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Aczel-Alsina Aggregation Operators Based on Interval-valued Complex Single-valued Neutrosophic Information and Their Application in Decision-Making Problems

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ABSTRACT

In this article, we evaluate the novel theory of Aczel-Alsina operational laws under the consideration of the interval-valued complex single-valued neutrosophic (IVCSVN) values by using the idea of Aczel-Alsina t-norm and t-conorm which was derived by Aczel and Alsina in 1982. Moreover, we derive the concept of the IVCSVN Aczel-Alsina weighted averaging (IVCSVNAAWA) operator, IVCSVN Aczel-Alsina ordered weighted averaging (IVCSVNAAOWA) operator, IVCSVN Aczel-Alsina hybrid averaging (IVCSVNAAHA) operator, and examined or presented their major properties, such as idempotency, boundedness, and monotonicity. Additionally, we demonstrate a MADM "multi-attribute decision-making" scenario in the availability of the derived operators for IVCSVN information and illustrates various numerical example to show the practicality and supremacy of the presented theory by comparing the derived work with some prevailing techniques.

Keywords: Interval-valued complex single-valued neutrosophic sets; Aczel-Alsina aggregation operators; Decision-making problems.

1 | INTRODUCTION

For depicting unreliable and vague information, various scholars have used the theory of clustering analysis, pattern recognition, medical diagnosis, artificial intelligence, machine learning, and decision-making problems. Furthermore, the MADM technique is also very famous and reliable for examining the finest preference from the collection of preferences, which is a major part of the decision-making technique. Under the presence of the classical set theory, we have only two opinions such as zero or one, but not between them, and in many situations, we have needed to use some information between zero and one. For this, Zadeh [1] derived the fuzzy set (FS) theory in 1965 by improving the function of classical information and exposing or defining a new function, called truth grade whose domine is a universal set and range is the unit interval. Furthermore, a lot of modifications and generalizations of FSs have been derived by different scholars such as intuitionistic FS (IFS) [2], interval-valued IFS (IVIFS) [3], neutrosophic set (NS) [4], interval-valued NS (IVNS) [5], single-valued NS (SVNS) [6], and interval-valued SVNS (IVSVNS) [7]. These ideas have also a lot of utilization in different fields such as similarity measures [8], [9], aggregation operators [10]–[12], and different kinds of methods [13]–[16].

Utilization of the phase information is a very famous and challenging task for scholars, because when we utilize the phase term in the field of fuzzy set theory, then we can easily evaluate a lot of real-life problems in a practical situation. In various situations, we faced two-dimensional information, for instance, during purchasing a new car, we faced two types of opinions such as the name and production date of the car which are stated in the real and unreal parts of the complex numbers. Therefore, for handling such types of problems, the idea of complex FS (CFS) was derived by Ramot et al.[17]. Furthermore, the theory of CFS has been extended by different scholars, such as interval-valued CFS (IVCFS) [18], complex IFS (CIFS) [19], interval-valued CIFS (IVCIFS) [20] complex NS (CNS) [21] interval-valued CNS (IVCNS) [22], complex SVNS (CSVNS) [23], and interval-valued CSVNS (IVCSVNS) [24]. These ideas have also a lot of utilization in different fields such as similarity measures [25], aggregation operators [26], and different kinds of methods [27].

Aczel-Alsina t-norm and t-conorm were evaluated by Aczel and Alsina [28] in 1982. After successfully investigating Aczel-Alsina information, certain utilizations have been neglected by different scholars, such as Aczel-Alsina aggregation operators for IFSs [29] Aczel-Alsina prioritized aggregation operators for IFSs [30], Aczel-Alsina geometric aggregation operators for IFSs [31], Aczel-Alsina aggregation operators for z-neutrosophic sets[32], Aczel-Alsina aggregation operators for CIFSs [33], Aczel-Alsina aggregation operators for SVNSs [34], and power Aczel-Alsina aggregation operators for IFSs [35] Under consideration of the above motivation, in this article, we derive the following information, such as:

1. To evaluate the novel theory of Aczel-Alsina operational laws under the consideration of the IVCSVN values by using the idea of Aczel-Alsina t-norm and t-conorm.
2. To derive the concept of the IVCSVNAAWA operator, IVCSVNAAOWA operator, and IVCSVNAAHA operator, and examined or presented their major properties, such as idempotency, boundedness, and monotonicity.
3. To demonstrate a MADM scenario in the availability of the derived operators for IVCSVN information and illustrate various numerical examples.
4. To show the practicality and supremacy of the presented theory by comparing the derived work with some prevailing techniques.

This theory is arranged in the following ways: In Section 2, we included the basics of CSVNSs, Algebraic and Aczel-Alsina t-norm and t-conorm. In Section 3, we evaluated the novel theory of Aczel-Alsina operational laws under the consideration of the IVCSVN values by using the idea of Aczel-Alsina t-norm and t-conorm. In Section 4, we derived the concept of the IVCSVNAAWA operator, IVCSVNAAOWA operator, and IVCSVNAAHA operator, and examined or presented their major properties, such as idempotency, boundedness, and monotonicity. In Section 5, we demonstrated a MADM scenario in the availability of the derived operators for IVCSVN information and illustrated various numerical examples. In Section 6, we show the practicality and supremacy of the presented theory by comparing the derived work with some prevailing techniques. The concluding remarks are stated in Section 7.

2 | PRELIMINARIES

In this section, we included the basics of IVCSVNSs, Algebraic and Aczel-Alsina t-norm and t-conorm.

Definition 1: [7] A well-known theory of IVSVNS $\overline{N}$ is arranged in the form:

$$\overline{\mathrm{N}} = \{\langle ŧ, \mathrm{m}_{\overline{\mathrm{N}}}(ŧ), a_{\overline{\mathrm{N}}}(ŧ), \mathrm{n}_{\overline{\mathrm{N}}}(ŧ)\rangle | ŧ\epsilon \mathrm{U}\} \quad (1)$$

Noticed that $m_{\overline{N}}(ŧ) = [m_{\overline{N}}^{l}(ŧ), m_{\overline{N}}^{u}(ŧ)], a_{\overline{N}}(ŧ) = [a_{\overline{N}}^{l}(ŧ), a_{\overline{N}}^{u}(ŧ)]$, and $n_{\overline{N}}(ŧ) = [n_{\overline{N}}^{l}(ŧ), n_{\overline{N}}^{u}(ŧ)]$ shown the grade of truth, abstinence, and falsity, where $m_{\overline{N}}^{u}(ŧ), a_{\overline{N}}^{u}(ŧ), n_{\overline{N}}^{u}(ŧ), m_{\overline{N}}^{l}(ŧ), a_{\overline{N}}^{l}(ŧ), n_{\overline{N}}^{l}(ŧ) \in [0,1]$, with $0 \le m_{\overline{N}}^{u}(ŧ) + a_{\overline{N}}^{u}(ŧ) + n_{\overline{N}}^{u}(ŧ) \le 3$.

Definition 2: [24] A well-known theory of IVCSVNS $\mathfrak{N}$ is arranged in the form:

$$\mathfrak{N} = \{(m_{\mathfrak{N}}(ŧ), a_{\mathfrak{N}}(ŧ), n_{\mathfrak{N}}(ŧ)) | ŧ\epsilon U\} \quad (2)$$

Noticed that $m_{\mathfrak{N}}(ŧ), a_{\mathfrak{N}}(ŧ), n_{\mathfrak{N}}(ŧ)$ shown the grade of truth, abstinence, and falsity, where $m_{\mathfrak{N}}(ŧ) = [x_{m_{\mathfrak{N}}}^{l}(ŧ), x_{m_{\mathfrak{N}}}^{u}(ŧ)].exp^{j[\omega_{m_{\mathfrak{N}}}^{l}(ŧ), \omega_{m_{\mathfrak{N}}}^{u}(ŧ)]}, a_{\mathfrak{N}}(ŧ) = [x_{a_{\mathfrak{N}}}^{l}(ŧ), x_{a_{\mathfrak{N}}}^{u}(ŧ)].exp^{j[\varphi_{a_{\mathfrak{N}}}^{l}(ŧ), \varphi_{a_{\mathfrak{N}}}^{u}(ŧ)]}, n_{\mathfrak{N}}(ŧ) = [x_{n_{\mathfrak{N}}}^{l}(ŧ), x_{n_{\mathfrak{N}}}^{u}(ŧ)].exp^{j[\psi_{n_{\mathfrak{N}}}^{l}(ŧ), \psi_{n_{\mathfrak{N}}}^{u}(ŧ)]}$. Noticed that $j = 2\pi i$, with $0 \le x_{m_{\mathfrak{N}}}^{u}(ŧ) + x_{a_{\mathfrak{N}}}^{u}(ŧ) + x_{n_{\mathfrak{N}}}^{u}(ŧ) \le 3$ and $0 \le \omega_{m_{\mathfrak{N}}}^{u}(ŧ) + \varphi_{a_{\mathfrak{N}}}^{u}(ŧ) + \psi_{n_{\mathfrak{N}}}^{u}(ŧ) \le 3\pi$. The simple form of IVCSVNV is stated by:

$$\mathfrak{N}_j = \left([x_{m_{\mathfrak{N}_j}}^{l}, x_{m_{\mathfrak{N}_j}}^{u}].exp^{j[\omega_{m_{\mathfrak{N}_j}}^{l}, \omega_{m_{\mathfrak{N}_j}}^{u}]}, [x_{a_{\mathfrak{N}_j}}^{l}, x_{a_{\mathfrak{N}_j}}^{u}].exp^{j[\varphi_{a_{\mathfrak{N}_j}}^{l}, \varphi_{a_{\mathfrak{N}_j}}^{u}]}, [x_{n_{\mathfrak{N}_j}}^{l}, x_{n_{\mathfrak{N}_j}}^{u}].exp^{j[\psi_{n_{\mathfrak{N}_j}}^{l}, \psi_{n_{\mathfrak{N}_j}}^{u}]}\right), j = 1,2, \dots n.$$

Definition 3: [24] The well-known theory of score function is arranged by:

$$\mathfrak{S}(\mathfrak{N}) = \frac{1}{2}\left(\frac{1}{6}(m_{\mathfrak{N}}^{l} + a_{\mathfrak{N}}^{l} + n_{\mathfrak{N}}^{l} + m_{\mathfrak{N}}^{u} + a_{\mathfrak{N}}^{u} + n_{\mathfrak{N}}^{u}) + \frac{1}{6}(\omega_{m_{\mathfrak{N}}}^{l} + \varphi_{a_{\mathfrak{N}}}^{l} + \psi_{n_{\mathfrak{N}}}^{l} + \omega_{m_{\mathfrak{N}}}^{u} + \varphi_{a_{\mathfrak{N}}}^{u} + \psi_{n_{\mathfrak{N}}}^{u})\right) \quad \mathbf{(3)}$$

