=1}^{ꞩ} Ꞓ_{₱} + 1\right)^{\frac{1}{Ȼ}} \end{pmatrix}$$

Proof: Let $\aleph_{₮} = \left(ƀ_{₮_{₱}}(ϐ), Ч_{₮_{₱}}(ϐ), Ƀ_{₮_{₱}}(ϐ)\right), ₱ = 1,2,3,\dots,ꞩ$ be the collection of PFVs. Then, PFSSPA can be proved using the technique of mathematical induction.

First, we suppose that the above equation is true for $₱ = 2$. Therefore,

$$Ꞓ_1\aleph_1 = \left(1-\left(Ꞓ_1\left(1-ƀ_{₮_1}\right)^{Ȼ} - (Ꞓ_1 - 1)\right)^{\frac{1}{Ȼ}}, \left(Ꞓ_1 Ч_{₮_1}^{Ȼ} - (Ꞓ_1 - 1)\right)^{\frac{1}{Ȼ}}, \left(Ꞓ_1 Ƀ_{₮_1}^{Ȼ} - (Ꞓ_1 - 1)\right)^{\frac{1}{Ȼ}}\right)$$

$$Ꞓ_2\aleph_2 = \left(1-\left(Ꞓ_2\left(1-ƀ_{₮_2}\right)^{Ȼ} - (Ꞓ_2 - 1)\right)^{\frac{1}{Ȼ}}, \left(Ꞓ_2 Ч_{₮_2}^{Ȼ} - (Ꞓ_2 - 1)\right)^{\frac{1}{Ȼ}}, \left(Ꞓ_2 Ƀ_{₮_2}^{Ȼ} - (Ꞓ_2 - 1)\right)^{\frac{1}{Ȼ}}\right)$$

$PFSSPA(\aleph_1,\aleph_2) = Ꞓ_2\aleph_2 \oplus Ꞓ_2\aleph_2$

$$= \begin{pmatrix} 1-\left(Ꞓ_1\left(1-ƀ_{₮_1}\right)^{Ȼ} - (Ꞓ_1 - 1)\right)^{\frac{1}{Ȼ}}, \\ \left(Ꞓ_1 Ч_{₮_1}^{Ȼ} - (Ꞓ_1 - 1)\right)^{\frac{1}{Ȼ}}, \\ \left(Ꞓ_1 Ƀ_{₮_1}^{Ȼ} - (Ꞓ_1 - 1)\right)^{\frac{1}{Ȼ}} \end{pmatrix} \oplus \begin{pmatrix} 1-\left(Ꞓ_2\left(1-ƀ_{₮_2}\right)^{Ȼ} - (Ꞓ_2 - 1)\right)^{\frac{1}{Ȼ}}, \\ \left(Ꞓ_2 Ч_{₮_2}^{Ȼ} - (Ꞓ_2 - 1)\right)^{\frac{1}{Ȼ}}, \\ \left(Ꞓ_2 Ƀ_{₮_2}^{Ȼ} - (Ꞓ_2 - 1)\right)^{\frac{1}{Ȼ}} \end{pmatrix}$$

$PFSSPA(\aleph_1,\aleph_2)$

$$= \begin{pmatrix} 1-\left(\left(1-\left(1-\left(Ꞓ_1\left(1-ƀ_{₮_1}\right)^{Ȼ} - (Ꞓ_1 - 1)\right)^{\frac{1}{Ȼ}}\right)\right)^{Ȼ} + \left(1-\left(1-\left(Ꞓ_2\left(1-ƀ_{₮_2}\right)^{Ȼ} - (Ꞓ_2 - 1)\right)^{\frac{1}{Ȼ}}\right)^{Ȼ} - 1\right)\right)^{\frac{1}{Ȼ}}, \\ \left(\left(\left(Ꞓ_1 Ч_{₮_1}^{Ȼ} - (Ꞓ_1 - 1)\right)^{\frac{1}{Ȼ}}\right)^{Ȼ} + \left(\left(Ꞓ_2 Ч_{₮_2}^{Ȼ} - (Ꞓ_2 - 1)\right)^{\frac{1}{Ȼ}}\right)^{Ȼ} - 1\right)^{\frac{1}{Ȼ}}, \\ \left(\left(\left(Ꞓ_1 Ƀ_{₮_1}^{Ȼ} - (Ꞓ_1 - 1)\right)^{\frac{1}{Ȼ}}\right)^{Ȼ} + \left(\left(Ꞓ_2 Ƀ_{₮_2}^{Ȼ} - (Ꞓ_2 - 1)\right)^{\frac{1}{Ȼ}}\right)^{Ȼ} - 1\right)^{\frac{1}{Ȼ}} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\left(\left(\left(1-1+\left(₲_1\left(1-ƀ_{Ŧ_1}\right)^{₡}-(₲_1-1)\right)^{\frac{1}{₡}}\right)^{₡}+\left(1-1+\left(₲_2\left(1-ƀ_{Ŧ_2}\right)^{₡}-(₲_2-1)\right)^{\frac{1}{₡}}\right)^{₡}-1\right)^{\frac{1}{₡}}\right), \\ \left(\left(₲_1Ч_{Ŧ_1}^{₡}-(₲_1-1)\right)+\left(₲_2Ч_{Ŧ_2}^{₡}-(₲_2-1)\right)-1\right)^{\frac{1}{₡}}, \\ \left(\left(₲_1Ƀ_{Ŧ_1}^{₡}-(₲_1-1)\right)+\left(₲_2Ƀ_{Ŧ_2}^{₡}-(₲_2-1)\right)-1\right)^{\frac{1}{₡}} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\left(\left(\left(\left(₲_1\left(1-ƀ_{Ŧ_1}\right)^{₡}-(₲_1-1)\right)^{\frac{1}{₡}}\right)^{₡}+\left(\left(₲_2\left(1-ƀ_{Ŧ_2}\right)^{₡}-(₲_2-1)\right)^{\frac{1}{₡}}\right)^{₡}-1\right)^{\frac{1}{₡}}\right), \\ \left(₲_1Ч_{Ŧ_1}^{₡}-₲_1+1+₲_2Ч_{Ŧ_2}^{₡}-₲_2+1-1\right)^{\frac{1}{₡}}, \left(₲_1Ƀ_{Ŧ_1}^{₡}-₲_1+1+Ƀ_2Ч_{Ŧ_2}^{₡}-₲_2+1-1\right)^{\frac{1}{₡}} \end{pmatrix}$$

$$PFSSPA(ℵ_1,ℵ_2)= \begin{pmatrix} 1-\left(\left(₲_1\left(1-ƀ_{Ŧ_1}\right)^{₡}+₲_2\left(1-ƀ_{Ŧ_2}\right)^{₡}-(₲_1-1)-(₲_2-1)-1\right)\right)^{\frac{1}{₡}}, \\ \left(₲_1Ч_{Ŧ_1}^{₡}+₲_2Ч_{Ŧ_2}^{₡}-₲_1-₲_2+1\right)^{\frac{1}{₡}}, \\ \left(₲_1Ƀ_{Ŧ_1}^{₡}+₲_2Ƀ_{Ŧ_2}^{₡}-₲_1-₲_2+1\right)^{\frac{1}{₡}} \end{pmatrix}$$

$$PFSSPA(ℵ_1,ℵ_2)= \begin{pmatrix} 1-\left(₲_1\left(1-ƀ_{Ŧ_1}\right)^{₡}+₲_2\left(1-ƀ_{Ŧ_2}\right)^{₡}-₲_1+1-₲_2+1-1\right)^{\frac{1}{₡}}, \\ \left(₲_1Ч_{Ŧ_1}^{₡}+₲_2Ч_{Ŧ_2}^{₡}-₲_1-₲_2+1\right)^{\frac{1}{₡}}, \\ \left(₲_1Ƀ_{Ŧ_1}^{₡}+₲_2Ƀ_{Ŧ_2}^{₡}-₲_1-₲_2+1\right)^{\frac{1}{₡}} \end{pmatrix}$$

$$PFSSPA(ℵ_1,ℵ_2)= \begin{pmatrix} 1-\left(₲_1\left(1-ƀ_{Ŧ_1}\right)^{₡}+₲_2\left(1-ƀ_{Ŧ_2}\right)^{₡}-₲_1-₲_2+1\right)^{\frac{1}{₡}}, \\ \left(₲_1Ч_{Ŧ_1}^{₡}+₲_2Ч_{Ŧ_2}^{₡}-₲_1-₲_2+1\right)^{\frac{1}{₡}}, \\ \left(₲_1Ƀ_{Ŧ_1}^{₡}+₲_2Ƀ_{Ŧ_2}^{₡}-₲_1-₲_2+1\right)^{\frac{1}{₡}} \end{pmatrix}$$

$$PFSSPA(ℵ_1,ℵ_2)= \begin{pmatrix} 1-\left(\sum_{₱=1}^{2}₲_{₱}\left(1-ƀ_{Ŧ_{₱}}\right)^{₡}-\sum_{₱=1}^{2}₲_{₱}+1\right)^{\frac{1}{₡}}, \\ \left(\sum_{₱=1}^{2}₲_{₱}Ч_{Ŧ_{₱}}^{₡}-\sum_{₱=1}^{2}₲_{₱}+1\right)^{\frac{1}{₡}}, \\ \left(\sum_{₱=1}^{2}₲_{₱}Ƀ_{Ŧ_{₱}}^{₡}-\sum_{₱=1}^{2}₲_{₱}+1\right)^{\frac{1}{₡}} \end{pmatrix}$$

For $₱ = 2$, we proved that it is true. Further, we suppose that it holds for $₱ = ķ$.