Definition 3: [24] Any two simple IVCSVNVs are stated by:

$$\mathfrak{N}_j = \left([x_{m_{\mathfrak{N}_j}}^{l}, x_{m_{\mathfrak{N}_j}}^{u}].exp^{j[\omega_{m_{\mathfrak{N}_j}}^{l}, \omega_{m_{\mathfrak{N}_j}}^{u}]}, [x_{a_{\mathfrak{N}_j}}^{l}, x_{a_{\mathfrak{N}_j}}^{u}].exp^{j[\varphi_{a_{\mathfrak{N}_j}}^{l}, \varphi_{a_{\mathfrak{N}_j}}^{u}]}, [x_{n_{\mathfrak{N}_j}}^{l}, x_{n_{\mathfrak{N}_j}}^{u}].exp^{j[\psi_{n_{\mathfrak{N}_j}}^{l}, \psi_{n_{\mathfrak{N}_j}}^{u}]}\right), j = 1,2, \text{ then}$$

we have

$$\mathfrak{N}_1 \cap \mathfrak{N}_2 = \begin{pmatrix} [\min(x_{m_{\mathfrak{N}_1}}^{l}, x_{m_{\mathfrak{N}_2}}^{l}), \min(x_{m_{\mathfrak{N}_1}}^{u}, x_{m_{\mathfrak{N}_2}}^{u})].exp^{j[\min(\omega_{m_{\mathfrak{N}_1}}^{l}, \omega_{m_{\mathfrak{N}_2}}^{l}), \min(\omega_{m_{\mathfrak{N}_1}}^{u}, \omega_{m_{\mathfrak{N}_2}}^{u})]}, \\ [\min(x_{a_{\mathfrak{N}_1}}^{l}, x_{a_{\mathfrak{N}_2}}^{l}), \min(x_{a_{\mathfrak{N}_1}}^{u}, x_{a_{\mathfrak{N}_2}}^{u})].exp^{j[\min(\varphi_{a_{\mathfrak{N}_1}}^{l}, \varphi_{a_{\mathfrak{N}_2}}^{l}), \min(\varphi_{a_{\mathfrak{N}_1}}^{u}, \varphi_{a_{\mathfrak{N}_2}}^{u})]}, \\ [\max(x_{n_{\mathfrak{N}_1}}^{l}, x_{n_{\mathfrak{N}_2}}^{l}), \max(x_{n_{\mathfrak{N}_1}}^{u}, x_{n_{\mathfrak{N}_2}}^{u})].exp^{j[\max(\psi_{n_{\mathfrak{N}_1}}^{l}, \psi_{n_{\mathfrak{N}_2}}^{l}), \max(\psi_{n_{\mathfrak{N}_1}}^{u}, \psi_{n_{\mathfrak{N}_2}}^{u})]} \end{pmatrix} \quad (4)$$

$$\mathfrak{N}_1 \cup \mathfrak{N}_2 = \begin{pmatrix} \left[\max\left(x^l_{m_{\mathfrak{N}_1}}, x^l_{m_{\mathfrak{N}_2}}\right), \max\left(x^u_{m_{\mathfrak{N}_1}}, x^u_{m_{\mathfrak{N}_2}}\right)\right].exp^{j\left[\max\left(\omega^l_{m_{\mathfrak{N}_1}},\omega^l_{m_{\mathfrak{N}_2}}\right),\max\left(\omega^u_{m_{\mathfrak{N}_1}},\omega^u_{m_{\mathfrak{N}_2}}\right)\right]}, \\ \left[\min\left(x^l_{a_{\mathfrak{N}_1}}, x^l_{a_{\mathfrak{N}_2}}\right), \min\left(x^u_{a_{\mathfrak{N}_1}}, x^u_{a_{\mathfrak{N}_2}}\right)\right].exp^{j\left[\min\left(\varphi^l_{a_{\mathfrak{N}_1}},\varphi^l_{a_{\mathfrak{N}_2}}\right),\min\left(\varphi^u_{a_{\mathfrak{N}_1}},\varphi^u_{a_{\mathfrak{N}_2}}\right)\right]}, \\ \left[\min\left(x^l_{n_{\mathfrak{N}_1}}, x^l_{n_{\mathfrak{N}_2}}\right), \min\left(x^u_{n_{\mathfrak{N}_1}}, x^u_{n_{\mathfrak{N}_2}}\right)\right].exp^{j\left[\min\left(\psi^l_{n_{\mathfrak{N}_1}},\psi^l_{n_{\mathfrak{N}_2}}\right),\min\left(\psi^u_{n_{\mathfrak{N}_1}},\psi^u_{n_{\mathfrak{N}_2}}\right)\right]} \end{pmatrix} \tag{5}$$

Definition 4: [28] The well-known theory of AA t-norms T_A^{λ} and t-conorm $\mathcal{L}_A^{\lambda}$ is stated by:

$$\mathrm{T}_A^{\lambda}(\sigma,\varsigma) = \begin{Bmatrix} \mathrm{T}_D(\sigma,\varsigma) & jff\ \lambda = 0 \\ \min(\sigma,\varsigma) & jff\ \lambda = \infty \\ exp^{-\left((-\ln\sigma)^{\lambda}+(-\ln\varsigma)^{\lambda}\right)^{1/\lambda}} & otherwise \end{Bmatrix} \tag{6}$$

$$\mathcal{L}_A^{\lambda}(\sigma,\varsigma) = \begin{Bmatrix} \mathcal{L}_D(\sigma,\varsigma) & jff\ \lambda = 0 \\ \min(\sigma,\varsigma) & jff\ \lambda = \infty \\ 1 - exp^{-\left((-\ln(1-\sigma))^{\lambda}+(-\ln(1-\varsigma))^{\lambda}\right)^{1/\lambda}} & otherwise \end{Bmatrix} \tag{7}$$

3 | AA Operational Laws for IVCSVNVs

In this section, we evaluated the novel theory of Aczel-Alsina operational laws under the consideration of the IVCSVN values by using the idea of Aczel-Alsina t-norm and t-conorm.

Definition 5: Any two simple IVCSVNVs are stated by:

$$\mathfrak{N}_j = \left(\left[x^l_{m_{\mathfrak{N}_j}}, x^u_{m_{\mathfrak{N}_j}}\right].exp^{j\left[\omega^l_{m_{\mathfrak{N}_j}},\omega^u_{m_{\mathfrak{N}_j}}\right]}, \left[x^l_{a_{\mathfrak{N}_j}}, x^u_{a_{\mathfrak{N}_j}}\right].exp^{j\left[\varphi^l_{a_{\mathfrak{N}_j}},\varphi^u_{a_{\mathfrak{N}_j}}\right]}, \left[x^l_{n_{\mathfrak{N}_j}}, x^u_{n_{\mathfrak{N}_j}}\right].exp^{j\left[\psi^l_{n_{\mathfrak{N}_j}},\psi^u_{n_{\mathfrak{N}_j}}\right]}\right), j = 1,2,$$

then we have

$$\mathfrak{N}_1 \oplus \mathfrak{N}_2 = \begin{pmatrix} \left[\left(1 - exp^{-\left(\left(-\ln\left(1-\left(x^l_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^l_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right), \left(1 - exp^{-\left(\left(-\ln\left(1-\left(x^u_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^u_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right)\right] \\ exp^{j\left[1-exp^{-\left(\left(-\ln\left(1-\left(\omega^l_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\omega^l_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\left(-\ln\left(1-\left(\omega^u_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\omega^u_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\left(-\ln\left(x^l_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(x^l_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(x^u_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(x^u_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\left(-\ln\left(\varphi^l_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(\varphi^l_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\left(-\ln\left(\varphi^u_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(\varphi^u_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\left(-\ln\left(x^l_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(x^l_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(x^u_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(x^u_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\left(-\ln\left(\psi^l_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(\psi^l_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\left(-\ln\left(\psi^u_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(\psi^u_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]} \end{pmatrix} \tag{8}$$

$$
\mathfrak{N}_1 \otimes \mathfrak{N}_2
= \begin{pmatrix}
\left[exp^{-\left(\left(-\ln\left(x^{l}_{m_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(x^{l}_{m_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(x^{u}_{m_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(x^{u}_{m_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}} \right] exp^{j\left[exp^{-\left(\left(-\ln\left(\omega^{l}_{m_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(\omega^{l}_{m_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(\omega^{u}_{m_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(\omega^{u}_{m_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}} \right]}, \\
\left[1-exp^{-\left(\left(-\ln\left(1-\left(x^{l}_{a_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{l}_{a_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(x^{u}_{a_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{u}_{a_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}} \right] exp^{j\left[1-exp^{-\left(\left(-\ln\left(1-\left(\varphi^{l}_{a_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\varphi^{l}_{a_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(\varphi^{u}_{a_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\varphi^{u}_{a_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}} \right]}, \\
\left[1-exp^{-\left(\left(-\ln\left(1-\left(x^{l}_{n_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{l}_{n_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(x^{u}_{n_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{u}_{n_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}} \right] exp^{j\left[1-exp^{-\left(\left(-\ln\left(1-\left(\psi^{l}_{n_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\psi^{l}_{n_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(\psi^{u}_{n_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\psi^{u}_{n_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}} \right]}
\end{pmatrix} \quad (9)
$$

$$
\lambda\mathfrak{N} = \begin{pmatrix}
\left[1-exp^{-\left(\lambda\left(-\ln\left(1-\left(x^{l}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\lambda\left(-\ln\left(1-\left(x^{u}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}} \right] exp^{j\left[1-exp^{-\left(\lambda\left(-\ln\left(1-\left(\omega^{l}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\lambda\left(-\ln\left(1-\left(\omega^{u}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}} \right]}, \\
\left[exp^{-\left(\lambda\left(-\ln\left(x^{l}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\lambda\left(-\ln\left(x^{u}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}} \right] exp^{j\left[exp^{-\left(\lambda\left(-\ln\left(\varphi^{l}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\lambda\left(-\ln\left(\varphi^{u}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}} \right]}, \\
\left[exp^{-\left(\lambda\left(-\ln\left(x^{l}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\lambda\left(-\ln\left(x^{u}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}} \right] exp^{j\left[exp^{-\left(\lambda\left(-\ln\left(\psi^{l}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\lambda\left(-\ln\left(\psi^{u}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}} \right]}
\end{pmatrix}
$$

$$\lambda\mathfrak{N} = \begin{pmatrix} \left[1-exp^{-\left(\lambda\left(-ln\left(1-\left(x^{l}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\lambda\left(-ln\left(1-\left(x^{u}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[1-exp^{-\left(\lambda\left(-ln\left(1-\left(\omega^{l}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\lambda\left(-ln\left(1-\left(\omega^{u}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\lambda\left(-ln\left(x^{l}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\lambda\left(-ln\left(x^{u}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] exp^{j\left[exp^{-\left(\lambda\left(-ln\left(\varphi^{l}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\lambda\left(-ln\left(\varphi^{u}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\lambda\left(-ln\left(x^{l}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\lambda\left(-ln\left(x^{u}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] exp^{j\left[exp^{-\left(\lambda\left(-ln\left(\psi^{l}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\lambda\left(-ln\left(\psi^{u}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]} \end{pmatrix} \quad (10)$$

$$\mathfrak{N}^{\lambda} = \begin{pmatrix} \left[exp^{-\left(\lambda\left(-ln\left(x^{l}_{m_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\lambda\left(-ln\left(x^{u}_{m_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] exp^{j\left[exp^{-\left(\lambda\left(-ln\left(\omega^{l}_{m_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\lambda\left(-ln\left(\omega^{u}_{m_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[1-exp^{-\left(\lambda\left(-ln\left(1-\left(x^{l}_{a_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\lambda\left(-ln\left(1-\left(x^{u}_{a_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[1-exp^{-\left(\lambda\left(-ln\left(1-\left(\varphi^{l}_{a_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\lambda\left(-ln\left(1-\left(\varphi^{u}_{a_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[1-exp^{-\left(\lambda\left(-ln\left(1-\left(x^{l}_{n_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\lambda\left(-ln\left(1-\left(x^{u}_{n_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[1-exp^{-\left(\lambda\left(-ln\left(1-\left(\psi^{l}_{n_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\lambda\left(-ln\left(1-\left(\psi^{u}_{n_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]} \end{pmatrix} \quad (11)$$

Theorem 1: Any two simple IVCSVNVs are stated by:

$$\mathfrak{N}_j = \left(\left[x^{l}_{m_{\mathfrak{N}_j}}, x^{u}_{m_{\mathfrak{N}_j}}\right].exp^{j\left[\omega^{l}_{m_{\mathfrak{N}_j}}, \omega^{u}_{m_{\mathfrak{N}_j}}\right]}, \left[x^{l}_{a_{\mathfrak{N}_j}}, x^{u}_{a_{\mathfrak{N}_j}}\right].exp^{j\left[\varphi^{l}_{a_{\mathfrak{N}_j}}, \varphi^{u}_{a_{\mathfrak{N}_j}}\right]}, \left[x^{l}_{n_{\mathfrak{N}_j}}, x^{u}_{n_{\mathfrak{N}_j}}\right].exp^{j\left[\psi^{l}_{n_{\mathfrak{N}_j}}, \psi^{u}_{n_{\mathfrak{N}_j}}\right]}\right), j = 1,2, \text{ then}$$

we have

1. $\mathfrak{N}_1 \oplus \mathfrak{N}_2 = \mathfrak{N}_2 \oplus \mathfrak{N}_1$
2. $\mathfrak{N}_1 \otimes \mathfrak{N}_2 = \mathfrak{N}_2 \otimes \mathfrak{N}_1$
3. $\lambda(\mathfrak{N}_1 \oplus \mathfrak{N}_2) = \lambda\mathfrak{N}_1 \oplus \lambda\mathfrak{N}_2$
4. $(\lambda_1 \oplus \lambda_2)\mathfrak{N}_1 = \lambda_1\mathfrak{N}_1 \oplus \lambda_2\mathfrak{N}_1$
5. $(\mathfrak{N}_1 \otimes \mathfrak{N}_2)^{\lambda} = \mathfrak{N}_1{}^{\lambda} \otimes \mathfrak{N}_2{}^{\lambda}$
6. $\mathfrak{N}_1^{\lambda_1} \otimes \mathfrak{N}_1^{\lambda_2} = \mathfrak{N}_1^{(\lambda_1 \oplus \lambda_2)}$

Proof:

$$\mathfrak{N}_1 \oplus \mathfrak{N}_2 = \begin{pmatrix} \left[1-exp^{-\left(\left(-\ln\left(1-\left(x^{l}_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{l}_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(x^{u}_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{u}_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[1-exp^{-\left(\left(-\ln\left(1-\left(\omega^{l}_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\omega^{l}_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(\omega^{u}_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\omega^{u}_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\left(-\ln\left(x^{l}_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(x^{l}_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(x^{u}_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(x^{u}_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\left(-\ln\left(\varphi^{l}_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(\varphi^{l}_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(\varphi^{u}_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(\varphi^{u}_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\left(-\ln\left(x^{l}_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(x^{l}_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(x^{u}_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(x^{u}_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\left(-\ln\left(\psi^{l}_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(\psi^{l}_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(\psi^{u}_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(\psi^{u}_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]} \end{pmatrix}$$

$$= \begin{pmatrix} \left[1-exp^{-\left(\left(-\ln\left(1-\left(x_{m_{\mathfrak{N}_2}}(t)\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x_{m_{\mathfrak{N}_1}}(t)\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(x_{m_{\mathfrak{N}_2}}(t)\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x_{m_{\mathfrak{N}_1}}(t)\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[1-exp^{-\left(\left(-\ln\left(1-\left(\omega^{l}_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\omega^{l}_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(\omega^{u}_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\omega^{u}_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\left(-\ln\left(x^{l}_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}+\left(-\ln\left(x^{l}_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(x^{u}_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}+\left(-\ln\left(x^{u}_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\left(-\ln\left(\varphi^{l}_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}+\left(-\ln\left(\varphi^{l}_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(\varphi^{u}_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}+\left(-\ln\left(\varphi^{u}_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\left(-\ln\left(x^{l}_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}+\left(-\ln\left(x^{l}_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(x^{u}_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}+\left(-\ln\left(x^{u}_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\left(-\ln\left(\psi^{l}_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}+\left(-\ln\left(\psi^{l}_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(\psi^{l}_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}+\left(-\ln\left(\psi^{l}_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]} \end{pmatrix}$$

$$= \mathfrak{N}_2 \oplus \mathfrak{N}_1$$

$$
\mathfrak{N}_1 \otimes \mathfrak{N}_2 = \left(
\begin{array}{c}
\left[exp^{-\left(\left(-\ln\left(x^{l}_{m\mathfrak{N}_1}\right)\right)^{\lambda}+\left(-\ln\left(x^{l}_{m\mathfrak{N}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(x^{u}_{m\mathfrak{N}_1}\right)\right)^{\lambda}+\left(-\ln\left(x^{u}_{m\mathfrak{N}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[exp^{-\left(\left(-\ln\left(\omega^{l}_{m\mathfrak{N}_1}\right)\right)^{\lambda}+\left(-\ln\left(\omega^{l}_{m\mathfrak{N}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(\omega^{u}_{m\mathfrak{N}_1}\right)\right)^{\lambda}+\left(-\ln\left(\omega^{u}_{m\mathfrak{N}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\
\left[1-exp^{-\left(\left(-\ln\left(1-\left(x^{l}_{a\mathfrak{N}_1}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{l}_{a\mathfrak{N}_2}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(x^{u}_{a\mathfrak{N}_1}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{u}_{a\mathfrak{N}_2}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[1-exp^{-\left(\left(-\ln\left(1-\left(\varphi^{l}_{a\mathfrak{N}_1}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\varphi^{l}_{a\mathfrak{N}_2}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(\varphi^{u}_{a\mathfrak{N}_1}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\varphi^{u}_{a\mathfrak{N}_2}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\
\left[1-exp^{-\left(\left(-\ln\left(1-\left(x^{l}_{n\mathfrak{N}_1}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{l}_{n\mathfrak{N}_2}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(x^{u}_{n\mathfrak{N}_1}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{u}_{n\mathfrak{N}_2}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[1-exp^{-\left(\left(-\ln\left(1-\left(\psi^{l}_{n\mathfrak{N}_1}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\psi^{l}_{n\mathfrak{N}_2}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(\psi^{u}_{n\mathfrak{N}_1}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\psi^{u}_{n\mathfrak{N}_2}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}
\end{array}
\right)
$$

$$
\left(
\begin{array}{c}
\left[exp^{-\left(\left(-\ln\left(x^{l}_{m\mathfrak{N}_2}\right)\right)^{\lambda}+\left(-\ln\left(x^{l}_{m\mathfrak{N}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(x^{u}_{m\mathfrak{N}_2}\right)\right)^{\lambda}+\left(-\ln\left(x^{u}_{m\mathfrak{N}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[exp^{-\left(\left(-\ln\left(\omega^{l}_{m\mathfrak{N}_2}\right)\right)^{\lambda}+\left(-\ln\left(\omega^{l}_{m\mathfrak{N}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(\omega^{u}_{m\mathfrak{N}_2}\right)\right)^{\lambda}+\left(-\ln\left(\omega^{u}_{m\mathfrak{N}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\
\left[1-exp^{-\left(\left(-\ln\left(1-\left(x^{l}_{a\mathfrak{N}_2}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{l}_{a\mathfrak{N}_1}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(x^{u}_{a\mathfrak{N}_2}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{u}_{a\mathfrak{N}_1}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[1-exp^{-\left(\left(-\ln\left(1-\left(\varphi^{l}_{a\mathfrak{N}_2}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\varphi^{l}_{a\mathfrak{N}_1}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(\varphi^{u}_{a\mathfrak{N}_2}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\varphi^{u}_{a\mathfrak{N}_1}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\
\left[1-exp^{-\left(\left(-\ln\left(1-\left(x^{l}_{n\mathfrak{N}_2}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{l}_{n\mathfrak{N}_1}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(x^{u}_{n\mathfrak{N}_2}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^{u}_{n\mathfrak{N}_1}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[1-exp^{-\left(\left(-\ln\left(1-\left(\psi^{l}_{n\mathfrak{N}_2}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\psi^{l}_{n\mathfrak{N}_1}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(\psi^{u}_{n\mathfrak{N}_2}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\psi^{u}_{n\mathfrak{N}_1}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}
\end{array}
\right)
$$

$$
= \mathfrak{N}_2 \otimes \mathfrak{N}_1
$$

$$\lambda(\mathfrak{N}_1 \oplus \mathfrak{N}_2) = \lambda \begin{pmatrix} \left[1 - exp^{-\left(\left(-\ln\left(1-\left(x^l_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda} + \left(-\ln\left(1-\left(x^l_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1 - exp^{-\left(\left(-\ln\left(1-\left(x^u_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda} + \left(-\ln\left(1-\left(x^u_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}} \right] \\ exp^{j\left[1-exp^{-\left(\left(-\ln\left(1-\left(\omega^l_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda} + \left(-\ln\left(1-\left(\omega^l_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(\omega^u_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda} + \left(-\ln\left(1-\left(\omega^u_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\left(-\ln\left(x^l_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(x^l_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(x^u_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(x^u_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}} \right] \\ exp^{j\left[exp^{-\left(\left(-\ln\left(\varphi^l_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(\varphi^l_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(\varphi^u_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(\varphi^u_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\left(-\ln\left(x^l_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(x^l_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(x^u_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(x^u_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}} \right] \\ exp^{j\left[exp^{-\left(\left(-\ln\left(\psi^l_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(\psi^l_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(\psi^u_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(\psi^u_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]} \end{pmatrix}$$

$$= \begin{pmatrix} \left[1 - exp^{-\left(\lambda\left(\left(-\ln\left(1-\left(x^l_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda} + \left(-\ln\left(1-\left(x^l_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, 1 - exp^{-\left(\lambda\left(\left(-\ln\left(1-\left(x^u_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda} + \left(-\ln\left(1-\left(x^u_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)} \right] \\ exp^{j\left[1-exp^{-\left(\lambda\left(\left(-\ln\left(1-\left(\omega^l_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda} + \left(-\ln\left(1-\left(\omega^l_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, 1-exp^{-\left(\lambda\left(\left(-\ln\left(1-\left(\omega^u_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda} + \left(-\ln\left(1-\left(\omega^u_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right]}, \\ \left[exp^{-\left(\lambda\left(\left(-\ln\left(x^l_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(x^l_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, exp^{-\left(\lambda\left(\left(-\ln\left(x^u_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(x^u_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)} \right] \\ exp^{j\left[exp^{-\left(\lambda\left(\left(-\ln\left(\varphi^l_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(\varphi^l_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, exp^{-\left(\lambda\left(\left(-\ln\left(\varphi^u_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(\varphi^u_{a_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right]}, \\ \left[exp^{-\left(\lambda\left(\left(-\ln\left(x^l_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(x^l_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, exp^{-\left(\lambda\left(\left(-\ln\left(x^u_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(x^u_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)} \right] \\ exp^{j\left[exp^{-\left(\lambda\left(\left(-\ln\left(\psi^l_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(\psi^l_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, exp^{-\left(\lambda\left(\left(-\ln\left(\psi^u_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda} + \left(-\ln\left(\psi^u_{n_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right]} \end{pmatrix}$$

$$
= \begin{pmatrix}
\left[1-exp^{-\left(\gimel\left(\left(-\ln\left(1-\left(x^{l}_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\gimel}\right)^{1/\gimel}\right)}, 1-exp^{-\left(\gimel\left(\left(-\ln\left(1-\left(x^{u}_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\gimel}\right)^{1/\gimel}\right)}\right] \\
exp^{j\left[1-exp^{-\left(\gimel\left(\left(-\ln\left(1-\left(\omega^{l}_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\gimel}\right)^{1/\gimel}\right)}, 1-exp^{-\left(\gimel\left(\left(-\ln\left(1-\left(\omega^{u}_{m_{\mathfrak{N}_1}}\right)\right)\right)^{\gimel}\right)^{1/\gimel}\right)}\right]}, \\
\left[exp^{-\left(\gimel\left(\left(-\ln\left(x^{l}_{a_{\mathfrak{N}_1}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}, exp^{-\left(\gimel\left(\left(-\ln\left(x^{u}_{a_{\mathfrak{N}_1}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}\right] exp^{j\left[exp^{-\left(\gimel\left(\left(-\ln\left(\varphi^{l}_{a_{\mathfrak{N}_1}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}, exp^{-\left(\gimel\left(\left(-\ln\left(\varphi^{u}_{a_{\mathfrak{N}_1}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}\right]}, \\
\left[exp^{-\left(\gimel\left(\left(-\ln\left(x^{l}_{n_{\mathfrak{N}_1}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}, exp^{-\left(\gimel\left(\left(-\ln\left(x^{u}_{n_{\mathfrak{N}_1}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}\right] exp^{j\left[exp^{-\left(\gimel\left(\left(-\ln\left(\psi^{l}_{n_{\mathfrak{N}_1}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}, exp^{-\left(\gimel\left(\left(-\ln\left(\psi^{u}_{n_{\mathfrak{N}_1}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}\right]}
\end{pmatrix}
$$