$$PFSSPA(ℵ_1, ℵ_2, \dots, ℵ_{ķ}) = \begin{pmatrix} 1 - \left(\sum_{₱=1}^{ķ} ₢_{₱} \left(1 - ѣ_{Ŧ_{₱}}\right)^{₡} - \sum_{₱=1}^{ķ} ₢_{₱} + 1 \right)^{\frac{1}{₡}}, \\ \left(\sum_{₱=1}^{ķ} ₢_{₱} Ч_{Ŧ_{₱}}^{₡} - \sum_{₱=1}^{ķ} ₢_{₱} + 1 \right)^{\frac{1}{₡}}, \\ \left(\sum_{₱=1}^{ķ} ₢_{₱} Ъ_{Ŧ_{₱}}^{₡} - \sum_{₱=1}^{ķ} ₢_{₱} + 1 \right)^{\frac{1}{₡}} \end{pmatrix}$$

Now, we are to prove that it is also true for $₱ = ķ + 1$, such that:

$$PFSSPA(ℵ_1, ℵ_2, \dots, ℵ_{ķ+1}) = ₢_1 ℵ_{Ŧ_1} \oplus ₢_2 ℵ_{Ŧ_2} \oplus \dots \oplus ₢_{ķ} ℵ_{Ŧ_{ķ}} \oplus ₢_{ķ+1} ℵ_{Ŧ_{ķ+1}} = \bigoplus_{₱=1}^{ķ} ₢_{ķ} ℵ_{Ŧ_{ķ}} \oplus ₢_{ķ+1} ℵ_{Ŧ_{ķ+1}}$$

$$= \bigoplus_{₱=1}^{ķ} ₢_{₱} ℵ_{Ŧ_{₱}} \oplus ₢_{ķ+1} ℵ_{Ŧ_{ķ+1}}$$

$$= \begin{pmatrix} 1 - \left(\sum_{₱=1}^{ķ} ₢_{₱} \left(1 - ѣ_{Ŧ_{₱}}\right)^{₡} - \sum_{₱=1}^{ķ} ₢_{₱} + 1 \right)^{\frac{1}{₡}}, \\ \left(\sum_{₱=1}^{ķ} ₢_{₱} Ч_{Ŧ_{₱}}^{₡} - \sum_{₱=1}^{ķ} ₢_{₱} + 1 \right)^{\frac{1}{₡}}, \\ \left(\sum_{₱=1}^{ķ} ₢_{₱} Ъ_{Ŧ_{₱}}^{₡} - \sum_{₱=1}^{ķ} ₢_{₱} + 1 \right)^{\frac{1}{₡}} \end{pmatrix} \oplus \begin{pmatrix} 1 - \left(₢_{ķ+1} \left(1 - ѣ_{Ŧ_{ķ+1}}\right)^{₡} - \left(₢_{ķ+1} + 1\right) \right)^{\frac{1}{₡}}, \\ \left(₢_{ķ+1} Ч_{Ŧ_{ķ+1}}^{₡} - \left(₢_{ķ+1} + 1\right) \right)^{\frac{1}{₡}}, \\ \left(₢_{ķ+1} Ъ_{Ŧ_{ķ+1}}^{₡} - \left(₢_{ķ+1} + 1\right) \right)^{\frac{1}{₡}} \end{pmatrix}$$

$$PFSSPA(ℵ_1, ℵ_2, \dots, ℵ_{ķ+1}) = \begin{pmatrix} 1 - \left(\sum_{₱=1}^{ķ+1} ₢_{₱} \left(1 - ѣ_{Ŧ_{₱}}\right)^{₡} - \sum_{₱=1}^{ķ+1} ₢_{₱} + 1 \right)^{\frac{1}{₡}}, \quad \left(\sum_{₱=1}^{ķ+1} ₢_{₱} Ч_{Ŧ_{₱}}^{₡} - \sum_{₱=1}^{ķ+1} ₢_{₱} + 1 \right)^{\frac{1}{₡}}, \\ \left(\sum_{₱=1}^{ķ+1} ₢_{₱} Ъ_{Ŧ_{₱}}^{₡} - \sum_{₱=1}^{ķ+1} ₢_{₱} + 1 \right)^{\frac{1}{₡}} \end{pmatrix}$$

Proposition 1: Let $ℵ_{Ŧ_{₱}} = \left(ѣ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, Ъ_{Ŧ_{₱}}\right), ₱ = 1, 2, 3, \dots, \mathcal{S}$ be the collection of any identical PFVs, that is $ℵ_{Ŧ_{₱}} = ℵ$. Then, we can say that:

$$PFSSPA(ℵ_1, ℵ_2, \dots, ℵ_{\mathcal{S}}) = ℵ$$

Proof: Since $ℵ_{Ŧ} = \left(ѣ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, Ъ_{Ŧ_{₱}}\right)$ be the collection of PFVs. So,

$$PFSSPA(ℵ_1, ℵ_2, \dots, ℵ_{\mathcal{S}}) = \begin{pmatrix} 1 - \left(\sum_{₱=1}^{\mathcal{S}} ₢_{₱} \left(1 - ѣ_{Ŧ_{₱}}\right)^{₡} - \sum_{₱=1}^{\mathcal{S}} ₢_{₱} + 1 \right)^{\frac{1}{₡}}, \\ \left(\sum_{₱=1}^{\mathcal{S}} ₢_{₱} Ч_{Ŧ_{₱}}^{₡} - \sum_{₱=1}^{\mathcal{S}} ₢_{₱} + 1 \right)^{\frac{1}{₡}}, \left(\sum_{₱=1}^{\mathcal{S}} ₢_{₱} Ъ_{Ŧ_{₱}}^{₡} - \sum_{₱=1}^{\mathcal{S}} ₢_{₱} + 1 \right)^{\frac{1}{₡}} \end{pmatrix}$$

$$=\left(\begin{array}{c} 1-\left(\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}(1-\text{ƀ}_{\text{₱}})^{\text{₡}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\right)^{\frac{1}{\text{₡}}}, \\ \left(\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\text{Ч}_{\text{₱}}^{\text{₡}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\right)^{\frac{1}{\text{₡}}}, \left(\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\text{ƃ}_{\text{₱}}^{\text{₡}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\right)^{\frac{1}{\text{₡}}} \end{array}\right)$$

$$PFSSPA(\aleph_1,\aleph_2,\dots,\aleph_{\text{₰}})=\left(1-\left((1-\text{ƀ}_{\text{₱}})^{\text{₡}}-1+1\right)^{\frac{1}{\text{₡}}},\left(\text{Ч}_{\text{₱}}^{\text{₡}}-1+1\right)^{\frac{1}{\text{₡}}},\left(\text{ƃ}_{\text{₱}}^{\text{₡}}-1+1\right)^{\frac{1}{\text{₡}}}\right) \text{ As } \textstyle\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}=1$$

$$PFSSPA(\aleph_1,\aleph_2,\dots,\aleph_{\text{₰}})=\left(1-\left((1-\text{ƀ}_{\text{₱}})^{\text{₡}}\right)^{\frac{1}{\text{₡}}},\left(\text{Ч}_{\text{₱}}^{\text{₡}}\right)^{\frac{1}{\text{₡}}},\left(\text{ƃ}_{\text{₱}}^{\text{₡}}\right)^{\frac{1}{\text{₡}}}\right)=\aleph_{\text{Ŧ}}$$

Hence, it is obvious that the property holds.

Proposition 2: Let $\aleph_{\text{₱}}=\left(\text{ƀ}_{\text{Ŧ}_{\text{₱}}},\text{Ч}_{\text{Ŧ}_{\text{₱}}},\text{ƃ}_{\text{Ŧ}_{\text{₱}}}\right)$ and $\aleph'_{\text{₱}}=\left(\text{ƀ}'_{\text{Ŧ}_{\text{₱}}},\text{Ч}'_{\text{Ŧ}_{\text{₱}}},\text{ƃ}'_{\text{Ŧ}_{\text{₱}}}\right)$ be the two sets of PFVs. Then, the property of boundedness is given as:

$$PFSSPA(\aleph_1,\aleph_2,\dots,\aleph_{\text{₰}})\le PFSSPA(\aleph'_1,\aleph'_2,\dots,\aleph'_{\text{₰}})$$

Proof: As $\aleph_{\text{Ŧ}}=\left(\text{ƀ}_{\text{Ŧ}_{\text{₱}}},\text{Ч}_{\text{Ŧ}_{\text{₱}}},\text{ƃ}_{\text{Ŧ}_{\text{₱}}}\right)$ and $\aleph'_{\text{Ŧ}}=\left(\text{ƀ}'_{\text{Ŧ}_{\text{₱}}},\text{Ч}'_{\text{Ŧ}_{\text{₱}}},\text{ƃ}'_{\text{Ŧ}_{\text{₱}}}\right)$ $\text{₱},\text{₱}'=1,2,3,\dots,\text{₰}$ be the collection of two PFVs. Then, we prove this inequality using Theorem 1. Assume that if $\aleph_{\text{Ŧ}}=\left(\text{ƀ}_{\text{Ŧ}_{\text{₱}}},\text{Ч}_{\text{Ŧ}_{\text{₱}}},\text{ƃ}_{\text{Ŧ}_{\text{₱}}}\right)\le\aleph'_{\text{Ŧ}}=\left(\text{ƀ}'_{\text{Ŧ}_{\text{₱}}},\text{Ч}'_{\text{Ŧ}_{\text{₱}}},\text{ƃ}'_{\text{Ŧ}_{\text{₱}}}\right)$ that is $\text{ƀ}_{\text{Ŧ}_{\text{₱}}}\le\text{ƀ}'_{\text{Ŧ}_{\text{₱}}}$, $\text{Ч}_{\text{Ŧ}_{\text{₱}}}\ge\text{Ч}'_{\text{Ŧ}_{\text{₱}}}$ and $\text{ƃ}_{\text{Ŧ}_{\text{₱}}}\ge\text{ƃ}'_{\text{Ŧ}_{\text{₱}}}$. Then, using the following steps

$$\text{ƀ}_{\text{Ŧ}_{\text{₱}}}\le\text{ƀ}'_{\text{Ŧ}_{\text{₱}}}\Rightarrow 1-\text{ƀ}_{\text{Ŧ}_{\text{₱}}}\ge 1-\text{ƀ}'_{\text{Ŧ}_{\text{₱}}}\Rightarrow\left(1-\text{ƀ}_{\text{Ŧ}_{\text{₱}}}\right)^{\text{₡}}\ge\left(1-\text{ƀ}'_{\text{Ŧ}_{\text{₱}}}\right)^{\text{₡}}\Rightarrow\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\left(1-\text{ƀ}_{\text{Ŧ}_{\text{₱}}}\right)^{\text{₡}}\ge\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\left(1-\text{ƀ}'_{\text{Ŧ}_{\text{₱}}}\right)^{\text{₡}}$$

$$\Rightarrow\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\left(1-\text{ƀ}_{\text{Ŧ}_{\text{₱}}}\right)^{\text{₡}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\ge\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\left(1-\text{ƀ}'_{\text{Ŧ}_{\text{₱}}}\right)^{\text{₡}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1$$

$$\Rightarrow\left(\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\left(1-\text{ƀ}_{\text{Ŧ}_{\text{₱}}}\right)^{\text{₡}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\right)^{\frac{1}{\text{₡}}}\ge\left(\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\left(1-\text{ƀ}'_{\text{Ŧ}_{\text{₱}}}\right)^{\text{₡}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\right)^{\frac{1}{\text{₡}}}$$

$$\Rightarrow 1-\left(\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\left(1-\text{ƀ}_{\text{Ŧ}_{\text{₱}}}\right)^{\text{₡}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\right)^{\frac{1}{\text{₡}}}\le 1-\left(\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\left(1-\text{ƀ}'_{\text{Ŧ}_{\text{₱}}}\right)^{\text{₡}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\right)^{\frac{1}{\text{₡}}}$$

Now, undertake that $\text{Ч}_{\text{Ŧ}_{\text{₱}}}\ge\text{Ч}'_{\text{Ŧ}_{\text{₱}}}$. Then,

$$\text{Ч}_{\text{Ŧ}_{\text{₱}}}\ge\text{Ч}'_{\text{Ŧ}_{\text{₱}}}\Rightarrow\text{Ч}_{\text{Ŧ}_1}\ge\text{Ч}'_{\text{Ŧ}_1}\ge\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\text{Ч}_{\text{₱}}^{\text{₡}}\ge\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\text{Ч}'^{\text{₡}}_{\text{₱}}\Rightarrow\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\text{Ч}_{\text{₱}}^{\text{₡}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\ge\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\text{Ч}'^{\text{₡}}_{\text{₱}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1$$

$$\Rightarrow\left(\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\text{Ч}_{\text{₱}}^{\text{₡}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\right)^{\frac{1}{\text{₡}}}\ge\left(\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\text{Ч}'^{\text{₡}}_{\text{₱}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\right)^{\frac{1}{\text{₡}}}$$

Similarly, in the same way:

$$\left(\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\text{ƃ}_{\text{₱}}^{\text{₡}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\right)^{\frac{1}{\text{₡}}}\ge\left(\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}\text{ƃ}'^{\text{₡}}_{\text{₱}}-\sum_{\text{₱}=1}^{\text{₰}}\text{₠}_{\text{₱}}+1\right)^{\frac{1}{\text{₡}}}$$

From the above discussion, it is proven that:

$$PFSSPA(ℵ_1, ℵ_2, \dots, ℵ_{₰}) \le PFSSPA(ℵ'_1, ℵ'_2, \dots, ℵ'_{₰})$$

Proposition 3: Let $ℵ_{Ŧ} = (ѣ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƀ_{Ŧ_{₱}}), ₱ = 1, 2, 3, \dots, ₰$ be the collection of PFVs. Let $ℵ_{Ŧ}^{-} = (mini ѣ_{Ŧ_{₱}}, maxi Ч_{Ŧ_{₱}}, maxi ƀ_{Ŧ_{₱}})$ and $ℵ_{Ŧ}^{+} = (maxi ѣ_{Ŧ_{₱}}, mini Ч_{Ŧ_{₱}}, mini ƀ_{Ŧ_{₱}})$. Then,

$$ℵ_{Ŧ}^{-} \le PFSSPA(ℵ_1, ℵ_2, \dots, ℵ_{₰}) \le ℵ_{Ŧ}^{+}$$

Proof: Let $PFSSPA(ℵ_1, ℵ_2, \dots, ℵ_{₰}) \le PFSSPA(ℵ_1^{+}, ℵ_2^{+}, \dots, ℵ_{₰}^{+}) = ℵ_{Ŧ}^{+}$ and $PFSSPA(ℵ_1, ℵ_2, \dots, ℵ_{₰}) \ge PFSSPA(ℵ_1^{-}, ℵ_2^{-}, \dots, ℵ_{₰}^{-}) = ℵ_{Ŧ}^{-}$. Then,

It is obvious that:

$ℵ_{Ŧ}^{-} \le PFSSPA(ℵ_1, ℵ_2, \dots, ℵ_{₰}) \le ℵ_{Ŧ}^{+}$ Holds.

Definition 8: Let $ℵ_{Ŧ} = (ѣ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƀ_{Ŧ_{₱}}), ₱ = 1, 2, 3, \dots, ₰$ be the collection of PFVs with corresponding weight vector $\psi = (\psi_1, \psi_2, \dots, \psi_{₰})$ and $\psi_{₱} \in [0, 1]$ such that $\sum_{₱=1}^{₰} \psi_{₱}$. The PFSSPWA operator is defined as follows:

$$PFSSPWA(ℵ_1, ℵ_2, \dots, ℵ_{₰}) = ₣_1 ℵ_{Ŧ_1} \oplus ₣_2 ℵ_{Ŧ_2} \oplus \dots \oplus ₣_{₰} ℵ_{Ŧ_{₰}} = \bigoplus_{₱=1}^{₰} ₣_{₱} ℵ_{Ŧ_{₱}}$$

Remember that $ℭ_{₱} = \frac{\psi_{₱}\varepsilon_{₱}}{\sum_{₱=1}^{₰} \psi_{₱}\varepsilon_{₱}}, \varepsilon_{₱} = \prod_{ķ=1}^{₱-1} ₵(ℵ_{ķ}), ķ = 2, 3, \dots, ₰$. The initial values are $\varepsilon_1 = 1$ and $₵(ℵ_{ķ})$ denotes score values of $ķ^{th}$ PFVs.

Theorem 2: Let $ℵ_{Ŧ} = \left(ѣ_{Ŧ_{₱}}(в), Ч_{Ŧ_{₱}}(в), ƀ_{Ŧ_{₱}}(в)\right), ₱ = 1, 2, 3, \dots, ₰$ be the collection of PFVs. Then, PFSSPWA is be proved as:

$$PFSSPWA(ℵ_1, ℵ_2, \dots, ℵ_{₰}) = \begin{pmatrix} 1 - \left(\sum_{₱=1}^{₰} ℭ_{₱}(1 - ѣ_{Ŧ_{₱}})^{Ȼ} - \sum_{₱=1}^{₰} ℭ_{₱} + 1\right)^{\frac{1}{Ȼ}}, \\ \left(\sum_{₱=1}^{₰} ℭ_{₱} Ч_{Ŧ_{₱}}^{Ȼ} - \sum_{₱=1}^{₰} ℭ_{₱} + 1\right)^{\frac{1}{Ȼ}}, \left(\sum_{₱=1}^{₰} ℭ_{₱} ƀ_{Ŧ_{₱}}^{Ȼ} - \sum_{₱=1}^{₰} ℭ_{₱} + 1\right)^{\frac{1}{Ȼ}} \end{pmatrix}$$

Proposition 4: Let $ℵ_{Ŧ_{₱}} = (ѣ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƀ_{Ŧ_{₱}}), ₱ = 1, 2, 3, \dots, ₰$ be the collection of any identical PFVs, that is $ℵ_{Ŧ_{₱}} = ℵ$. Then, we can say that:

$$PFSSPWA(ℵ_1, ℵ_2, \dots, ℵ_{₰}) = ℵ$$

Proposition 5: Let $ℵ_{₱} = (ѣ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƀ_{Ŧ_{₱}})$ and $ℵ'_{₱} = (ѣ'_{Ŧ_{₱}}, Ч'_{Ŧ_{₱}}, ƀ'_{Ŧ_{₱}})$ be the two sets of PFVs. Then, the property of boundedness is given as:

$$PFSSPWA(ℵ_1, ℵ_2, \dots, ℵ_{₰}) \le PFSSPWA(ℵ'_1, ℵ'_2, \dots, ℵ'_{₰})$$

Proposition 6: Let $ℵ_{Ŧ} = (ѣ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƀ_{Ŧ_{₱}}), ₱ = 1, 2, 3, \dots, ₰$ be the collection of PFVs. Let $ℵ_{Ŧ}^{-} = (min ѣ_{Ŧ_{₱}}, max Ч_{Ŧ_{₱}}, max ƀ_{Ŧ_{₱}})$ and $ℵ_{Ŧ}^{+} = (max ѣ_{Ŧ_{₱}}, min Ч_{Ŧ_{₱}}, min ƀ_{Ŧ_{₱}})$. Then,

$$ℵ_{Ŧ}^{-} \le PFSSPWA(ℵ_1, ℵ_2, \dots, ℵ_{₰}) \le ℵ_{Ŧ}^{+}$$

5 | PICTURE FUZZY SCHWEIZER-SKLAR PRIORITIZED GEOMETRIC AGGREGATION OPERATORS

In this section, we presented a series of new approaches with specific degrees of weights, like PFSSPG and PFSSPWG operators.