$$
\oplus \begin{pmatrix}
\left[1-exp^{-\left(\gimel\left(\left(-\ln\left(1-\left(x^{l}_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\gimel}\right)^{1/\gimel}\right)}, 1-exp^{-\left(\gimel\left(\left(-\ln\left(1-\left(x^{u}_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\gimel}\right)^{1/\gimel}\right)}\right] \\
exp^{j\left[1-exp^{-\left(\gimel\left(\left(-\ln\left(1-\left(\omega^{l}_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\gimel}\right)^{1/\gimel}\right)}, 1-exp^{-\left(\gimel\left(\left(-\ln\left(1-\left(\omega^{u}_{m_{\mathfrak{N}_2}}\right)\right)\right)^{\gimel}\right)^{1/\gimel}\right)}\right]}, \\
\left[exp^{-\left(\gimel\left(\left(-\ln\left(x^{l}_{a_{\mathfrak{N}_2}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}, exp^{-\left(\gimel\left(\left(-\ln\left(x^{u}_{a_{\mathfrak{N}_2}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}\right] exp^{j\left[exp^{-\left(\gimel\left(\left(-\ln\left(\varphi^{l}_{a_{\mathfrak{N}_2}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}, exp^{-\left(\gimel\left(\left(-\ln\left(\varphi^{u}_{a_{\mathfrak{N}_2}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}\right]}, \\
\left[exp^{-\left(\gimel\left(\left(-\ln\left(x^{l}_{n_{\mathfrak{N}_2}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}, exp^{-\left(\gimel\left(\left(-\ln\left(x^{u}_{n_{\mathfrak{N}_2}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}\right] exp^{j\left[exp^{-\left(\gimel\left(\left(-\ln\left(\psi^{l}_{n_{\mathfrak{N}_2}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}, exp^{-\left(\gimel\left(\left(-\ln\left(\psi^{u}_{n_{\mathfrak{N}_2}}\right)\right)^{\gimel}\right)^{1/\gimel}\right)}\right]}
\end{pmatrix}
$$

$$= \gimel\mathfrak{N}_1 \oplus \gimel\mathfrak{N}_2$$

$$(\lambda_1 \oplus \lambda_2)\mathfrak{N} = \begin{pmatrix} \left[1 - exp^{-\left(\lambda_1\left(\left(-\ln\left(1-\left(x^{l}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, 1 - exp^{-\left(\lambda_1\left(\left(-\ln\left(1-\left(x^{u}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right] \\ exp^{j\left[1-exp^{-\left(\lambda_1\left(\left(-\ln\left(1-\left(\omega^{l}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)},1-exp^{-\left(\lambda_1\left(\left(-\ln\left(1-\left(\omega^{u}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right]}, \\ \left[exp^{-\left(\lambda_1\left(\left(-\ln\left(x^{l}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, exp^{-\left(\lambda_1\left(\left(-\ln\left(x^{u}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right] exp^{j\left[exp^{-\left(\lambda_1\left(\left(-\ln\left(\varphi^{l}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)},exp^{-\left(\lambda_1\left(\left(-\ln\left(\varphi^{u}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right]}, \\ \left[exp^{-\left(\lambda_1\left(\left(-\ln\left(x^{l}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, exp^{-\left(\lambda_1\left(\left(-\ln\left(x^{u}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right] exp^{j\left[exp^{-\left(\lambda_1\left(\left(-\ln\left(\psi^{l}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)},exp^{-\left(\lambda_1\left(\left(-\ln\left(\psi^{u}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right]} \end{pmatrix}$$

$$\oplus \begin{pmatrix} \left[1 - exp^{-\left(\lambda_2\left(\left(-\ln\left(1-\left(x^{l}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, 1 - exp^{-\left(\lambda_2\left(\left(-\ln\left(1-\left(x^{u}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right] \\ exp^{j\left[1-exp^{-\left(\lambda_2\left(\left(-\ln\left(1-\left(\omega^{l}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)},1-exp^{-\left(\lambda_2\left(\left(-\ln\left(1-\left(\omega^{u}_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right]}, \\ \left[exp^{-\left(\lambda_2\left(\left(-\ln\left(x^{l}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, exp^{-\left(\lambda_2\left(\left(-\ln\left(x^{u}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right] exp^{j\left[exp^{-\left(\lambda_2\left(\left(-\ln\left(\varphi^{l}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)},exp^{-\left(\lambda_2\left(\left(-\ln\left(\varphi^{u}_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right]}, \\ \left[exp^{-\left(\lambda_2\left(\left(-\ln\left(x^{l}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, exp^{-\left(\lambda_2\left(\left(-\ln\left(x^{u}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right] exp^{j\left[exp^{-\left(\lambda_2\left(\left(-\ln\left(\psi^{l}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)},exp^{-\left(\lambda_2\left(\left(-\ln\left(\psi^{u}_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right]} \end{pmatrix}$$

$$= \begin{pmatrix} \left[1-exp^{-\left((\lambda_1+\lambda_2)\left(\left(-\ln\left(1-\left(x^l_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, 1-exp^{-\left((\lambda_1+\lambda_2)\left(\left(-\ln\left(1-\left(x^u_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right] exp^{j\left[1-exp^{-\left((\lambda_1+\lambda_2)\left(\left(-\ln\left(1-\left(\omega^l_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, 1-exp^{-\left((\lambda_1+\lambda_2)\left(\left(-\ln\left(1-\left(\omega^u_{m_{\mathfrak{N}}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right]}, \\ \left[exp^{-\left((\lambda_1+\lambda_2)\left(\left(-\ln\left(x^l_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, exp^{-\left((\lambda_1+\lambda_2)\left(\left(-\ln\left(x^u_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right] exp^{j\left[exp^{-\left((\lambda_1+\lambda_2)\left(\left(-\ln\left(\varphi^l_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, exp^{-\left((\lambda_1+\lambda_2)\left(\left(-\ln\left(\varphi^u_{a_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right]}, \\ \left[exp^{-\left((\lambda_1+\lambda_2)\left(\left(-\ln\left(x^l_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, exp^{-\left((\lambda_1+\lambda_2)\left(\left(-\ln\left(x^u_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right] exp^{j\left[exp^{-\left((\lambda_1+\lambda_2)\left(\left(-\ln\left(\psi^l_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}, exp^{-\left((\lambda_1+\lambda_2)\left(\left(-\ln\left(\psi^u_{n_{\mathfrak{N}}}\right)\right)^{\lambda}\right)^{1/\lambda}\right)}\right]} \end{pmatrix}$$

$$= \lambda_1\mathfrak{N} \oplus \lambda_2\mathfrak{N}$$

$$(\mathfrak{N}_1 \otimes \mathfrak{N}_2)^{\lambda} = \begin{pmatrix} \left[exp^{-\left(\left(-\ln\left(x^l_{m_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(x^u_{m_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(x^l_{m_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(x^u_{m_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] exp^{j\left[exp^{-\left(\left(-\ln\left(\omega^l_{m_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(\omega^l_{m_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\left(-\ln\left(\omega^u_{m_{\mathfrak{N}_1}}\right)\right)^{\lambda}+\left(-\ln\left(\omega^u_{m_{\mathfrak{N}_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[1-exp^{-\left(\left(-\ln\left(1-\left(x^l_{a_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^l_{a_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(x^u_{a_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^u_{a_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right] exp^{j\left[1-exp^{-\left(\left(-\ln\left(1-\left(\varphi^l_{a_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\varphi^l_{a_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(\varphi^u_{a_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\varphi^u_{a_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[1-exp^{-\left(\left(-\ln\left(1-\left(x^l_{n_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^l_{n_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(x^u_{n_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(x^u_{n_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right] exp^{j\left[1-exp^{-\left(\left(-\ln\left(1-\left(\psi^l_{n_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\psi^l_{n_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\left(-\ln\left(1-\left(\psi^u_{n_{\mathfrak{N}_1}}\right)\right)\right)^{\lambda}+\left(-\ln\left(1-\left(\psi^u_{n_{\mathfrak{N}_2}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]} \end{pmatrix}^{\lambda}$$

$$= \mathfrak{N}^{\lambda_1} \otimes \mathfrak{N}^{\lambda_2}.$$

4 | AA AGGREGATION OPERATORS FOR IVCSVNVS

The well-known theory of score function is arranged by: In this section, we derived the concept of the IVCSVNAAWA operator, IVCSVNAAOWA operator, and IVCSVNAAHA operator, and examined or presented their major properties, such as idempotency, boundedness, and monotonicity.

Definition 6: Let $\mathfrak{N}_j = \begin{pmatrix} \left[x^l_{m_{\mathfrak{N}_j}}, x^u_{m_{\mathfrak{N}_j}}\right].exp^{j\left[\omega^l_{m_{\mathfrak{N}_j}},\omega^u_{m_{\mathfrak{N}_j}}\right]}, \left[x^l_{a_{\mathfrak{N}_j}}, x^u_{a_{\mathfrak{N}_j}}\right]. \\ exp^{j\left[\varphi^l_{a_{\mathfrak{N}_j}},\varphi^u_{a_{\mathfrak{N}_j}}\right]}, \left[x^l_{n_{\mathfrak{N}_j}}, x^u_{n_{\mathfrak{N}_j}}\right].exp^{j\left[\psi^l_{n_{\mathfrak{N}_j}},\psi^u_{n_{\mathfrak{N}_j}}\right]} \end{pmatrix}, j = 1,2,\dots,n$ be the finite family of IVCSVNVs. Then

$$\text{IVCSVNAAWA}_{\varpi}(\mathfrak{N}_1, \mathfrak{N}_2, \dots, \mathfrak{N}_n) = \sum_{j=1}^{n}\left(\varpi_j \mathfrak{N}_j\right) = (\varpi_1\mathfrak{N}_1 \oplus \varpi_2\mathfrak{N}_2 \oplus \dots \oplus \varpi_n\mathfrak{N}_n) \tag{12}$$

Represented the IVCSVNAAWA and noticed that $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_n)^T$ with $\varpi_j \in [0,1]$ and $\sum_{j=1}^{n}\varpi_j = 1$ stated the weight vectors.