Definition 9: Let $ℵ_{Ŧ} = \left(ƀ_{Ŧ_{₱}}(б), Ч_{Ŧ_{₱}}(б), ƅ_{Ŧ_{₱}}(б)\right), ₱ = 1, 2, 3, \dots, ₰$ be the collection of PFVs. Then, PF Schweizer-Sklar prioritized geometric (PFSSPG) is defined as:

$$PFSSPG(ℵ_1, ℵ_2, \dots, ℵ_{₰}) = ℵ_{Ŧ_1}^{₲_1} \otimes ℵ_{Ŧ_2}^{₲_2} \otimes \dots \otimes ℵ_{Ŧ_{₰}}^{₲_{₰}} = \bigotimes_{₱=1}^{₰} ℵ_{Ŧ_{₱}}^{₲_{₱}}$$

Remember that $₲_{₱} = \frac{\varepsilon_{₱}}{\sum_{₱=1}^{₰} \varepsilon_{₱}}, \varepsilon_{₱} = \prod_{ķ=1}^{₱-1} ₵(ℵ_{ķ}), ķ = 2, 3, \dots, ₰$. The initial values are $\varepsilon_1 = 1$ and $₵(ℵ_{ķ})$ denotes score values of $ķ^{th}$ PFVs.

Theorem 3: Let $ℵ_{Ŧ} = (ƀ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƅ_{Ŧ_{₱}}), ₱ = 1, 2, 3, \dots, ₰$ be the collection of PFVs. Then, PFSSPA is proved as:

$$PFSSPG(ℵ_1, ℵ_2, \dots, ℵ_{₰}) = \begin{pmatrix} \left(\sum_{₱=1}^{₰} ₲_{₱} ƀ_{Ŧ_{₱}}^{Ȼ} - \sum_{₱=1}^{₰} ₲_{₱} + 1\right)^{\frac{1}{Ȼ}}, \\ 1 - \left(\sum_{₱=1}^{₰} ₲_{₱} \left(1 - Ч_{Ŧ_{₱}}\right)^{Ȼ} - \sum_{₱=1}^{₰} ₲_{₱} + 1\right)^{\frac{1}{Ȼ}}, \\ 1 - \left(\sum_{₱=1}^{₰} ₲_{₱} \left(1 - ƅ_{Ŧ_{₱}}\right)^{Ȼ} - \sum_{₱=1}^{₰} ₲_{₱} + 1\right)^{\frac{1}{Ȼ}} \end{pmatrix}$$

Proposition 7: Let $ℵ_{Ŧ_{₱}} = (ƀ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƅ_{Ŧ_{₱}}), ₱ = 1, 2, 3, \dots, ₰$ be the collection of any identical PFVs, that is $ℵ_{Ŧ_{₱}} = ℵ$. Then, we can say that:

$$PFSSPG(ℵ_1, ℵ_2, \dots, ℵ_{₰}) = ℵ$$

Proposition 8: Let $ℵ_{₱} = (ƀ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƅ_{Ŧ_{₱}})$ and $ℵ'_{₱} = (ƀ'_{Ŧ_{₱}}, Ч'_{Ŧ_{₱}}, ƅ'_{Ŧ_{₱}})$ be the two sets of PFVs. Then, the property of boundedness is given as:

$$PFSSPG(ℵ_1, ℵ_2, \dots, ℵ_{₰}) \leq PFSSPG(ℵ'_1, ℵ'_2, \dots, ℵ'_{₰})$$

Proposition 9: Let $ℵ_{Ŧ} = (ƀ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƅ_{Ŧ_{₱}}), ₱ = 1, 2, 3, \dots, ₰$ be the collection of PFVs. Let $ℵ_{Ŧ}^{-} = \left(\min_i ƀ_{Ŧ_{₱}}, \max_i Ч_{Ŧ_{₱}}, \max_i ƅ_{Ŧ_{₱}}\right)$ and $ℵ_{Ŧ}^{+} = \left(\max_i ƀ_{Ŧ_{₱}}, \min_i Ч_{Ŧ_{₱}}, \min_i ƅ_{Ŧ_{₱}}\right)$. Then,

$$ℵ_{Ŧ}^{-} \leq PFSSPG(ℵ_1, ℵ_2, \dots, ℵ_{₰}) \leq ℵ_{Ŧ}^{+}$$

Definition 11: Let $ℵ_{Ŧ} = (ƀ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƅ_{Ŧ_{₱}}), ₱ = 1, 2, 3, \dots, ₰$ be the collection of PFVs with corresponding weight vector $\psi = (\psi_1, \psi_2, \dots, \psi_{₰})$ and $\psi_{₱} \in [0, 1]$ such that $\sum_{₱=1}^{₰} \psi_{₱}$. Then, PF Schweizer-Sklar prioritized weighted geometric (PFSSPWG) can be defined as:

$$PFSSPWG(ℵ_1, ℵ_2, \dots, ℵ_{₰}) = ℵ_{Ŧ_1}^{ℭ_1} \otimes ℵ_{Ŧ_2}^{ℭ_1} \otimes \dots \otimes ℵ_{Ŧ_{₰}}^{ℭ_{₰}} = \bigotimes_{₱=1}^{₰} ℵ_{Ŧ_{₱}}^{ℭ_{₱}}$$

Remember that $ℭ_{₱} = \frac{\psi_{₱}\varepsilon_{₱}}{\sum_{₱=1}^{₰} \psi_{₱}\varepsilon_{₱}}, \varepsilon_{₱} = \prod_{ķ=1}^{₱-1} ₵(ℵ_{ķ}), ķ = 2, 3, \dots, ₰$. The initial values are $\varepsilon_1 = 1$ and $₵(ℵ_{ķ})$ denotes score values of $ķ^{th}$ PFVs.

Theorem 4: Let $ℵ_{Ŧ} = \left(ƀ_{Ŧ_{₱}}(ɞ), Ч_{Ŧ_{₱}}(ɞ), ƃ_{Ŧ_{₱}}(ɞ)\right), ₱ = 1, 2, 3, \dots, ₰$ be the collection of PFVs. Then, PFSSPWG is be proved as:

$$PFSSPWG(ℵ_1, ℵ_2, \dots, ℵ_{₰}) = \begin{pmatrix} \left(\sum_{₱=1}^{₰} ℭ_{₱} ƀ_{Ŧ_{₱}}^{¢} - \sum_{₱=1}^{₰} ℭ_{₱} + 1\right)^{\frac{1}{¢}}, \\ 1 - \left(\sum_{₱=1}^{₰} ℭ_{₱} \left(1 - Ч_{Ŧ_{₱}}\right)^{¢} - \sum_{₱=1}^{₰} ℭ_{₱} + 1\right)^{\frac{1}{¢}}, \\ 1 - \left(\sum_{₱=1}^{₰} ℭ_{₱} \left(1 - ƃ_{Ŧ_{₱}}\right)^{¢} - \sum_{₱=1}^{₰} ℭ_{₱} + 1\right)^{\frac{1}{¢}} \end{pmatrix}$$

Proposition 10: Let $ℵ_{Ŧ_{₱}} = \left(ƀ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƃ_{Ŧ_{₱}}\right), ₱ = 1, 2, 3, \dots, ₰$ be the collection of any identical PFVs, that is $ℵ_{Ŧ_{₱}} = ℵ$. Then, we can say that:

$$PFSSPWG(ℵ_1, ℵ_2, \dots, ℵ_{₰}) = ℵ$$

Proposition 11: Let $ℵ_{₱} = \left(ƀ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƃ_{Ŧ_{₱}}\right)$ and $ℵ'_{₱} = \left(ƀ'_{Ŧ_{₱}}, Ч'_{Ŧ_{₱}}, ƃ'_{Ŧ_{₱}}\right)$ be the two sets of PFVs. Then, the property of boundedness is given as:

$$PFSSPWG(ℵ_1, ℵ_2, \dots, ℵ_{₰}) \le PFSSPWG(ℵ'_1, ℵ'_2, \dots, ℵ'_{₰})$$

Proposition 12: Let $ℵ_{Ŧ} = \left(ƀ_{Ŧ_{₱}}, Ч_{Ŧ_{₱}}, ƃ_{Ŧ_{₱}}\right), ₱ = 1, 2, 3, \dots, ₰$ be the collection of PFVs. Let $ℵ_{Ŧ}^{-} = \left(min\, ƀ_{Ŧ_{₱}}, max\, Ч_{Ŧ_{₱}}, max\, ƃ_{Ŧ_{₱}}\right)$ and $ℵ_{Ŧ}^{+} = \left(max\, ƀ_{Ŧ_{₱}}, min\, Ч_{Ŧ_{₱}}, min\, ƃ_{Ŧ_{₱}}\right)$. Then,

$$ℵ_{Ŧ}^{-} \le PFSSPWG(ℵ_1, ℵ_2, \dots, ℵ_{₰}) \le ℵ_{Ŧ}^{+}$$

6 | ASSESSMENT OF MADM PROBLEM BASED ON PICTURE FUZZY INFORMATION

MADM is a decision-making approach that evaluates and compares multiple choices based on a set of criteria or attributes. It is often used to help decision-making processes in fields such as business, engineering, and project management. Each alternative in MADM is evaluated and graded based on its performance across multiple parameters. The characteristics might be quantitative (cost, reliability, and size) or qualitative (quality, dependability, and user happiness). The purpose is to choose the best option or to rank the alternatives in order of preference based on the relative value of the features. In this study, we develop an algorithm for the MADM problem under consideration of PF information. Assume $ɮ = (ɮ_{\mathbf{1}}, ɮ_{\mathbf{1}}, \dots, ɮ_n)$ shows the class of alternatives and a set of attributes $Ћ = (Ћ_{\mathbf{1}}, Ћ_{\mathbf{1}}, \dots, Ћ_{ṅ})$ with their corresponding weight vectors $\psi = (\psi_1, \psi_2, \dots, \psi_{ṅ})$ and $\psi_{₱} \in [0, 1]$ such that $\sum_{₱=1}^{₰} \psi_{₱}$. The decision maker constructs a decision matrix to cover PF information. A decision matrix $Ȓ = \left(Y_{\eta Ᵽ}\right)_{n \times ṅ}$ is organized as follows:

$$Ȓ = \left(Y_{\eta Ᵽ}\right)_{n \times ṅ} = \begin{pmatrix} (ƀ_{11}, Ч_{11}, ƃ_{11}) & (ƀ_{12}, Ч_{12}, ƃ_{12}) & \cdots & (ƀ_{1ṅ}, Ч_{1ṅ}, ƃ_{1ṅ}) \\ (ƀ_{21}, Ч_{21}, ƃ_{21}) & (ƀ_{22}, Ч_{22}, ƃ_{22}) & \cdots & (ƀ_{2ṅ}, Ч_{2ṅ}, ƃ_{2ṅ}) \\ \vdots & \vdots & \ddots & \vdots \\ (ƀ_{n1}, Ч_{n1}, ƃ_{n1}) & (ƀ_{n2}, Ч_{n2}, ƃ_{n2}) & \vdots & (ƀ_{nṅ}, Ч_{nṅ}, ƃ_{nṅ}) \end{pmatrix}$$

In this decision matrix, $(ᵬ_{\eta ℘}, Ч_{\eta ℘}, Ƀ_{\eta ℘})$ denotes the value of PFV, $ᵬ_{\eta ℘} \in [0,1]$, $Ч_{\eta ℘} \in [0,1]$ and $Ƀ_{\eta ℘} \in [0,1]$ such that $0 \leq ᵬ_{\eta ℘} + Ч_{\eta ℘} + Ƀ_{\eta ℘} \leq 1$. The assessment of the given information under considering the following algorithm of the MAMD problem.

6.1| Algorithm

The following steps of the MADM problem.

Step 1: Construct a decision matrix to cover information about any object in the form of PFVs.

Step 2: Transform the standard decision matrix into a normalized decision matrix if the given information has two types of information, beneficial and non-beneficial. All attributes should be the same type before the aggregation process by using the following expressions.

$$Ɽ = (Y_{\eta ℘})_{n \times ṅ} = \begin{cases} (ᵬ_{\eta ℘}, Ч_{\eta ℘}, Ƀ_{\eta ℘}) & \text{if beneficial attribute} \\ (ᵬ_{\eta ℘}, ᵬ_{\eta ℘}, Ƀ_{\eta ℘}) & \text{if non} - \text{beneficial attribute} \end{cases}$$

If all attributes are the same type, there is no need for this step.

Step 3: By using our invented approaches of the PFSSPA, PFSSPG, PFSSPWA, and PFSSPWG operators.

Step 4: Investigate the score values of all individuals by using definition 5.

Step 5: To choose an appropriate optimal option, rearrange all computed score values obtained by the proposed methodologies.

Step 5: End.

6.2| Experimental Case Study

In this experimental case, a multinational company wants to hire a general manager. For this purpose, several applicants applied again for this post. After initial scrutiny, five different applicants are short-listed. The decision maker evaluates the profile of these candidates under the following characteristics:

$Ћ_1$: academic and professional qualification

$Ћ_2$: Experience

$Ћ_3$: communication skills

$Ћ_4$: Personality and quality of the leadership

Some particular degrees assigned to the above-discussed characteristics $(0.25, 0.35, 0.15, 025)$. Decision makers aggregate given information related to shortlisted applicants based on invented methodologies. To find an appropriate applicant, the decision maker aggregates the given information by using the steps of the discussed algorithm of the MADM problem.

6.3| Procedure of the Decision-Making Process

In this section, the author aggregates given information by using proposed aggregation methodologies like PFSSPA, PFSSPG, PFSSPWA, and PFSSPWG operators.

Step 1: Firstly, the decision maker acquired information about all applicants in the form of PF environments and listed them in Table 1.

Table 1. Shows the PF decision matrix information.

Alternative/Attribute	Ћ_1	Ћ_2	Ћ_3	Ћ_4
ƺ_1	$(0.35, 0.22, 0.33)$	$(0.23, 0.45, 0.31)$	$(0.41, 0.10, 0.44)$	$(0.32, 0.25, 0.39)$
ƺ_2	$(0.42, 0.12, 0.29)$	$(0.34, 0.12, 0.4)$	$(0.5, 0.22, 0.1)$	$(0.19, 0.31, 0.43)$
ƺ_3	$(0.23, 0.39, 0.22)$	$(0.28, 0.33, 0.35)$	$(0.5, 0.27, 0.2)$	$(0.35, 0.25, 0.35)$
ƺ_4	$(0.4, 0.11, 0.35)$	$(0.11, 0.22, 0.33)$	$(0.37, 0.19, 0.42)$	$(0.41, 0.17, 0.33)$
ƺ_5	$(0.42, 0.24, 0.31)$	$(0.22, 0.3, 0.4)$	$(0.22, 0.33, 0.42)$	$(0.27, 0.33, 0.31)$

Step 2: Given information about all applicants, only one type of attribute, such as beneficial attribute, so there is no need to transform the standard decision matrix into a normalized decision matrix.

Step 3: For the evaluation of the given information by the PFSSPA and PFSSPG operators, all required results are listed in Table 2. After aggregating information, findings are drawn in Table 4. Similarly, all required results for the PFSSPWA and PFSSPWG operators are presented in Table 3. Table 4 also covered outcomes computed by the PFSSPWG operator.

Table 2. Shows the values of prioritization.

Alternative	Score Values	ε_∂	$\sum_{\partial=1}^{\text{ꞩ}} \varepsilon_\partial$	₠_∂
ƺ_1	$\text{Ȼ}_1 = 0.6000, \text{Ȼ}_2 = 0.4900, \text{Ȼ}_3 = 0.6233, \text{Ȼ}_4 = 0.5600$	$\varepsilon_1 = 1, \varepsilon_2 = 0.6000, \varepsilon_3 = 0.2940, \varepsilon_4 = 0.1833$	2.0773	$\bar{\text{₠}}_1 = 0.4814, \bar{\text{₠}}_2 = 0.2888, \bar{\text{₠}}_3 = 0.1415, \bar{\text{₠}}_4 = 0.0882$
ƺ_2	$\text{Ȼ}_1 = 0.6700, \text{Ȼ}_2 = 0.6067, \text{Ȼ}_3 = 0.7267, \text{Ȼ}_4 = 0.4833$	$\varepsilon_1 = 1, \varepsilon_2 = 0.6700, \varepsilon_3 = 0.4065, \varepsilon_4 = 0.2954$	2.3718	$\bar{\text{₠}}_1 = 0.4216, \bar{\text{₠}}_2 = 0.2825, \bar{\text{₠}}_3 = 0.1714, \bar{\text{₠}}_4 = 0.1245$
ƺ_3	$\text{Ȼ}_1 = 0.5400, \text{Ȼ}_2 = 0.5333, \text{Ȼ}_3 = 0.6767, \text{Ȼ}_4 = 0.5833$	$\varepsilon_1 = 1, \varepsilon_2 = 0.5400, \varepsilon_3 = 0.2880, \text{Ȼ}_4 = 0.1949$	2.0229	$\bar{\text{₠}}_1 = 0.4943, \bar{\text{₠}}_2 = 0.2669, \bar{\text{₠}}_3 = 0.1424, \bar{\text{₠}}_4 = 0.0963$
ƺ_4	$\text{Ȼ}_1 = 0.6467, \text{Ȼ}_2 = 0.5200, \text{Ȼ}_3 = 0.5867, \text{Ȼ}_4 = 0.6367$	$\varepsilon_1 = 1, \varepsilon_2 = 0.6467, \varepsilon_3 = 0.3363, \varepsilon_4 = 0.1973$	2.1802	$\bar{\text{₠}}_1 = 0.4587, \bar{\text{₠}}_2 = 0.2966, \bar{\text{₠}}_3 = 0.1542, \bar{\text{₠}}_4 = 0.0905$
ƺ_5	$\text{Ȼ}_1 = 0.6233, \text{Ȼ}_2 = 0.5067, \text{Ȼ}_3 = 0.4900, \text{Ȼ}_4 = 0.5433$	$\varepsilon_1 = 1, \varepsilon_2 = 0.6233, \varepsilon_3 = 0.3158, \varepsilon_4 = 0.1548$	2.0939	$\bar{\text{₠}}_1 = 0.4776, \bar{\text{₠}}_2 = 0.2977, \bar{\text{₠}}_3 = 0.1508, \bar{\text{₠}}_4 = 0.0739$

Table 3. Depicts the values of weighted prioritization.