Theorem 2: Let $\mathfrak{N}_j = \begin{pmatrix} \left[x^l_{m_{\mathfrak{N}_j}}, x^u_{m_{\mathfrak{N}_j}}\right].exp^{j\left[\omega^l_{m_{\mathfrak{N}_j}},\omega^u_{m_{\mathfrak{N}_j}}\right]}, \left[x^l_{a_{\mathfrak{N}_j}}, x^u_{a_{\mathfrak{N}_j}}\right]. \\ exp^{j\left[\varphi^l_{a_{\mathfrak{N}_j}},\varphi^u_{a_{\mathfrak{N}_j}}\right]}, \left[x^l_{n_{\mathfrak{N}_j}}, x^u_{n_{\mathfrak{N}_j}}\right].exp^{j\left[\psi^l_{n_{\mathfrak{N}_j}},\psi^u_{n_{\mathfrak{N}_j}}\right]} \end{pmatrix}, j = 1,2,\dots,n$ be the finite family of IVCSVNVs. Then in the presence of the data in Equation (12), we discover the theory in Equation (13), such as

$$\text{IVCSVNAAWA}_{\varpi}(\mathfrak{N}_1, \mathfrak{N}_2, \dots, \mathfrak{N}_n) = \sum_{j=1}^{n}\left(\varpi_j \mathfrak{N}_j\right) =$$

Proof: If $n = 2$, then we have

$$\varpi_1\mathfrak{N}_1 = \begin{pmatrix} \left[1 - exp^{-\left(\varpi_1\left(-ln\left(1-x^l_{m_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}, 1 - exp^{-\left(\varpi_1\left(-ln\left(1-x^u_{m_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[1-exp^{-\left(\varpi_1\left(-ln\left(1-\omega^l_{m_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\varpi_1\left(-ln\left(1-\omega^u_{m_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\varpi_1\left(-ln\left(x^l_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-ln\left(x^u_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\varpi_1\left(-ln\left(\varphi^l_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-ln\left(\varphi^u_{a_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\varpi_1\left(-ln\left(x^l_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-ln\left(x^u_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\varpi_1\left(-ln\left(\psi^l_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-ln\left(\psi^u_{n_{\mathfrak{N}_1}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]} \end{pmatrix}$$

$$\varpi_2\mathfrak{R}_2 = \begin{pmatrix} \left[1 - exp^{-\left(\varpi_2\left(-\ln\left(1-x^l_{m\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, 1 - exp^{-\left(\varpi_2\left(-\ln\left(1-x^u_{m\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[1-exp^{-\left(\varpi_1\left(-\ln\left(1-\omega^l_{m\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\varpi_1\left(-\ln\left(1-\omega^u_{m\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\varpi_1\left(-\ln\left(x^l_{a\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-\ln\left(x^u_{a\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\varpi_1\left(-\ln\left(\varphi^l_{a\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-\ln\left(\varphi^u_{a\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\varpi_1\left(-\ln\left(x^l_{n\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-\ln\left(x^u_{n\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\varpi_1\left(-\ln\left(\psi^l_{n\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-\ln\left(\psi^u_{n\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]} \end{pmatrix}$$

Then, we have

$$IVCSVNAAWA_{\varpi}(\mathfrak{R}_1, \mathfrak{R}_2) = \varpi_1\mathfrak{R}_1 \oplus \varpi_2\mathfrak{R}_2$$

$$= \begin{pmatrix} \left[1 - exp^{-\left(\varpi_1\left(-\ln\left(1-x^l_{m\mathfrak{R}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}, 1 - exp^{-\left(\varpi_1\left(-\ln\left(1-x^u_{m\mathfrak{R}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[1-exp^{-\left(\varpi_1\left(-\ln\left(1-\omega^l_{m\mathfrak{R}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\varpi_1\left(-\ln\left(1-\omega^u_{m\mathfrak{R}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\varpi_1\left(-\ln\left(x^l_{a\mathfrak{R}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-\ln\left(x^u_{a\mathfrak{R}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\varpi_1\left(-\ln\left(\varphi^l_{a\mathfrak{R}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-\ln\left(\varphi^u_{a\mathfrak{R}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\varpi_1\left(-\ln\left(x^l_{n\mathfrak{R}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-\ln\left(x^u_{n\mathfrak{R}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\varpi_1\left(-\ln\left(\psi^l_{n\mathfrak{R}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-\ln\left(\psi^u_{n\mathfrak{R}_1}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]} \end{pmatrix}$$

$$\oplus\left(\begin{array}{l}\left[1-exp^{-\left(\varpi_2\left(-\ln\left(1-x^{l}_{m_{\Re_2}}\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\varpi_2\left(-\ln\left(1-x^{u}_{m_{\Re_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]\\ exp^{j\left[1-exp^{-\left(\varpi_1\left(-\ln\left(1-\omega^{l}_{m_{\Re_2}}\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\varpi_1\left(-\ln\left(1-\omega^{u}_{m_{\Re_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]},\\ \left[exp^{-\left(\varpi_1\left(-\ln\left(x^{l}_{a_{\Re_2}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\varpi_1\left(-\ln\left(x^{u}_{a_{\Re_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]\\ exp^{j\left[exp^{-\left(\varpi_1\left(-\ln\left(\varphi^{l}_{a_{\Re_2}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\varpi_1\left(-\ln\left(\varphi^{u}_{a_{\Re_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]},\\ \left[exp^{-\left(\varpi_1\left(-\ln\left(x^{l}_{n_{\Re_2}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\varpi_1\left(-\ln\left(x^{u}_{n_{\Re_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]\\ exp^{j\left[exp^{-\left(\varpi_1\left(-\ln\left(\psi^{l}_{n_{\Re_2}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\varpi_1\left(-\ln\left(\psi^{u}_{n_{\Re_2}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}\end{array}\right)$$

$$=\left(\begin{array}{l}\left[1-exp^{-\left(\sum_{j=1}^{n}\varpi_j\left(-\ln\left(1-x^{l}_{m_{\Re_j}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\sum_{j=1}^{n}\varpi_j\left(-\ln\left(1-x^{u}_{m_{\Re_j}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]\\ exp^{j\left[1-exp^{-\left(\sum_{j=1}^{n}\varpi_j\left(-\ln\left(1-\omega^{l}_{m_{\Re_j}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\sum_{j=1}^{n}\varpi_j\left(-\ln\left(1-\omega^{u}_{m_{\Re_j}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]},\\ \left[exp^{-\left(\sum_{j=1}^{n}\varpi_j\left(-\ln\left(x^{l}_{a_{\Re_j}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{n}\varpi_j\left(-\ln\left(x^{u}_{a_{\Re_j}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]\\ exp^{j\left[exp^{-\left(\sum_{j=1}^{n}\varpi_j\left(-\ln\left(\varphi^{l}_{a_{\Re_j}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{n}\varpi_j\left(-\ln\left(\varphi^{u}_{a_{\Re_j}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]},\\ \left[exp^{-\left(\sum_{j=1}^{n}\varpi_j\left(-\ln\left(x^{l}_{n_{\Re_j}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{n}\varpi_j\left(-\ln\left(x^{u}_{n_{\Re_j}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]\\ exp^{j\left[exp^{-\left(\sum_{j=1}^{n}\varpi_j\left(-\ln\left(\psi^{l}_{n_{\Re_j}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{n}\varpi_j\left(-\ln\left(\psi^{u}_{n_{\Re_j}}\right)\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}\end{array}\right)$$

$$= \left(\begin{array}{c} \left[1 - exp^{-\left(\varpi_1\left(-\ln\left(1-x^l_{m\mathfrak{R}_1}\right)\right)^{\lambda}+\varpi_2\left(-\ln\left(1-x^l_{m\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, 1 - exp^{-\left(\varpi_1\left(-\ln\left(1-x^u_{m\mathfrak{R}_1}\right)\right)^{\lambda}+\varpi_2\left(-\ln\left(1-x^u_{m\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[1-exp^{-\left(\varpi_1\left(-\ln\left(1-\omega^l_{m\mathfrak{R}_1}\right)\right)^{\lambda}+\varpi_2\left(-\ln\left(1-\omega^l_{m\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\varpi_1\left(-\ln\left(1-\omega^u_{m\mathfrak{R}_1}\right)\right)^{\lambda}+\varpi_2\left(-\ln\left(1-\omega^u_{m\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\varpi_1\left(-\ln\left(x^l_{a\mathfrak{R}_1}\right)\right)^{\lambda}+\varpi_2\left(-\ln\left(x^l_{a\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-\ln\left(x^u_{a\mathfrak{R}_1}\right)\right)^{\lambda}+\varpi_2\left(-\ln\left(x^u_{a\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\varpi_1\left(-\ln\left(\varphi^l_{a\mathfrak{R}_1}\right)\right)^{\lambda}+\varpi_2\left(-\ln\left(\varphi^l_{a\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-\ln\left(\varphi^u_{a\mathfrak{R}_1}\right)\right)^{\lambda}+\varpi_2\left(-\ln\left(\varphi^u_{a\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\varpi_1\left(-\ln\left(x^l_{n\mathfrak{R}_1}\right)\right)^{\lambda}+\varpi_2\left(-\ln\left(x^l_{n\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-\ln\left(x^u_{n\mathfrak{R}_1}\right)\right)^{\lambda}+\varpi_2\left(-\ln\left(x^u_{n\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\varpi_1\left(-\ln\left(\psi^l_{n\mathfrak{R}_1}\right)\right)^{\lambda}+\varpi_2\left(-\ln\left(\psi^l_{n\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\varpi_1\left(-\ln\left(\psi^u_{n\mathfrak{R}_1}\right)\right)^{\lambda}+\varpi_2\left(-\ln\left(\psi^u_{n\mathfrak{R}_2}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]} \end{array} \right)$$

$$= \left(\begin{array}{c} \left[1 - exp^{-\left(\sum_{j=1}^{2}\varpi_j\left(-\ln\left(1-x^l_{m\mathfrak{R}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}, 1 - exp^{-\left(\sum_{j=1}^{2}\varpi_j\left(-\ln\left(1-x^u_{m\mathfrak{R}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[1-exp^{-\left(\sum_{j=1}^{2}\varpi_j\left(-\ln\left(1-\omega^l_{m\mathfrak{R}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}, 1-exp^{-\left(\sum_{j=1}^{2}\varpi_j\left(-\ln\left(1-\omega^u_{m\mathfrak{R}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\sum_{j=1}^{2}\varpi_j\left(-\ln\left(x^l_{a\mathfrak{R}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\sum_{j=1}^{2}\varpi_j\left(-\ln\left(x^u_{a\mathfrak{R}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\sum_{j=1}^{2}\varpi_j\left(-\ln\left(\varphi^l_{a\mathfrak{R}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\sum_{j=1}^{2}\varpi_j\left(-\ln\left(\varphi^u_{a\mathfrak{R}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\ \left[exp^{-\left(\sum_{j=1}^{2}\varpi_j\left(-\ln\left(x^l_{n\mathfrak{R}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\sum_{j=1}^{2}\varpi_j\left(-\ln\left(x^u_{n\mathfrak{R}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\ exp^{j\left[exp^{-\left(\sum_{j=1}^{2}\varpi_j\left(-\ln\left(\psi^l_{n\mathfrak{R}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}, exp^{-\left(\sum_{j=1}^{2}\varpi_j\left(-\ln\left(\psi^u_{n\mathfrak{R}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]} \end{array} \right)$$

We derive the correct theory; we considered that the data in Equation (13) is also correct for $n = \rho$, then we have

$$\text{IVCSVNAAWA}_{\varpi}\left(\mathfrak{R}_1, \mathfrak{R}_2, \dots, \mathfrak{R}_{\rho}\right) = \sum_{j=1}^{\rho}\left(\varpi_j \mathfrak{R}_j\right)$$

$$
=\left(\begin{array}{l}
\left[1-exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(1-x^{l}_{m\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(1-x^{u}_{m\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[1-exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(1-\omega^{l}_{m\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(1-\omega^{u}_{m\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\
\left[exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(x^{l}_{a\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(x^{u}_{a\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(\varphi^{l}_{a\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(\varphi^{u}_{a\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\
\left[exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(x^{l}_{n\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(x^{u}_{n\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(\psi^{l}_{n\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(\psi^{u}_{n\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}
\end{array}\right)
$$

Then, we evaluate it for $n=\rho+1$, such as

$$
\mathrm{IVCSVNAAWA}_{\varpi}(\mathfrak{N}_1,\mathfrak{N}_2,\dots,\mathfrak{N}_\rho,\mathfrak{N}_{\rho+1})=\sum_{j=1}^{\rho}(\varpi_j\mathfrak{N}_j)\oplus(\varpi_{\rho+1}\mathfrak{N}_{\rho+1})
$$

$$
=\left(\begin{array}{l}
\left[1-exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(1-x^{l}_{m\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(1-x^{u}_{m\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[1-exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(1-\omega^{l}_{m\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(1-\omega^{u}_{m\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\
\left[exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(x^{l}_{a\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(x^{u}_{a\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(\varphi^{l}_{a\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(\varphi^{u}_{a\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\
\left[exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(x^{l}_{n\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(x^{u}_{n\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(\psi^{l}_{n\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{\rho}\varpi_j\left(-\ln\left(\psi^{u}_{n\mathfrak{N}_j}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}
\end{array}\right)
$$

$$\oplus\left(\begin{array}{c}\left(1-exp^{-\left(\varpi_{\rho+1}\left(-\ln\left(1-x_{m_{\mathfrak{N}_{\rho+1}}}\right)\right)^{\gimel}\right)^{1/\gimel}}\right)exp^{j\left(1-exp^{-\left(\varpi_{\rho+1}\left(-\ln\left(1-\omega_{m_{\mathfrak{N}_{\rho+1}}}\right)\right)^{\gimel}\right)^{1/\gimel}}\right)},\\ \left(exp^{-\left(\varpi_{\rho+1}\left(-\ln\left(x_{a_{\mathfrak{N}_{\rho+1}}}\right)\right)^{\gimel}\right)^{1/\gimel}}\right)exp^{j\left(exp^{-\left(\varpi_{\rho+1}\left(-\ln\left(\varphi_{a_{\mathfrak{N}_{\rho+1}}}\right)\right)^{\gimel}\right)^{1/\gimel}}\right)},\\ \left(exp^{-\left(\varpi_{\rho+1}\left(-\ln\left(x_{n_{\mathfrak{N}_{\rho+1}}}\right)\right)^{\gimel}\right)^{1/\gimel}}\right)exp^{j\left(exp^{-\left(\varpi_{\rho+1}\left(-\ln\left(\psi_{n_{\mathfrak{N}_{\rho+1}}}\right)\right)^{\gimel}\right)^{1/\gimel}}\right)}\end{array}\right)$$

$$=\left(\begin{array}{c}\left[1-exp^{-\left(\sum_{j=1}^{\rho+1}\varpi_j\left(-ln\left(1-x^l_{m_{\mathfrak{N}_j}}\right)\right)\right)^{\gimel\,1/\gimel}},1-exp^{-\left(\sum_{j=1}^{\rho+1}\varpi_j\left(-ln\left(1-x^u_{m_{\mathfrak{N}_j}}\right)\right)\right)^{\gimel\,1/\gimel}}\right]\\ exp^{j\left[1-exp^{-\left(\sum_{j=1}^{\rho+1}\varpi_j\left(-ln\left(1-\omega^l_{m_{\mathfrak{N}_j}}\right)\right)\right)^{\gimel\,1/\gimel}},1-exp^{-\left(\sum_{j=1}^{\rho+!}\varpi_j\left(-ln\left(1-\omega^u_{m_{\mathfrak{N}_j}}\right)\right)\right)^{\gimel\,1/\gimel}}\right]},\\ \left[exp^{-\left(\sum_{j=1}^{\rho+1}\varpi_j\left(-ln\left(x^l_{a_{\mathfrak{N}_j}}\right)\right)\right)^{\gimel\,1/\gimel}},exp^{-\left(\sum_{j=1}^{\rho+1}\varpi_j\left(-ln\left(x^u_{a_{\mathfrak{N}_j}}\right)\right)\right)^{\gimel\,1/\gimel}}\right]\\ exp^{j\left[exp^{-\left(\sum_{j=1}^{\rho+1}\varpi_j\left(-ln\left(\varphi^l_{a_{\mathfrak{N}_j}}\right)\right)\right)^{\gimel\,1/\gimel}},exp^{-\left(\sum_{j=1}^{\rho+1}\varpi_j\left(-ln\left(\varphi^u_{a_{\mathfrak{N}_j}}\right)\right)\right)^{\gimel\,1/\gimel}}\right]},\\ \left[exp^{-\left(\sum_{j=1}^{\rho+1}\varpi_j\left(-ln\left(x^l_{n_{\mathfrak{N}_j}}\right)\right)\right)^{\gimel\,1/\gimel}},exp^{-\left(\sum_{j=1}^{\rho+1}\varpi_j\left(-ln\left(x^u_{n_{\mathfrak{N}_j}}\right)\right)\right)^{\gimel\,1/\gimel}}\right]\\ exp^{j\left[exp^{-\left(\sum_{j=1}^{\rho+1}\varpi_j\left(-ln\left(\psi^l_{n_{\mathfrak{N}_j}}\right)\right)\right)^{\gimel\,1/\gimel}},exp^{-\left(\sum_{j=1}^{\rho+1}\varpi_j\left(-ln\left(\psi^u_{n_{\mathfrak{N}_j}}\right)\right)\right)^{\gimel\,1/\gimel}}\right]}\end{array}\right)$$

Hence, we get the correct theory. Furthermore, we derive the theory of idempotency, monotonicity, and boundedness of the theory in Equation (11).

Theorem 3: If $\mathfrak{N}_j=\mathfrak{N}$, then

$$IVCSVNAAWA_{\varpi}(\mathfrak{N}_1,\mathfrak{N}_2,\dots,\mathfrak{N}_n)=\mathfrak{N} \tag{14}$$

Theorem 4: If $\mathfrak{N}^-$ =min$(\mathfrak{N}_1,\mathfrak{N}_2,\dots,\mathfrak{N}_n)$ and $\mathfrak{N}^+$ =max$(\mathfrak{N}_1,\mathfrak{N}_2,\dots,\mathfrak{N}_n)$, then

$$\mathfrak{N}^-\le IVCSVNAAWA_{\varpi}(\mathfrak{N}_1,\mathfrak{N}_2,\dots,\mathfrak{N}_n)\le\mathfrak{N}^+ \tag{15}$$

Theorem 5: If $\mathfrak{N}_j\le S_j\ \forall\ j$, then

$$IVCSVNAAWA_{\varpi}(\mathfrak{R}_1,\mathfrak{R}_2,\dots,\mathfrak{R}_n)\le IVCSVNAAWA_{\varpi}(S_1,S_2,\dots,S_n) \tag{16}$$

Definition 7: Let $\mathfrak{R}_j=\begin{pmatrix}\left[x^l_{m\mathfrak{R}_j},x^u_{m\mathfrak{R}_j}\right].exp^{j\left[\omega^l_{m\mathfrak{R}_j},\omega^u_{m\mathfrak{R}_j}\right]},\left[x^l_{a\mathfrak{R}_j},x^u_{a\mathfrak{R}_j}\right].\\ exp^{j\left[\varphi^l_{a\mathfrak{R}_j},\varphi^u_{a\mathfrak{R}_j}\right]},\left[x^l_{n\mathfrak{R}_j},x^u_{n\mathfrak{R}_j}\right].exp^{j\left[\psi^l_{n\mathfrak{R}_j},\psi^u_{n\mathfrak{R}_j}\right]}\end{pmatrix}, j=1,2,\dots,n$ be the finite family of IVCSVNVs. Then

$$\text{IVCSVNAAOWA}_{\varpi}(\mathfrak{R}_1,\mathfrak{R}_2,\dots,\mathfrak{R}_n)=\sum_{j=1}^{n}\left(\chi_j\mathfrak{R}_{\lambda(j)}\right)=\left(\varpi_1\mathfrak{R}_{\lambda(1)}\oplus\varpi_2\mathfrak{R}_{\lambda(2)}\oplus\dots\oplus\varpi_n\mathfrak{R}_{\lambda(n)}\right) \tag{17}$$

Represented the IVCSVNAAOWA and noticed that $\varpi=(\varpi_1,\varpi_2,\dots,\varpi_n)^T$ with $\varpi_j\in[0,1]$ and $\sum_{j=1}^{n}\varpi_j=1$ stated the weight vectors with $\lambda(j)\le\lambda(j-1)$.

Theorem 6: Let $\mathfrak{R}_j=\left(.exp^{j\left[\omega^l_{m\mathfrak{R}_j},\omega^u_{m\mathfrak{R}_j}\right]},\left[x^l_{a\mathfrak{R}_j},x^u_{a\mathfrak{R}_j}\right].exp^{j\left[\varphi^l_{a\mathfrak{R}_j},\varphi^u_{a\mathfrak{R}_j}\right]},\left[x^l_{n\mathfrak{R}_j},x^u_{n\mathfrak{R}_j}\right].exp^{j\left[\psi^l_{n\mathfrak{R}_j},\psi^u_{n\mathfrak{R}_j}\right]}\right), j=1,2,\dots,n$ be the finite family of IVCSVNVs. Then in the presence of the data in Equation (17), we discover the theory in Equation (18), such as

$$IVCSVNAAOWA_{\chi}(\mathfrak{R}_1,\mathfrak{R}_2,\dots,\mathfrak{R}_n)=\sum_{j=1}^{n}\left(\chi_j\mathfrak{R}_{\lambda(j)}\right)=$$
$$=\begin{pmatrix}\left[1-exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(1-x^l_{m\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(1-x^u_{m\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]\\ exp^{j\left[1-exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(1-\omega^l_{m\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(1-\omega^u_{m\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]},\\ \left[exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(x^l_{a\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(x^u_{a\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]\\ exp^{j\left[exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(\varphi^l_{a\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(\varphi^u_{a\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]},\\ \left[exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(x^l_{n\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(x^u_{n\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]\\ exp^{j\left[exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(\psi^l_{n\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(\psi^u_{n\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}\end{pmatrix} \tag{18}$$

Furthermore, we derive the theory of idempotency, monotonicity, and boundedness of the theory in Equation (18).

Theorem 7: If $\mathfrak{N}_j = \mathfrak{N}$, then

$$IVCSVNAAOWA_{\varpi}(\mathfrak{N}_1, \mathfrak{N}_2, \dots, \mathfrak{N}_n) = \mathfrak{N} \tag{19}$$

Theorem 8: If $\mathfrak{N}^- = \min(\mathfrak{N}_1, \mathfrak{N}_2, \dots, \mathfrak{N}_n)$ and $\mathfrak{N}^+ = \max(\mathfrak{N}_1, \mathfrak{N}_2, \dots, \mathfrak{N}_n)$, then

$$\mathfrak{N}^- \leq IVCSVNAAOWA_{\varpi}(\mathfrak{N}_1, \mathfrak{N}_2, \dots, \mathfrak{N}_n) \leq \mathfrak{N}^+ \tag{20}$$

Theorem 9: If $\mathfrak{N}_j \leq S_j \; \forall \, j$, then

$$IVCSVNAAOWA_{\varpi}(\mathfrak{N}_1, \mathfrak{N}_2, \dots, \mathfrak{N}_n) \leq IVCSVNAAOWA_{\varpi}(S_1, S_2, \dots, S_n) \tag{21}$$

Definition 8: Let $\mathfrak{N}_j = \left(\begin{array}{c} \left[x^l_{m_{\mathfrak{N}_j}}, x^u_{m_{\mathfrak{N}_j}}\right].exp^{j\left[\omega^l_{m_{\mathfrak{N}_j}}, \omega^u_{m_{\mathfrak{N}_j}}\right]}, \\ \left[x^l_{a_{\mathfrak{N}_j}}, x^u_{a_{\mathfrak{N}_j}}\right].exp^{j\left[\varphi^l_{a_{\mathfrak{N}_j}}, \varphi^u_{a_{\mathfrak{N}_j}}\right]}, \left[x^l_{n_{\mathfrak{N}_j}}, x^u_{n_{\mathfrak{N}_j}}\right].exp^{j\left[\psi^l_{n_{\mathfrak{N}_j}}, \psi^u_{n_{\mathfrak{N}_j}}\right]} \end{array} \right), j = 1,2, \dots, n$ be the finite family of IVCSVNVs. Then

$$IVCSVNAAHA_{\varpi,\chi}(\mathfrak{N}_1, \mathfrak{N}_2, \dots, \mathfrak{N}_n) = \sum_{j=1}^{n} \left(\chi_j \ddot{S}_{\lambda(j)}\right) = \left(\chi_1 \ddot{S}_{\lambda(1)} \oplus \chi_2 \ddot{S}_{\lambda(2)} \oplus \dots \oplus \chi_n \ddot{S}_{\lambda(n)}\right) \tag{22}$$

Represented the IVCSVNAAOWA and noticed that $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_n)^T$ with $\varpi_j \in [0,1]$ and $\sum_{j=1}^{n} \varpi_j = 1$ stated the weight vectors with $\ddot{S}_k = v\varpi_j \mathfrak{N}_j$ $(j = 1,2, \dots, n)$,$\left(\ddot{S}_{\lambda(1)}, \ddot{S}_{\lambda(2)}, \dots, \ddot{S}_{\lambda(n)}\right)$.