Alternative	Score Values	ε_∂	$\psi_\partial \varepsilon_\partial$	$\sum_{₱=1}^{\text{ꞩ}} \psi_₱ \varepsilon_₱$	$\mathfrak{C}_₱$
ƺ_1	$\text{Ȼ}_1 = 0.6000, \text{Ȼ}_2 = 0.4900, \text{Ȼ}_3 = 0.6233, \text{Ȼ}_4 = 0.5600$	$\varepsilon_1 = 1, \varepsilon_2 = 0.6000, \varepsilon_3 = 0.2940, \varepsilon_4 = 0.1833$	$\psi_1\varepsilon_1 = 0.2500, \psi_2\varepsilon_2 = 0.2100, \psi_3\varepsilon_3 = 0.0441, \psi_4\varepsilon_4 = 0.0458$	0.5499	$\mathfrak{C}_1 = 0.4546, \mathfrak{C}_2 = 0.3819, \mathfrak{C}_3 = 0.0802, \mathfrak{C}_4 = 0.0833$
ƺ_2	$\text{Ȼ}_1 = 0.6700, \text{Ȼ}_2 = 0.6067, \text{Ȼ}_3 = 0.7267, \text{Ȼ}_4 = 0.4833$	$\varepsilon_1 = 1, \varepsilon_2 = 0.6700, \varepsilon_3 = 0.4065, \varepsilon_4 = 0.2954$	$\psi_1\varepsilon_1 = 0.2500, \psi_2\varepsilon_2 = 0.2345, \psi_3\varepsilon_3 = 0.0610, \psi_4\varepsilon_4 = 0.0738$	0.6193	$\mathfrak{C}_1 = 0.4037, \mathfrak{C}_2 = 0.3786, \mathfrak{C}_3 = 0.0984, \mathfrak{C}_4 = 0.1192$

ɮ$_3$	$\mathfrak{C}_1 = 0.5400, \mathfrak{C}_2 = 0.5333, \mathfrak{C}_3 = 0.6767, \mathfrak{C}_4 = 0.5833$	$\varepsilon_1 = 1, \varepsilon_2 = 0.5400, \varepsilon_3 = 0.2880, \varepsilon_4 = 0.1949$	$\psi_1\varepsilon_1 = 0.2500, \psi_2\varepsilon_2 = 0.1890, \psi_3\varepsilon_3 = 0.0432, \psi_4\varepsilon_4 = 0.0487$	0.5309	$\mathfrak{C}_1 = 0.4709, \mathfrak{C}_2 = 0.3560, \mathfrak{C}_3 = 0.0814, \mathfrak{C}_4 = 0.0918$
ɮ$_4$	$\mathfrak{C}_1 = 0.6467, \mathfrak{C}_2 = 0.5200, \mathfrak{C}_3 = 0.5867, \mathfrak{C}_4 = 0.6367$	$\varepsilon_1 = 1, \varepsilon_2 = 0.6467, \varepsilon_3 = 0.3363, \varepsilon_4 = 0.1973$	$\psi_1\varepsilon_1 = 0.2500, \psi_2\varepsilon_2 = 0.2263, \psi_3\varepsilon_3 = 0.0504, \psi_4\varepsilon_4 = 0.0493$	0.5761	$\mathfrak{C}_1 = 0.4340, \mathfrak{C}_2 = 0.3929, \mathfrak{C}_3 = 0.0876, \mathfrak{C}_4 = 0.0856$
ɮ$_5$	$\mathfrak{C}_1 = 0.6233, \mathfrak{C}_2 = 0.5067, \mathfrak{C}_3 = 0.4900, \mathfrak{C}_4 = 0.5433$	$\varepsilon_1 = 1, \varepsilon_2 = 0.6233, \varepsilon_3 = 0.3158, \varepsilon_4 = 0.1548$	$\psi_1\varepsilon_1 = 0.2500, \psi_2\varepsilon_2 = 0.2182, \psi_3\varepsilon_3 = 0.0474, \psi_4\varepsilon_4 = 0.0387$	0.5542	$\bar{\mathrm{F}}_1 = 0.4511, \bar{\mathrm{F}}_2 = 0.3936, \bar{\mathrm{F}}_3 = 0.0855, \bar{\mathrm{F}}_4 = 0.0698$

Step 4: Compute score values of all individuals by using Definition 3, all computed results by the PFSSPA, PFSSPG, PFSSPWA, and PFSSPWG operators. All acquired results are listed in Table 5.

Table 4. Shows the aggregated values of Schweizer-Sklar prioritized AOs

	PFSSPA	PFSSPG	PFSSPWA	PFSSPWG
ɮ$_1$	$(0.3268, 0.2175, 0.3403)$	$(0.3075, 0.2944, 0.3483)$	$(0.3121, 0.2469, 0.3327)$	$(0.2928, 0.3231, 0.3385)$
ɮ$_2$	$(0.3945, 0.1419, 0.2402)$	$(0.3530, 0.1669, 0.3214)$	$(0.3803, 0.1360, 0.2778)$	$(0.3449, 0.1583, 0.3413)$
ɮ$_3$	$(0.3083, 0.3345, 0.2502)$	$(0.2730, 0.3474, 0.2704)$	$(0.2907, 0.3385, 0.2613)$	$(0.2671, 0.3494, 0.2828)$
ɮ$_4$	$(0.3315, 0.1457, 0.3508)$	$(0.2232, 0.1632, 0.3546)$	$(0.3098, 0.1494, 0.3450)$	$(0.1960, 0.1684, 0.3476)$
ɮ$_5$	$(0.3332, 0.2730, 0.3469)$	$(0.2899, 0.2801, 0.3571)$	$(0.3278, 0.2731, 0.3487)$	$(0.2849, 0.2793, 0.3583)$

Table 5. Depicts all investigated score values by our invented approaches.

Alternative/Score values	PFSSPA	PFSSPG	PFSSPWA	PFSSPWG
ɮ$_1$	0.5897	0.5549	0.5775	0.5437
ɮ$_2$	0.6708	0.6216	0.6555	0.6151
ɮ$_3$	0.5745	0.5517	0.5636	0.5450
ɮ$_4$	0.6117	0.5685	0.6051	0.5600
ɮ$_5$	0.5711	0.5509	0.5687	0.5491

Step 5: Finally, we rank all the alternatives, determine the best alternative from the above-aggregated values, and display it in Table 6.

Table 6. Displays the ranking and ordering of AOs.

Aggregation operators	Ranking and ordering
PFSSPA	ɮ$_2$ > ɮ$_4$ > ɮ$_1$ > ɮ$_3$ > ɮ$_5$
PFSSPG	ɮ$_2$ > ɮ$_4$ > ɮ$_1$ > ɮ$_3$ > ɮ$_5$
PFSSPWA	ɮ$_2$ > ɮ$_4$ > ɮ$_1$ > ɮ$_5$ > ɮ$_3$
PFSSPWG	ɮ$_2$ > ɮ$_4$ > ɮ$_5$ > ɮ$_3$ > ɮ$_1$

Figure 1 shows the graphical structure of computed score values which are listed in Table 6. We see a red color lead from all presented colors. So, the second alternative or applicants are more eligible for the required post.

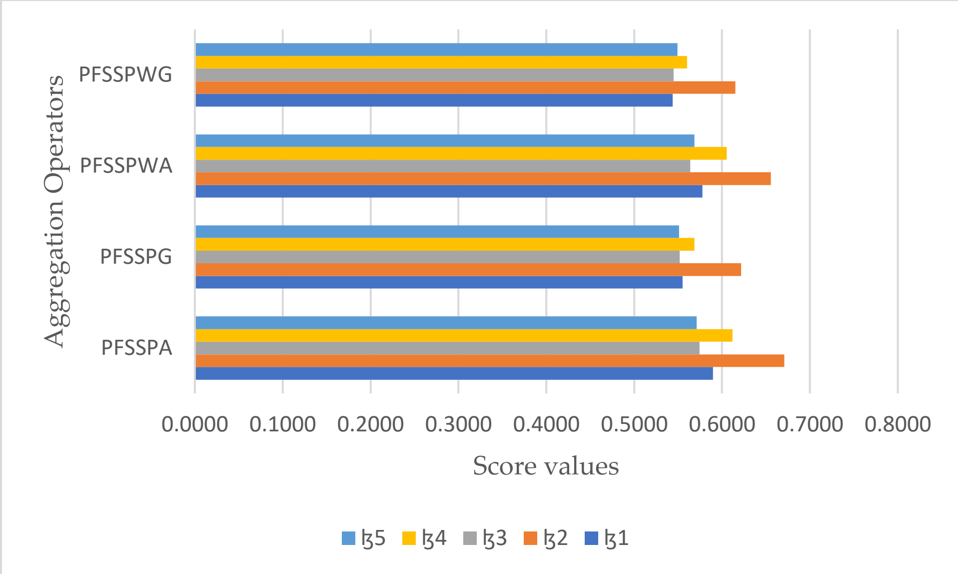

Figure 1. Displays the graphical representation of score values listed in Table 6.

6.3| Influence Study

This section shows the effect of different parametric values of the Schweizer-Sklar aggregation tools on the consequences of proposed aggregation approaches. By setting different parametric values $Ȼ$ of the Schweizer-Sklar aggregation tools, we gained different results of the score values by the PFSSPWA and PFSSPWG operators. Table 7 and Table 8, respectively. From Table 7, the ranking of score values $ƺ_2 > ƺ_3 > ƺ_1 > ƺ_4 > ƺ_5$ remain the same by the PFSSPWA operator for all parametric values of the Schweizer-Sklar aggregation tools. In Table 8, it is demonstrated that the best choice $ƺ_2$ remains unaltered when the parametric value $Ȼ$ lies between -1 and -35. After that, a slight variation occurs in the best choice as $ƺ_5$ becomes the best preference. From onward, ranking shows that $ƺ_3$ is the best alternative table 8 also covered the raking of score values $ƺ_3 > ƺ_5 > ƺ_4 > ƺ_2 > ƺ_1$ obtained by the PFSSPWG operators. These characteristics show the reliability and applicability of the proposed aggregation approaches.