Theorem 10: Let $\mathfrak{N}_j = \left(\begin{array}{c} \left[x^l_{m_{\mathfrak{N}_j}}, x^u_{m_{\mathfrak{N}_j}}\right].exp^{j\left[\omega^l_{m_{\mathfrak{N}_j}}, \omega^u_{m_{\mathfrak{N}_j}}\right]}, \\ \left[x^l_{a_{\mathfrak{N}_j}}, x^u_{a_{\mathfrak{N}_j}}\right].exp^{j\left[\varphi^l_{a_{\mathfrak{N}_j}}, \varphi^u_{a_{\mathfrak{N}_j}}\right]}, \left[x^l_{n_{\mathfrak{N}_j}}, x^u_{n_{\mathfrak{N}_j}}\right].exp^{j\left[\psi^l_{n_{\mathfrak{N}_j}}, \psi^u_{n_{\mathfrak{N}_j}}\right]} \end{array} \right), j = 1,2, \dots, n$ be the finite family of IVCSVNVs. Then in the presence of the data in Equation (22), we discover the theory in Equation (23), such as

$$IVCSVNAAHA_{\varpi,\chi}(\mathfrak{R}_1,\mathfrak{R}_2,\dots,\mathfrak{R}_n)=\sum_{j=1}^{n}\left(\chi_j\ddot{S}_{\lambda(j)}\right)=$$

$$=\left(\begin{array}{c}
\left[1-exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(1-x^{l}_{\ddot{m}\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(1-x^{u}_{\ddot{m}\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[1-exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(1-\omega^{l}_{\ddot{m}\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}},1-exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(1-\omega^{u}_{\ddot{m}\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\
\left[exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(x^{l}_{\ddot{a}\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(x^{u}_{\ddot{a}\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(\varphi^{l}_{\ddot{a}\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(\varphi^{u}_{\ddot{a}\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}, \\
\left[exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(x^{l}_{\ddot{n}\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(x^{u}_{\ddot{n}\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right] \\
exp^{j\left[exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(\psi^{l}_{\ddot{n}\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}},exp^{-\left(\sum_{j=1}^{n}\chi_j\left(-\ln\left(\psi^{u}_{\ddot{n}\mathfrak{R}_{\lambda(j)}}\right)\right)^{\lambda}\right)^{1/\lambda}}\right]}
\end{array}\right) \quad (23)$$

Furthermore, we derive the theory of idempotency, monotonicity, and boundedness of the theory in Equation (23).

Theorem 11: If $\mathfrak{R}_j=\mathfrak{R}$, then

$$IVCSVNAAHA_{\varpi}(\mathfrak{R}_1,\mathfrak{R}_2,\dots,\mathfrak{R}_n)=\mathfrak{R} \quad (24)$$

Theorem 12: If $\mathfrak{R}^-$ =min$(\mathfrak{R}_1,\mathfrak{R}_2,\dots,\mathfrak{R}_n)$ and $\mathfrak{R}^+$ =max$(\mathfrak{R}_1,\mathfrak{R}_2,\dots,\mathfrak{R}_n)$, then

$$\mathfrak{R}^-\le IVCSVNAAHA_{\varpi}(\mathfrak{R}_1,\mathfrak{R}_2,\dots,\mathfrak{R}_n)\le\mathfrak{R}^+ \quad (25)$$

Theorem 13: If $\mathfrak{R}_j\le S_j\ \forall\ j$, then

$$IVCSVNAAHA_{\varpi}(\mathfrak{R}_1,\mathfrak{R}_2,\dots,\mathfrak{R}_n)\le IVCSVNAAHA_{\varpi}(S_1,S_2,\dots,S_n) \quad (26)$$

5 | MADM APPLICATION FOR INVENTED THEORY

Then In this section, with the help of derived operators, we focus on evaluating the theory of the MADM problem to enhance the worth of the invented theory.

Assume that the finite family of alternatives $\mathfrak{R}_1,\mathfrak{R}_2,\dots,\mathfrak{R}_n$, furthermore, for each alternative, we were given a collection of attributes $\mathfrak{R}_1^*,\mathfrak{R}_2^*,\dots,\mathfrak{R}_m^*$ with weight vectors $\varpi=(\varpi_1,\varpi_2,\dots,\varpi_n)^T$ with $\varpi_j\in[0,1]$ and $\sum_{j=1}^{n}\varpi_j=1$. Additionally, we arranged the data in the form of the matrix, called the decision matrix whose every item will be computed in the form of IVCSVN information, noticed that $m_{\mathfrak{R}}(\text{ŧ}), a_{\mathfrak{R}}(\text{ŧ}), n_{\mathfrak{R}}(\text{ŧ})$ shown the grade of truth, abstinence, and falsity, where $m_{\mathfrak{R}}(\text{ŧ}) = [x^l_{m_{\mathfrak{R}}}(\text{ŧ}), x^u_{m_{\mathfrak{R}}}(\text{ŧ})].\boldsymbol{exp}^{j[\omega^l_{m_{\mathfrak{R}}}(\text{ŧ}),\omega^u_{m_{\mathfrak{R}}}(\text{ŧ})]}$, $a_{\mathfrak{R}}(\text{ŧ}) = [x^l_{a_{\mathfrak{R}}}(\text{ŧ}),\ x^u_{a_{\mathfrak{R}}}(\text{ŧ})].\ \boldsymbol{exp}^{j[\varphi^l_{a_{\mathfrak{R}}}(\text{ŧ}),\varphi^u_{a_{\mathfrak{R}}}(\text{ŧ})]}$, $n_{\mathfrak{R}}(\text{ŧ}) = [x^l_{n_{\mathfrak{R}}}(\text{ŧ}), x^u_{n_{\mathfrak{R}}}(\text{ŧ})].\boldsymbol{exp}^{j[\psi^l_{n_{\mathfrak{R}}}(\text{ŧ}),\psi^u_{n_{\mathfrak{R}}}(\text{ŧ})]}$. Noticed that $j = 2\pi i$, with $0 \le x^u_{m_{\mathfrak{R}}}(\text{ŧ}) + x^u_{a_{\mathfrak{R}}}(\text{ŧ}) + x^u_{n_{\mathfrak{R}}}(\text{ŧ}) \le 3$ and $0 \le \omega^u_{m_{\mathfrak{R}}}(\text{ŧ}) + \varphi^u_{a_{\mathfrak{R}}}(\text{ŧ}) + \psi^u_{n_{\mathfrak{R}}}(\text{ŧ}) \le 3\pi$. The simple form of IVCSVNV is stated by: $\mathfrak{R}_j = \begin{pmatrix} [x^l_{m_{\mathfrak{R}_j}}, x^u_{m_{\mathfrak{R}_j}}].exp^{j[\omega^l_{m_{\mathfrak{R}_j}},\omega^u_{m_{\mathfrak{R}_j}}]}, \\ [x^l_{a_{\mathfrak{R}_j}}, x^u_{a_{\mathfrak{R}_j}}].exp^{j[\varphi^l_{a_{\mathfrak{R}_j}},\varphi^u_{a_{\mathfrak{R}_j}}]},\ [x^l_{n_{\mathfrak{R}_j}}, x^u_{n_{\mathfrak{R}_j}}]\ .exp^{j[\psi^l_{n_{\mathfrak{R}_j}},\psi^u_{n_{\mathfrak{R}_j}}]} \end{pmatrix}, j = 1,2, \dots n$. To resolve some practical problems, we presented the decision-making procedure, which is discussed below:

Step 1: We compute the matrix of decision-making by including the IVCSVN information, during the construction of the matrix, we have two possibilities, if the data is cost type, then we normalized it with the help of the below theory:

$$Z = \begin{cases} \left(\left[x^l_{m_{\mathfrak{R}_j}}, x^u_{m_{\mathfrak{R}_j}}\right].exp^{j\left[\omega^l_{m_{\mathfrak{R}_j}},\omega^u_{m_{\mathfrak{R}_j}}\right]}, \left[x^l_{a_{\mathfrak{R}_j}}, x^u_{a_{\mathfrak{R}_j}}\right].exp^{j\left[\varphi^l_{a_{\mathfrak{R}_j}},\varphi^u_{a_{\mathfrak{R}_j}}\right]}, \left[x^l_{n_{\mathfrak{R}_j}}, x^u_{n_{\mathfrak{R}_j}}\right].exp^{j\left[\psi^l_{n_{\mathfrak{R}_j}},\psi^u_{n_{\mathfrak{R}_j}}\right]}\right) & benefit \\ \left(\left[x^l_{n_{\mathfrak{R}_j}}, x^u_{n_{\mathfrak{R}_j}}\right].exp^{j\left[\psi^l_{n_{\mathfrak{R}_j}},\psi^u_{n_{\mathfrak{R}_j}}\right]}, \left[x^l_{a_{\mathfrak{R}_j}}, x^u_{a_{\mathfrak{R}_j}}\right].exp^{j\left[\varphi^l_{a_{\mathfrak{R}_j}},\varphi^u_{a_{\mathfrak{R}_j}}\right]}, \left[x^l_{m_{\mathfrak{R}_j}}, x^u_{m_{\mathfrak{R}_j}}\right].exp^{j\left[\omega^l_{m_{\mathfrak{R}_j}},\omega^u_{m_{\mathfrak{R}_j}}\right]}\right) & cost \end{cases}$$

If the data is benefiting type, then we do to next step.

Step 2: Aggregate the data in a normalized matrix with the presence of the IVCSVNAAWA operator.

Step 3: Examine the value of the score function by using the aggregated information.

Step 4: Rank all the preferences based on the score values and evaluate the finest one.

5.1| MADM Application for Invented Theory

To select the director of the campus for any university, we considered five candidates which are applied for the post of director $\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3, \mathfrak{R}_4, \mathfrak{R}_5$. To decide which one is more perfect and more suitable for the director of the campus based on the following four features such as: $\mathfrak{R}^*_1$: Ph.D. degree, $\mathfrak{R}^*_2$: fifteen-year experience, $\mathfrak{R}^*_3$: more than twenty publications, and $\mathfrak{R}^*_4$: personality. For Each feature, we have the following weight vectors such as 0.3,0.3,0.1, and 0.3. To resolve some practical problems, we presented the decision-making procedure, which is discussed below:

Step 1: We compute the matrix (see Table 1) of decision-making by including the IVCSVN information, During the construction of the matrix, we have two possibilities, if the data is cost type, then we normalized it with the help of the below theory:

$$Z = \begin{cases} \left(\left[x^l_{m_{\mathfrak{R}_j}}, x^u_{m_{\mathfrak{R}_j}}\right].exp^{j\left[\omega^l_{m_{\mathfrak{R}_j}},\omega^u_{m_{\mathfrak{R}_j}}\right]}, \left[x^l_{a_{\mathfrak{R}_j}}, x^u_{a_{\mathfrak{R}_j}}\right].exp^{j\left[\varphi^l_{a_{\mathfrak{R}_j}},\varphi^u_{a_{\mathfrak{R}_j}}\right]}, \left[x^l_{n_{\mathfrak{R}_j}}, x^u_{n_{\mathfrak{R}_j}}\right].exp^{j\left[\psi^l_{n_{\mathfrak{R}_j}},\psi^u_{n_{\mathfrak{R}_j}}\right]}\right) & benefit \\ \left(\left[x^l_{n_{\mathfrak{R}_j}}, x^u_{n_{\mathfrak{R}_j}}\right].exp^{j\left[\psi^l_{n_{\mathfrak{R}_j}},\psi^u_{n_{\mathfrak{R}_j}}\right]}, \left[x^l_{a_{\mathfrak{R}_j}}, x^u_{a_{\mathfrak{R}_j}}\right].exp^{j\left[\varphi^l_{a_{\mathfrak{R}_j}},\varphi^u_{a_{\mathfrak{R}_j}}\right]}, \left[x^l_{m_{\mathfrak{R}_j}}, x^u_{m_{\mathfrak{R}_j}}\right].exp^{j\left[\omega^l_{m_{\mathfrak{R}_j}},\omega^u_{m_{\mathfrak{R}_j}}\right]}\right) & cost \end{cases}$$

If the data is benefiting type, then we do to next step, but the data in Table 1 does not need to be normalized.

Table 1: IVCSVN decision matrix.

	$\mathfrak{N}_1^*$	$\mathfrak{N}_2^*$	$\mathfrak{N}_3^*$	$\mathfrak{N}_4^*$
$\mathfrak{N}_1$	$\begin{pmatrix}[0.6, 0.7].[0.3,0.6], \\ [0.4, 0.6].[0.5, 0.6], \\ [0.7, 0.8].[0.1, 0.3]\end{pmatrix}$	$\begin{pmatrix}[0.61, 0.71].[0.31, 0.61], \\ [0.41, 0.61].[0.51, 0.61], \\ [0.71, 0.81].[0.11, 0.31]\end{pmatrix}$	$\begin{pmatrix}[0.62,0.72].[0.32, 0.62], \\ [0.42,0.62].[0.52, 0.62], \\ [0.72,0.82].[0.12, 0.32]\end{pmatrix}$	$\begin{pmatrix}[0.63,0.73].[0.33, 0.63], \\ [0.43,0.63].[0.53, 0.63], \\ [0.73,0.83].[0.13, 0.63]\end{pmatrix}$
$\mathfrak{N}_2$	$\begin{pmatrix}[0.1,0.3].[0.2, 0.3], \\ [0.2,0.5].[0.2, 0.3], \\ [0.1,0.7].[0.1, 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.11,0.31].[0.21, 0.31], \\ [0.21,0.51].[0.21, 0.31], \\ [0.11,0.71].[0.11, 0.41]\end{pmatrix}$	$\begin{pmatrix}[0.12,0.32].[0.22, 0.32], \\ [0.22,0.52].[0.22, 0.32], \\ [0.12,0.72].[0.12, 0.42]\end{pmatrix}$	$\begin{pmatrix}[0.13,0.33].[0.23, 0.33], \\ [0.23,0.53].[0.23, 0.33], \\ [0.13,0.73].[0.13, 0.43]\end{pmatrix}$
$\mathfrak{N}_3$	$\begin{pmatrix}[0.4,0.5].[0.3, 0.4], \\ [0.1,0.2].[0.2, 0.3], \\ [0.2,0.3].[0.2, 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.41,0.51].[0.31, 0.41], \\ [0.11,0.21].[0.21, 0.31] \\ [0.21,0.31].[0.21, 0.41]\end{pmatrix}$	$\begin{pmatrix}[0.42,0.52].[0.22, 0.42], \\ [0.12,0.22].[0.22, 0.32] \\ [0.22,0.32].[0.22, 0.42]\end{pmatrix}$	$\begin{pmatrix}[0.43,0.53].[0.33, 0.43], \\ [0.13,0.23].[0.23, 0.33], \\ [0.23,0.33].[0.23, 0.43]\end{pmatrix}$
$\mathfrak{N}_4$	$\begin{pmatrix}[0.2,0.3].[0.1,0.2], \\ [0.3,0.4].[0.1,0.2], \\ [0.4,0.5].[0.2,0.3]\end{pmatrix}$	$\begin{pmatrix}[0.21,0.31].[0.11, 0.21] \\ [0.31,0.41].[0.11, 0.21], \\ [0.41,0.51].[0.21, 0.31]\end{pmatrix}$	$\begin{pmatrix}[0.22,0.32].[0.12, 0.22] \\ [0.32,0.42].[0.12, 0.22] \\ [0.42,0.52].[0.22, 0.32]\end{pmatrix}$	$\begin{pmatrix}[0.23,0.33].[0.13, 0.23], \\ [0.33,0.43].[0.13, 0.23], \\ [0.43,0.53].[0.23, 0.33]\end{pmatrix}$
$\mathfrak{N}_5$	$\begin{pmatrix}[0.1,0.2].[0.4, 0.5], \\ [0.2,0.3].[0.5, 0.6], \\ [0.3,0.4].[0.6, 0.7]\end{pmatrix}$	$\begin{pmatrix}[0.11,0.21].[0.41, 0.51], \\ [0.21,0.31].[0.51, 0.61], \\ [0.31,0.41].[0.61, 0.71]\end{pmatrix}$	$\begin{pmatrix}[0.12,0.22].[0.42, 0.52], \\ [0.22,0.32].[0.52, 0.62] \\ [0.32,0.42].[0.62, 0.72]\end{pmatrix}$	$\begin{pmatrix}[0.13,0.23].[0.43, 0.53], \\ [0.23,0.33].[0.53, 0.63], \\ [0.33,0.43].[0.63, 0.73]\end{pmatrix}$

Step 2: Aggregate the data in a normalized matrix with the presence of the IVCSVNAAWA operator, see Table 2.

Table 2: IVCSVN aggregated matrix

	$IVCSVNAAWA\ Operator$
$\mathfrak{N}_1$	$\begin{pmatrix}[0.6143,0.7144].[0.3143,0.6143], \\ [0.4136,0.6136].[0.5136,0.6136], \\ [0.7136,0.8134].[0.1130,0.3135]\end{pmatrix}$
$\mathfrak{N}_2$	$\begin{pmatrix}[0.1147,0.3143].[0.2144,0.3143], \\ [0.2143,0.2134].[0.2134,0.3135], \\ [0.1130,0.7136].[0.1130,0.4136]\end{pmatrix}$
$\mathfrak{N}_3$	$\begin{pmatrix}[0.4143,0.5143].[0.3143,0.4143], \\ [0.1130,0.2134].[0.2134,0.3135], \\ [0.2134,0.3135].[0.2134,0.4136]\end{pmatrix}$
$\mathfrak{N}_4$	$\begin{pmatrix}[0.2144,0.3143].[0.1147,0.2144], \\ [0.3135,0.4136].[0.1130,0.2134], \\ [0.4136,0.5136].[0.2134,0.3135]\end{pmatrix}$
$\mathfrak{N}_5$	$\begin{pmatrix}[0.1147,0.2144].[0.4143,0.5143], \\ [0.2134,0.3135].[0.5136,0.6136], \\ [0.3135,0.4136].[0.6136,0.7136]\end{pmatrix}$

Step 3: Examine the value of the score function by using the aggregated information, see Table 3.

Table 3: IVCSVN score values

	$IVCSVNAAWA\ Operator$
$\mathfrak{N}_1$	0.53049
$\mathfrak{N}_2$	0.29713
$\mathfrak{N}_3$	0.30543
$\mathfrak{N}_4$	0.28051
$\mathfrak{N}_5$	0.41389

Step 4: Rank all the preferences based on the score values and evaluate the finest one, such as

$$\mathfrak{N}_1 > \mathfrak{N}_5 > \mathfrak{N}_3 > \mathfrak{N}_2 > \mathfrak{N}_4$$

The finest optimal is $\mathfrak{N}_1$. Furthermore, we compare the proposed work with some existing operators to enhance the worth of the invented theory.

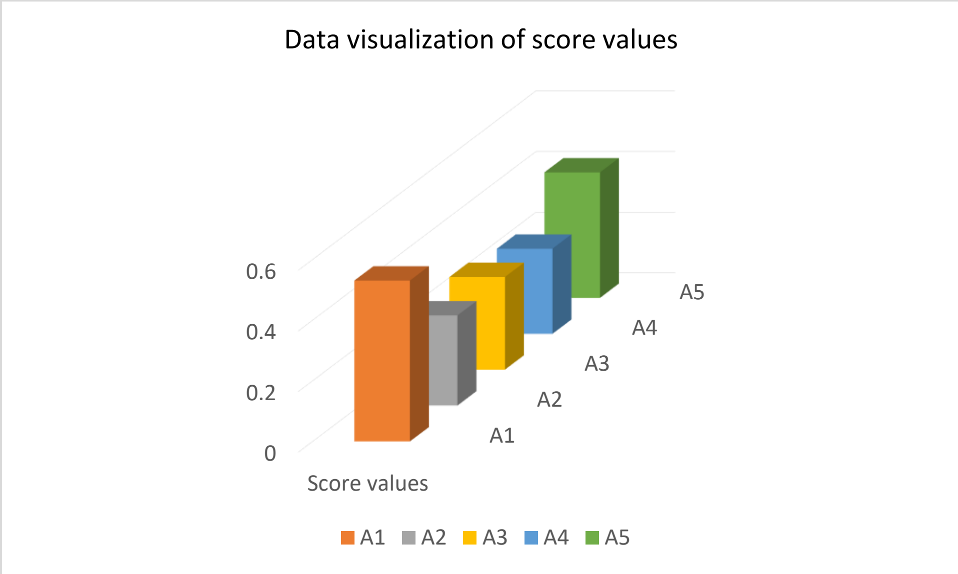

Figure 1: Displaying Data using Table 3

6 | COMPARATIVE ANALYSIS

In this section, we compare the invented theory with some existing operators by using the data in Table 1 under the consideration of various valuable prevailing operators. For this, we considered some prevailing operators which are described as follows: Aczel-Alsina aggregation operators for IFSs [29], Aczel-Alsina prioritized aggregation operators for IFSs [30], Aczel-Alsina geometric aggregation operators for IFSs [31], Aczel-Alsina aggregation operators for z-neutrosophic sets [32], Aczel-Alsina aggregation operators for CIFSs [33], Aczel-Alsina aggregation operators for SVNSs [34], and power Aczel-Alsina aggregation operators for IFSs [35], but the above all theories are failed because they are not able to evaluate the theory in Table 1 because these all information are the special cases of the invented theory.

7 | CONCLUSION

Using the theory of IVCSVN information and Aczel-Alsina aggregation operators, we derived the following information, such as:

1. We evaluated the novel theory of Aczel-Alsina operational laws under the consideration of the IVCSVN values by using the idea of Aczel-Alsina t-norm and t-conorm.
2. We derived the concept of the IVCSVNAAWA operator, IVCSVNAAOWA operator, and IVCSVNAAHA operator, and examined or presented their major properties, such as idempotency, boundedness, and monotonicity.
3. We demonstrated a MADM scenario in the availability of the derived operators for IVCSVN information and illustrated various numerical examples.

4. We show the practicality and supremacy of the presented theory by comparing the derived work with some prevailing techniques.

In the future, we will derive some new ideas such as interval-valued complex single-valued neutrosophic hesitant fuzzy sets and their extensions, and then we try to employ them in the areas of decision-making, pattern recognition, and clustering analysis.

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