Table 7. Show the effect of parametric value $Ȼ$ on PFSSPWA

Parametric values	Score values	Ranking and ordering
$Ȼ = -1$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_1) > \mathfrak{C}(ƺ_5) > \mathfrak{C}(ƺ_3)$	$ƺ_2 > ƺ_4 > ƺ_1 > ƺ_5 > ƺ_3$
$Ȼ = -15$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_1) > \mathfrak{C}(ƺ_5) > \mathfrak{C}(ƺ_3)$	$ƺ_2 > ƺ_4 > ƺ_1 > ƺ_5 > ƺ_3$
$Ȼ = -35$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5)$	$ƺ_2 > ƺ_3 > ƺ_1 > ƺ_4 > ƺ_5$
$Ȼ = -55$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5)$	$ƺ_2 > ƺ_3 > ƺ_1 > ƺ_4 > ƺ_5$
$Ȼ = -75$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5)$	$ƺ_2 > ƺ_3 > ƺ_1 > ƺ_4 > ƺ_5$
$Ȼ = -100$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5)$	$ƺ_2 > ƺ_3 > ƺ_1 > ƺ_4 > ƺ_5$
$Ȼ = -135$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5)$	$ƺ_2 > ƺ_3 > ƺ_1 > ƺ_4 > ƺ_5$

$₵ = -155$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5)$	$ƺ_2 > ƺ_3 > ƺ_1 > ƺ_4 > ƺ_5$
$₵ = -175$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5)$	$ƺ_2 > ƺ_3 > ƺ_1 > ƺ_4 > ƺ_5$
$₵ = -185$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5)$	$ƺ_2 > ƺ_3 > ƺ_1 > ƺ_4 > ƺ_5$
$₵ = -200$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5)$	$ƺ_2 > ƺ_3 > ƺ_1 > ƺ_4 > ƺ_5$

Table 8. Show the effect of parametric value $₵$ on PFSSPWG

Parametric values	Score values	Ranking and ordering
$₵ = -1$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1)$	$ƺ_2 > ƺ_4 > ƺ_5 > ƺ_3 > ƺ_1$
$₵ = -15$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1)$	$ƺ_2 > ƺ_4 > ƺ_5 > ƺ_3 > ƺ_1$
$₵ = -35$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1)$	$ƺ_2 > ƺ_4 > ƺ_5 > ƺ_3 > ƺ_1$
$₵ = -55$	$\mathfrak{C}(ƺ_5) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_1)$	$ƺ_5 > ƺ_3 > ƺ_4 > ƺ_2 > ƺ_1$
$₵ = -75$	$\mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_5) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_1)$	$ƺ_3 > ƺ_5 > ƺ_4 > ƺ_2 > ƺ_1$
$₵ = -100$	$\mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_5) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_1)$	$ƺ_3 > ƺ_5 > ƺ_4 > ƺ_2 > ƺ_1$
$₵ = -135$	$\mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_5) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_1)$	$ƺ_3 > ƺ_5 > ƺ_4 > ƺ_2 > ƺ_1$
$₵ = -155$	$\mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_5) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_1)$	$ƺ_3 > ƺ_5 > ƺ_4 > ƺ_2 > ƺ_1$
$₵ = -175$	$\mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_5) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_1)$	$ƺ_3 > ƺ_5 > ƺ_4 > ƺ_2 > ƺ_1$
$₵ = -185$	$\mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_5) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_1)$	$ƺ_3 > ƺ_5 > ƺ_4 > ƺ_2 > ƺ_1$
$₵ = -200$	$\mathfrak{C}(ƺ_2) > \mathfrak{C}(ƺ_3) > \mathfrak{C}(ƺ_1) > \mathfrak{C}(ƺ_4) > \mathfrak{C}(ƺ_5)$	$ƺ_3 > ƺ_5 > ƺ_4 > ƺ_2 > ƺ_1$

Figure 2 and Figure 3 portray the graphical behavior of obtained score values by the PFSSPWA and PFSSPWG operators, respectively.

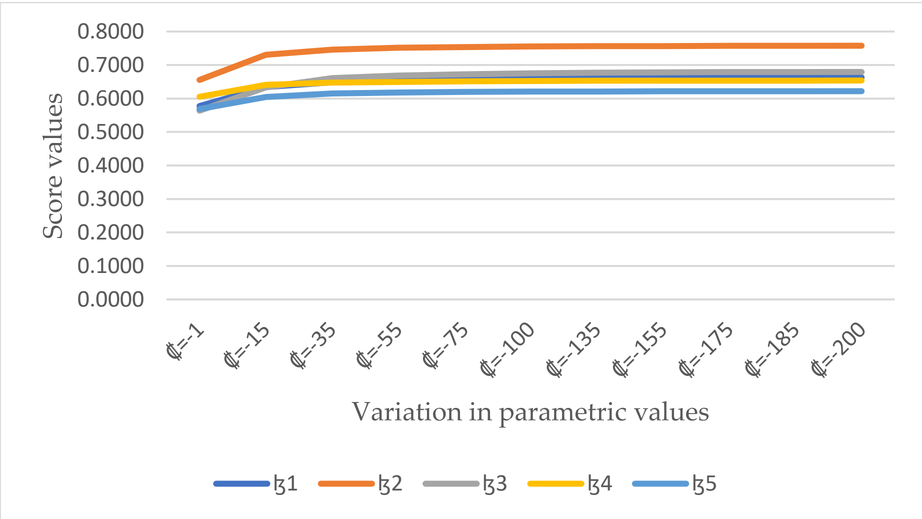

Figure 2. Displays the graphical representation of score values listed in Table 6.

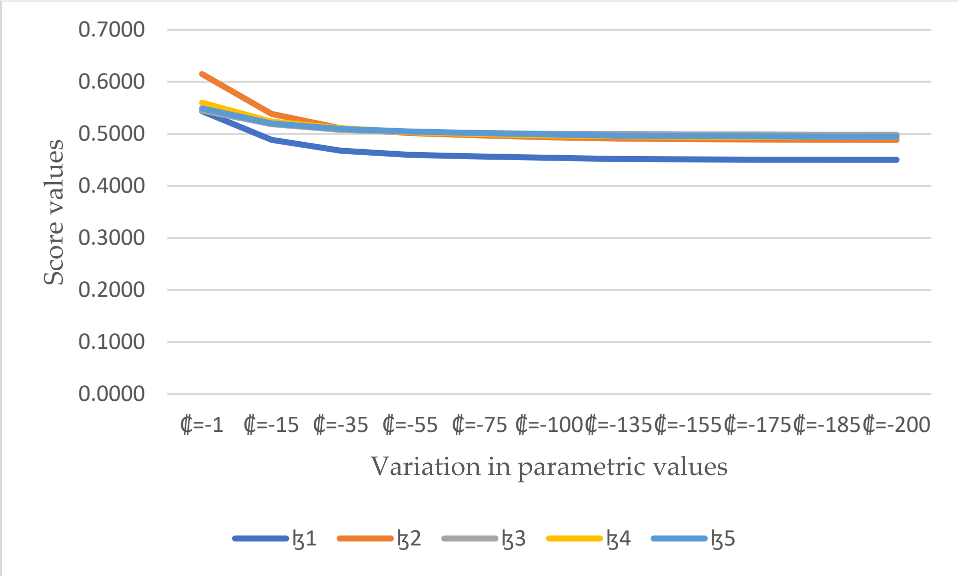

Figure 3. Displays the graphical representation of score values listed in Table 6.

7 | CONCLUSION

The PFS is a more powerful and efficient agnation model to mitigate the impact of unpredictable information of human opinions. The PFS is the modified version of an IFS and FSs. Several mathematicians illustrated different aggregation models under different fuzzy domains to handle vague information about any object of human opinions. The theory of prioritization among different attributes makes decisions very easy for the decision-maker. The Schweizer-Sklar aggregation tools attained a lot of attraction from numerous scientists. The main aim of this article is to propose a series of new methodologies under the system of PF information. We developed some new approaches based on PF information, including PFSSPA, PFSSPG, PFSSPWA, and PFSSPWG operators. To expose the applicability and effectiveness of developed approaches, we discussed some reliable characteristics and particular cases of invented approaches. One of the most important characteristics of the created operators is their capacity to make the right choices by removing the impact of erroneous data given by the affected decision-maker by utilizing a prioritized aggregation operator. A unique strategy for dealing with MADM using PF information has been shown using the suggested operators. The created technique is used to address a decision-making issue involving the selection of the most optimal option. Furthermore, the influence of parameter value of the Schweizer-Sklar aggregation tools on the ranking results computed by established technique.

In the future, we will extend our proposed research work in different fuzzy circumstances like spherical fuzzy sets [47], t-spherical fuzzy sets [48], Dual Hesitant bipolar fuzzy sets [49], and linear diophantine fuzzy sets [50]. We also try to resolve complex real-life situations by using our proposed aggregation approaches, such as computer programming, renewable energy, waste management, and artificial intelligence.

Acknowledgment: We are thankful to the editors and anonymous reviewers for their constructive suggestions. All authors contributed equally to this manuscript.

Declaration of Conflicting Interest: The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding: The author(s) received no financial support for the research, authorship, and/or publication of this article.

REFERENCES

[1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965, doi: 10.1016/S0019-9958(65)90241-X.

[2] Z. Yang and J. Chang, "Interval-valued Pythagorean normal fuzzy information aggregation operators for multi-attribute decision making," *IEEE Access*, vol. 8, pp. 51295–51314, 2020.

[3] K. Ullah, T. Mahmood, Z. Ali, and N. Jan, "On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition, Complex Intell. Syst. 6 (2020), 15–27."

[4] T. Mahmood, K. Ullah, Q. Khan, and N. Jan, "An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets," *Neural Comput & Applic*, vol. 31, no. 11, pp. 7041–7053, Nov. 2019, doi: 10.1007/s00521-018-3521-2.

[5] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, Aug. 1986, doi: 10.1016/S0165-0114(86)80034-3.

[6] X. Chen, C. Suo, and Y. Li, "Distance measures on intuitionistic hesitant fuzzy set and its application in decision-making," *Computational and Applied Mathematics*, vol. 40, no. 3, pp. 1–21, 2021.

[7] S. Li, X. Peng, and Y. Li, "Choquet integrals of weighted generalized and group generalized intuitionistic fuzzy soft sets," *Soft Computing*, vol. 24, pp. 745–760, 2020.

[8] W.-L. Hung and M.-S. Yang, "Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance," *Pattern recognition letters*, vol. 25, no. 14, pp. 1603–1611, 2004.

[9] K. T. Atanassov, "Interval-valued intuitionistic fuzzy sets," in *Intuitionistic fuzzy sets*, Springer, 1999, pp. 139–177.

[10] R. Yager, "Pythagorean fuzzy subsets," *2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*, 2013, doi: 10.1109/IFSA-NAFIPS.2013.6608375.

[11] A. Hussain, M. I. Ali, and T. Mahmood, "Pythagorean fuzzy soft rough sets and their applications in decision-making," *Journal of Taibah University for Science*, vol. 14, no. 1, pp. 101–113, 2020.

[12] X. Peng, Y. Yang, J. Song, and Y. Jiang, "Pythagorean fuzzy soft set and its application," *Computer engineering*, vol. 41, no. 7, pp. 224–229, 2015.

[13] R. R. Yager, "Generalized Orthopair Fuzzy Sets," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 5, pp. 1222–1230, Oct. 2017, doi: 10.1109/TFUZZ.2016.2604005.

[14] M. I. Ali, "Another view on q-rung orthopair fuzzy sets," *International Journal of Intelligent Systems*, vol. 33, no. 11, pp. 2139–2153, 2018, doi: 10.1002/int.22007.

[15] P. Wang, J. Wang, G. Wei, and C. Wei, "Similarity measures of q-rung orthopair fuzzy sets based on cosine function and their applications," *Mathematics*, vol. 7, no. 4, p. 340, 2019.

[16] X. Tian, M. Niu, W. Zhang, L. Li, and E. Herrera Viedma, "A novel TODIM based on prospect theory to select green supplier with q-rung orthopair fuzzy set," 2020.

[17] D. Liu, X. Chen, and D. Peng, “Some cosine similarity measures and distance measures between q-rung orthopair fuzzy sets,” *International Journal of Intelligent Systems*, vol. 34, no. 7, pp. 1572–1587, 2019, doi: 10.1002/int.22108.
[18] B. C. Cuong, “Picture fuzzy sets,” *JCC*, vol. 30, no. 4, pp. 409–420, Feb. 2015, doi: 10.15625/1813-9663/30/4/5032.
[19] B. C. Cuong and V. H. Pham, “Some fuzzy logic operators for picture fuzzy sets,” in *2015 seventh international conference on knowledge and systems engineering (KSE)*, IEEE, 2015, pp. 132–137.
[20] P. Dutta and S. Ganju, “Some aspects of picture fuzzy set,” *Transactions of A. Razmadze Mathematical Institute*, vol. 172, no. 2, pp. 164–175, Aug. 2018, doi: 10.1016/j.trmi.2017.10.006.
[21] B. C. Cuong, V. Kreinovitch, and R. T. Ngan, “A classification of representable t-norm operators for picture fuzzy sets,” in *2016 Eighth International Conference on Knowledge and Systems Engineering (KSE)*, Oct. 2016, pp. 19–24. doi: 10.1109/KSE.2016.7758023.
[22] H. Garg, “Some picture fuzzy aggregation operators and their applications to multicriteria decision-making,” *Arabian Journal for Science and Engineering*, vol. 42, no. 12, pp. 5275–5290, 2017.
[23] D. Ahmed and B. Dai, “Picture Fuzzy Rough Set and Rough Picture Fuzzy Set on Two Different Universes and Their Applications,” *Journal of Mathematics*, vol. 2020, p. e8823580, Nov. 2020, doi: 10.1155/2020/8823580.
[24] A. Hussain, A. Alsanad, K. Ullah, Z. Ali, M. K. Jamil, and M. A. Mosleh, “Investigating the Short-Circuit Problem Using the Planarity Index of Complex q-Rung Orthopair Fuzzy Planar Graphs,” *Complexity*, vol. 2021, 2021.
[25] M. J. Khan, P. Kumam, S. Ashraf, and W. Kumam, “Generalized picture fuzzy soft sets and their application in decision support systems,” *Symmetry*, vol. 11, no. 3, p. 415, 2019.
[26] F. E. Boran and D. Akay, “A biparametric similarity measure on intuitionistic fuzzy sets with applications to pattern recognition,” *Information Sciences*, vol. 255, pp. 45–57, Jan. 2014, doi: 10.1016/j.ins.2013.08.013.
[27] T. Senapati and R. R. Yager, “Some New Operations Over Fermatean Fuzzy Numbers and Application of Fermatean Fuzzy WPM in Multiple Criteria Decision Making,” *Informatica*, vol. 30, no. 2, pp. 391–412, Jan. 2019.
[28] A. Dey, T. Senapati, M. Pal, and G. Chen, “A novel approach to hesitant multi-fuzzy soft set based decision-making,” *AIMS Mathematics*, vol. 5, no. 3, pp. 1985–2008, 2020.
[29] T. Senapati and R. R. Yager, “Fermatean fuzzy weighted averaging/geometric operators and its application in multi-criteria decision-making methods,” *Engineering Applications of Artificial Intelligence*, vol. 85, pp. 112–121, 2019.
[30] M. Lu, G. Wei, F. E. Alsaadi, T. Hayat, and A. Alsaedi, “Hesitant pythagorean fuzzy hamacher aggregation operators and their application to multiple attribute decision making,” *Journal of Intelligent & Fuzzy Systems*, vol. 33, no. 2, pp. 1105–1117, 2017.
[31] M. Riaz, W. Sa\labun, H. M. Athar Farid, N. Ali, and J. Wątróbski, “A robust q-rung orthopair fuzzy information aggregation using Einstein operations with application to sustainable energy planning decision management,” *Energies*, vol. 13, no. 9, p. 2155, 2020.
[32] Y. Liu and W. Jiang, “A new distance measure of interval-valued intuitionistic fuzzy sets and its application in decision making,” *Soft Comput*, vol. 24, no. 9, pp. 6987–7003, May 2020, doi: 10.1007/s00500-019-04332-5.
[33] M. Riaz, D. Pamucar, H. M. Athar Farid, and M. R. Hashmi, “q-Rung Orthopair Fuzzy Prioritized Aggregation Operators and Their Application Towards Green Supplier Chain Management,” *Symmetry*, vol. 12, no. 6, Art. no. 6, Jun. 2020, doi: 10.3390/sym12060976.
[34] P. Liu and H. Gao, “Multicriteria Decision Making Based on Generalized Maclaurin Symmetric Means with Multi-Hesitant Fuzzy Linguistic Information,” *Symmetry*, vol. 10, no. 4, Art. no. 4, Apr. 2018, doi: 10.3390/sym10040081.
[35] H. Zhao and N. Li, “Optimal Siting of Charging Stations for Electric Vehicles Based on Fuzzy Delphi and Hybrid Multi-Criteria Decision Making Approaches from an Extended Sustainability Perspective,” *Energies*, vol. 9, no. 4, Art. no. 4, Apr. 2016, doi: 10.3390/en9040270.
[36] B. Schweizer and A. Sklar, “Statistical metric spaces,” *Pacific J. Math*, vol. 10, no. 1, pp. 313–334, 1960.
[37] H. Garg, “Intuitionistic fuzzy hamacher aggregation operators with entropy weight and their applications to multi-criteria decision-making problems,” *Iranian Journal of Science and Technology, Transactions of Electrical Engineering*, vol. 43, no. 3, pp. 597–613, 2019.
[38] Q. Khan, J. Gwak, M. Shahzad, and M. K. Alam, “A novel approached based on T-spherical fuzzy Schweizer-Sklar power Heronian mean operator for evaluating water reuse applications under uncertainty,” *Sustainability*, vol. 13, no. 13, p. 7108, 2021.
[39] L. Zhang, “Intuitionistic fuzzy averaging Schweizer-Sklar operators based on interval-valued intuitionistic fuzzy numbers and its applications,” in *2018 Chinese control and decision conference (CCDC)*, IEEE, 2018, pp. 2194–2197.
[40] P. Liu and P. Wang, “Some interval-valued intuitionistic fuzzy Schweizer–Sklar power aggregation operators and their application to supplier selection,” *International Journal of Systems Science*, vol. 49, no. 6, pp. 1188–1211, 2018.

[41] P. Wang and P. Liu, "Some Maclaurin symmetric mean aggregation operators based on Schweizer-Sklar operations for intuitionistic fuzzy numbers and their application to decision making," *Journal of Intelligent & Fuzzy Systems*, vol. 36, no. 4, pp. 3801–3824, Jan. 2019, doi: 10.3233/JIFS-18801.
[42] D. Zindani, S. R. Maity, and S. Bhowmik, "Interval-valued intuitionistic fuzzy TODIM method based on Schweizer–Sklar power aggregation operators and their applications to group decision making," *Soft Computing*, vol. 24, pp. 14091–14133, 2020.
[43] B. Cuong, "Picture fuzzy sets-first results. Part 1, in: seminar," *Neuro-Fuzzy Systems with Applications*, 2013.
[44] C. Jana, T. Senapati, M. Pal, and R. R. Yager, "Picture fuzzy Dombi aggregation operators: application to MADM process," *Applied Soft Computing*, vol. 74, pp. 99–109, 2019.
[45] R. R. Yager, "Prioritized aggregation operators," *International Journal of Approximate Reasoning*, vol. 48, no. 1, pp. 263–274, Apr. 2008, doi: 10.1016/j.ijar.2007.08.009.
[46] T. Mahmood and Z. Ali, "Schweizer–Sklar muirhead mean aggregation operators based on pythagorean fuzzy sets and their application in multi-criteria decision-making," *Pythagorean Fuzzy Sets: Theory and Applications*, pp. 235–259, 2021.
[47] Z. Ali, T. Mahmood, and M.-S. Yang, "TOPSIS method based on complex spherical fuzzy sets with Bonferroni mean operators," *Mathematics*, vol. 8, no. 10, p. 1739, 2020.
[48] E. Ozceylan, B. Ozkan, M. Kabak, and M. Dagdeviren, "A Survey on spherical fuzzy sets and clustering the literature," in *International Conference on Intelligent and Fuzzy Systems*, Springer, 2020, pp. 87–97.
[49] H. Gao, G. Wei, and Y. Huang, "Dual hesitant bipolar fuzzy Hamacher prioritized aggregation operators in multiple attribute decision making," *Ieee Access*, vol. 6, pp. 11508–11522, 2017.
[50] M. Riaz and M. R. Hashmi, "Linear Diophantine fuzzy set and its applications towards multi-attribute decision-making problems," *Journal of Intelligent & Fuzzy Systems*, vol. 37, no. 4, pp. 5417–5439, 2019.